Online Ramsey numbers

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Consider the following graph-building game played between two players, Builder and Painter. Starting from an infinite empty graph, Builder places a new edge on each turn and Painter colors it red or blue. The online Ramsey number $\tilde{r}(n)$ is the least number of turns Builder needs (under optimal play) to guarantee the existence of a monochromatic n-clique. Online Ramsey numbers were originally introduced as a possible method for improving bounds on the classical Ramsey number $r(n)$. We prove using a random Painter strategy that $\tilde{r}(n) > c \cdot 2^{(2-\sqrt{2})n}$, where c is a positive constant. This is the first exponential improvement on the trivial bound $\tilde{r}(n) > 2^{n/2}$ given by directly comparing it with $r(n)$. Our method also gives lower bounds on the algorithmic problem of quickly finding copy of a fixed graph H in a hidden random graph by revealing one edge at a time.

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