Non-concentration of the chromatic number of a random graph

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A classic result of Shamir and Spencer states that the chromatic number of $G_{n,p}$ is whp concentrated on about $\sqrt{n}$ consecutive values for any function $p = p(n)$. For $p = \frac{1}{2}$, this can be improved slightly to about $\sqrt{n}/\log n$ values . For sparse random graphs, much sharper concentration results are known. In particular, Alon and Krivelevich showed in 1997 that for $p < n^{\frac{1}{2}-\varepsilon}$, $\chi(G_{n,p})$ is whp concentrated on only *two* consecutive values.

However, until now there have been no results showing that we do *not* always have extremely narrow concentration, other than in the trivial case where $p \to 1$ quickly enough. In 2004, Bollobás asked for any non-trivial results asserting a lack of concentration, suggesting the dense random graph $G_{n,m}$ with $m = \lfloor n^2/4 \rfloor$ as a candidate.

We sketch an argument which shows that the chromatic number of $G_{n,\frac{1}{2}}$ is *not* whp concentrated on fewer than $n^{\frac{1}{4}-\varepsilon}$ consecutive values. As a corollary, this also holds for the chromatic number of $G_{n,m}$ with $m = \lfloor n^2/4 \rfloor$.