Algorithms for graphs without induced linear forests

Many graph problems that are NP-hard in general are solvable on polynomial time when we restrict ourselves to a class of H-free graphs, that is, graphs without an induced copy of a fixed graph H. In this talk I will focus on two such problems - Maximum Weight Independent Set and k-Colouring. Both problems are known to be NP-hard even for graphs of large girth, thus, one can only hope to find a polynomial time algorithm for H-free graphs when H is a linear forest.

In the classic Maximum Weight Independent Set problem we are given a graph G with a nonnegative weight function on vertices, and the goal is to find an independent set in G of maximum possible weight. We give a polynomial-time algorithm working on any P_6-free graph, that is, a graph that has no path on 6 vertices as an induced subgraph. This improves the polynomial-time algorithm on P_5-free graphs of Lokshtanov et al. (SODA 2014).

The k-Colouring problem is to decide if the vertices of a graph can be coloured with at most k colours for a fixed integer k such that no two adjacent vertices are coloured alike. We prove that 3-Colouring is polynomial-time solvable for $(P_2 + P_5)$-free graphs and for $(P_3 + P_4)$-free graphs, where $(P_r + P_s)$ is the disjoint union of P_r and P_s. Combining our results with previously known results yields complete complexity classification of 3-Colouring on H-free graphs for all graphs H on up to seven vertices.

The talk includes joint results with Andrzej Grzesik, Marcin Pilipczuk and Michał Pilipczuk and with Josef Malík, Tomáš Masařík, Jana Novotná, Daniël Paulusma and Veronika Slívová.