BIVARIATE FLUCTUATIONS FOR THE NUMBER OF ARITHMETIC PROGRESSIONS IN RANDOM SETS

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We study arithmetic progressions $\{a, a+b, a+2b, \ldots, a+(\ell-1)b\}$, with $\ell \geq 3$, in random subsets of the initial segment of natural numbers $[n] := \{1, 2, \ldots, n\}$. Given $p \in [0,1]$ we denote by $[n]_p$ the random subset of $[n]$ which includes every number with probability p, independently of one another. The focus lies on sparse random subsets, i.e. when $p = p(n) = o(1)$ with respect to $n \to \infty$.

Let X_ℓ denote the number of distinct arithmetic progressions of length ℓ which are contained in $[n]_p$. We determine the limiting distribution for X_ℓ not only for fixed $\ell \geq 3$ but also when $\ell = \ell(n) \to \infty$ sufficiently slowly. Moreover, we prove a central limit theorem for the joint distribution of the pair $(X_\ell, X_{\ell'})$ for a wide range of p. Our proofs are based on the method of moments and combinatorial arguments, such as an algorithmic enumeration of collections of arithmetic progressions.

This is joint work with Yacine Barhoumi-Andréani and Hong Liu.