Large homogeneous submatrices

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Abstract

A matrix is homogeneous if all of its entries are equal. Let P be a 2×2 zero-one matrix that is not homogeneous. We prove that if an $n\times n$ zero-one matrix A does not contain P as a submatrix, then A has an $cn\times cn$ homogeneous submatrix for a suitable constant $c>0$. We further provide an almost complete characterization of the matrices P (missing only finitely many cases) such that forbidding P in A guarantees an $n^{1-o(1)}\times n^{1-o(1)}$ homogeneous submatrix. We apply our results to chordal bipartite graphs, totally balanced matrices, halfplane-arrangements and string graphs.

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