Turán numbers of long cycles in random graphs

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For a graph G on n vertices and a graph H, denote by $ex(G, H)$ the maximal number of edges in an H-free subgraph of G. We consider a random graph $G \sim G(n, p)$ where $p = \Theta\left(\frac{1}{n}\right)$, and study the typical value of $ex(G, H)$, where H is a long cycle.

We determine the asymptotic value of $ex(G, C_t)$, where $G \sim G(n, p)$, $p \geq \frac{C}{n}$ and $A \log n \leq t \leq (1 - \varepsilon)n$. The behavior of $ex(G, C_t)$ can depend substantially on the parity of t. In particular, our results match the classical result of Woodall on the Turán number of long cycles, and can be seen as its random version. In fact, our techniques apply in a more general sparse pseudo-random setting.

We also prove a robustness-type result, showing the likely existence of cycles of prescribed lengths in a random subgraph of a graph with a nearly optimal density.

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