Odd cycles in subgraphs of sparse pseudorandom graphs

Joonkyung Lee

Joonkyung.Lee@uni-hamburg.de

Universität Hamburg

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We answer two extremal questions about odd cycles that naturally arise in the study of sparse pseudorandom graphs. Let Γ be an (n, d, λ)-graph, i.e., n-vertex, d-regular graphs with all nontrivial eigenvalues in the interval $[-\lambda, \lambda]$. Krivelevich, Lee, and Sudakov conjectured that, whenever $\lambda^{2k-1} \ll d^{2k}/n$, every subgraph G of Γ with $(1/2 + o(1))e(\Gamma)$ edges contains an odd cycle C_{2k+1}. Aigner-Horev, Hàn, and Schacht proved a weaker statment by allowing an extra polylogarithmic factor in the assumption $\lambda^{2k-1} \ll d^{2k}/n$, but we completely remove it and hence settle the conjecture. This also generalises Sudakov, Szabo, and Vu's Turán-type theorem for triangles.

Secondly, we obtain a Ramsey multiplicity result for odd cycles. Namely, in the same range of parameters, we prove that every 2-edge-colouring of Γ contains at least $(1 - o(1))2^{-2k}d^{2k+1}$ monochromatic copies of C_{2k+1}. Both results are asymptotically best possible by Alon and Kahale's construction of C_{2k+1}-free pseudorandom graphs.