Edge-ordered Ramsey numbers

Ray Li

Stanford University

An edge-ordered graph is a graph with a linear ordering of its edges. Two edge-ordered graphs are *equivalent* if their is an isomorphism between them preserving the ordering of the edges. The *edge-ordered Ramsey number* $r_{edge}(H)$ of an edge-ordered graph H is the smallest N such that there exists an edge-ordered graph G on N vertices such that, for every two-coloring of the edges of G, there is a monochromatic subgraph of G equivalent to H. Recently, Balko and Vizer announced that $r_{edge}(H)$ exists. However, their proof uses the Graham-Rothschild theorem and consequently gives an enormous upper bound on these numbers. We discuss a new proof giving a much better bound of exponential type. We also discuss how the proof can be adapted to give a polynomial bound for edge-ordered Ramsey number of edge-ordered graphs of bounded degeneracy.

Joint work with Jacob Fox.