New bounds for the n-queens problem

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Abstract

The famous n-queens problem asks: In how many ways can n nonattacking queens be placed on an $n \times n$ chessboard? This question also makes sense on the toroidal chessboard, in which opposite sides of the board are identified. In this setting, the n-queens problem counts the number of perfect matchings in a certain regular hypergraph. We give an extremely general upper bound for such counting problems, which include Sudoku squares and designs.

Our lower bound, which confirms a conjecture of Vardi and Rivin, is based on an algebraic construction which is similar to the algebraic absorbers used by Peter Keevash in his construction of designs.

We will describe a general algebraic framework that includes the toroidal n-queens problem, as well as the enumeration problem for partite designs, and discuss which of the bounds can be generalized for the general case.