On weakly distinguishing graph polynomials: Adddable graph classes and graphs of fixed genus

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B. Bollobás, L.Pebody and O. Riordan (2000) conjecture that almost all graphs are unique for the Tutte polynomial, and even the chromatic polynomial. E. van Dam and W. Haemers (2009) ask whether this is also true for the characteristic and the Laplacian polynomial.

We say that a graph invariant $\mathcal{F}$ is *weakly distinguishing on a class of graphs* $\mathcal{P}$ if for almost all graphs $G \in \mathcal{P}$ there is a graph $H \in \mathcal{P}$ with $\mathcal{F}(G) = \mathcal{F}(H)$. The degree sequence is clearly weakly distinguishing on all finite graphs. We have studied graph polynomials weakly distinguishing on all finite graphs previously in (*On Weakly Distinguishing Graph Polynomials*, Discrete Mathematics & Theoretical Computer Science, 2019).

In this paper we show, using results by C. McDiarmid, A. Steger and D. Welsh (2005), that the Tutte polynomial (and hence the chromatic polynomial), the dominating polynomial, and the characteristic polynomial are weakly distinguishing on planar graphs. Using results by McDiarmid (2008, 2009) we extend this to graphs of fixed genus k and to addable graph properties. A graph property $\mathcal{P}$ is addable if for any two disjoint graphs G and H the graph $G \sqcup H \in \mathcal{P}$ iff both $G, H \in \mathcal{P}$ and if both $G, H \in \mathcal{P}$ and $e = (u, v)$ with $u \in V(G)$ and $v \in V(H)$ then $(V(G \sqcup H), E(G) \cup E(H) \cup \{e\}) \in \mathcal{P}$.