Sharp inequalities for moments with respect to the Ewens Sampling Formula

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We deal with the ubiquitous Ewens distribution defined on the set of vectors $\omega = (\omega_1, \ldots, \omega_n) \in \mathbf{N}_0^n$ by

$$P_\theta(\{\omega\}) = \frac{\mathbf{1}\{\ell(\omega) = n\}}{\Theta(n)} \prod_{j=1}^{n} \Big(\frac{\theta}{j}\Big)^{\omega_j} \frac{1}{\omega_j!}.$$

Here $\theta > 0$ is a parameter, $\Theta(k) = \theta(\theta+1)\cdots(\theta+k-1)/k!$, $\mathbf{1}\{\cdot\}$ stands for an indicator, and $\ell(\omega) := 1\omega_1 + \cdots + n\omega_n$. The main target is the variance Var_θ with respect to P_θ of the linear statistics $h(\omega) = a_1\omega_1 + \cdots + a_n\omega_n$. The following sharp inequality

$$\mathrm{Var}_\theta h \le \frac{\theta(\theta+2)}{\theta+1} \sum_{j=1}^{n} \frac{a_j^2}{j} \frac{\Theta(n-j)}{\Theta(n)}$$

holds for all $n \ge 2$. The proof is built upon complete spectral analysis of the involved matrices. The discrete orthogonal Hahn's polynomials naturally appear in the definition of canonical basis. Applications to random permutations will be also discussed.