Invertibility of Random Symmetric Matrices

A well-known conjecture states that a random symmetric $n \times n$ matrix with entries in $\{-1, 1\}$ is singular with probability $\Theta(n^2 2^{-n})$. In this talk I will describe some recent work where we prove that the probability of this event is at most $\exp(-\Omega(\sqrt{n}))$. This improves the previous best known bound of $\exp(-\Omega(n^{1/4}\sqrt{\log n}))$, which was obtained by Ferber and Jain. Our main theorem is an inverse Littlewood-Offord theorem in \mathbb{Z}_p^n , which is inspired by the method of hypergraph containers.

This is joint work with Marcelo Campos, Letícia Mattos and Rob Morris.