Complete minors in graphs without sparse cuts

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Abstract

We show that if G is a graph on n vertices, with all degrees comparable to some $d = d(n)$, and without a sparse cut, for a suitably chosen notion of sparseness, then it contains a complete minor of order

$$\Omega\left(\sqrt{\frac{nd}{\log d}}\right).$$

As a corollary we determine the order of a largest complete minor one can guarantee in d-regular graphs for which the second largest eigenvalue is bounded away from $d/2$, in $(d/n, o(d))$-jumbled graphs, and in random d-regular graphs, for almost all $d = d(n)$. The proof is based on a 'garbage' argument of Plotkin, Rao, and Smith and utilises random walks on expanders.

Joint work with Michael Krivelevich.