UNIVERSALITY IN RANDOMLY PERTURBED GRAPHS

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Abstract. We denote by $\mathcal{F}(n,\Delta)$ the family of all graphs on n vertices with maximum degree Δ and call a graph $\mathcal{F}(n,\Delta)$-universal if it contains all graphs from the family simultaneously. It is belived, that the threshold for $\mathcal{F}(n,\Delta)$-universality in the binomial random graph $G(n,p)$ is determined by the $K_{\Delta+1}$-factor, which gives $(n^{-1}\log^{1/\Delta} n)^{2/(\Delta+1)}$. This was confirmed by Ferber, Kronenberg, and Luh for $\Delta = 2$, while for larger Δ already the single containment problem is open.

We study the model of randomly perturbed dense graphs, that is, for any constant $\alpha > 0$, the union of some n-vertex graph G_α with minimum degree at least αn and $G(n,p)$. Together with Böttcher, Montgomery, and Person we resolved the single containment problem for $\mathcal{F}(n,\Delta)$ in $G_\alpha \cup G(n,p)$, showing that $n^{-2/(\Delta+1)}$ is sufficient. This log-term difference in comparison to the threshold in $G(n,p)$ is optimal and typical for this model. We believe that $n^{-2/(\Delta+1)}$ also gives the threshold for $\mathcal{F}(n,\Delta)$-universality in $G_\alpha \cup G(n,p)$. We can prove it for $\Delta = 2$ and for larger Δ we get $n^{-1/(\Delta-1)}$, which is optimal for $\Delta = 3$.