

ZERO-FREE REGIONS OF GRAPH POLYNOMIALS AND COMPUTATIONAL COUNTING

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In the area of computational counting one seeks to find polynomial-time algorithms to (approximately) count certain combinatorial objects such as independent sets, proper colourings, matchings, etc in a graph. More generally, each combinatorial counting problem has an associated generating function, namely the independence polynomial for independent sets, the chromatic and more generally Tutte polynomial for proper graph colourings, and the matching polynomial for matchings. A fundamental question asks for which graphs and at which numerical values can one approximately evaluate such polynomials efficiently?

Extending work of Barvinok, we establish an intimate connection between the locations of the zeros of the graph polynomials and the locations at which these graph polynomials can be approximately evaluated by a polynomial-time algorithm. The result applies to a large class of functions that includes the independence, matching and Tutte polynomials. It also allows complex evaluations, whereas previous methods have often been restricted to real or even integer evaluations. We discuss new and existing results about the locations of zeros of various graph polynomials and how these translate into algorithms. This is based on joint works with authors including Matthew Coulson, Ewan Davies, Ferenc Bencs, and Guus Regts