Glauber dynamics for edges colourings of trees

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Abstract

Let T be a tree on n vertices and with maximum degree Δ . We show that for $k \geq \Delta + 1$ the Glauber dynamics for k-edge-colourings of T mixes in polynomial time in n. The bound on the number of colours is best possible as the chain is not even ergodic for $k \leq \Delta$. Our proof uses a recursive decomposition of the tree into subtrees; we bound the relaxation time of the original tree in terms of the relaxation time of its subtrees using block dynamics and chain comparison techniques. Of independent interest, we also introduce a monotonicity result for Glauber dynamics that simplifies our proof.