Partitioning 2-coloured complete 3-graphs into two monochromatic tight cycles

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(This talk is based on joint work with Frederik Garbe, Richard Lang, Allan Lo, Richard Mycroft.)

In 1979, Lehel conjectured that the vertex set of every 2-edge-coloured complete graph can be partitioned into two monochromatic cycles of distinct colours. This was answered in the affirmative by Bessy and Thomassé in 2010.

We generalise the question of Lehel to the setting of 3-uniform hypergraphs (3-graphs). More precisely, we show that every sufficiently large 2-edge-coloured complete 3-graph admits a vertex partition into two monochromatic tight cycles, possibly of the same colour. We also present examples showing that (in contrast to the graph case) it is not always possible to ask for partitions into two monochromatic tight cycles of different colours.

From our proof, we also show that in the same setting (large 2-edge-coloured complete 3-graphs) we can always find a vertex partition into a tight cycle and a tight path, both of which are monochromatic and have different colours; or cover all but at most 2 vertices with two vertex-disjoint monochromatic tight cycles of different colours. This answers questions of Gyárfás and of Bustamante, Hàn and Stein.