Successive minimum spanning trees

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In a complete graph K_n with edge weights drawn independently from a uniform distribution $U(0,1)$ (or alternatively an exponential distribution $\mathrm{Exp}(1)$), let T_1 be the MST (the spanning tree of minimum weight) and let T_k be the MST after deletion of the edges of all previous trees T_i, $i < k$. We show that each tree's weight $w(T_k)$ converges in probability to a constant γ_k with $2k - 2\sqrt{k} < \gamma_k < 2k + 2\sqrt{k}$, and we conjecture that $\gamma_k = 2k - 1 + o(1)$. The problem is distinct from one of Frieze and Johansson (2018), finding k MSTs of combined minimum weight, and the combined cost for two trees in their problem is strictly smaller than our $\gamma_1 + \gamma_2$.

Our results also hold (and mostly are derived) in a multigraph model where edge weights for each vertex pair follow a Poisson process; here we additionally have $\mathbb{E}(w(T_k)) \to \gamma_k$. Thinking of an edge of weight w as arriving at time $t = nw$, Kruskal's algorithm defines forests $F_k(t)$, each initially empty and eventually equal to T_k, with each arriving edge added to the first $F_k(t)$ where it does not create a cycle. Using tools of inhomogeneous random graphs we obtain structural results including that $C_1(F_k(t))/n$, the fraction of vertices in the largest component of $F_k(t)$, converges in probability to a function $\rho_k(t)$, uniformly for all t, and that a giant component appears in $F_k(t)$ at a time $t = \sigma_k$. We conjecture that the functions ρ_k tend to time translations of a single function, $\rho_k(2k + x) \to \rho_\infty(x)$ as $k \to \infty$, uniformly in $x \in \mathbb{R}$.

Simulations and numerical computations give estimated values of γ_k for small k, and support the conjectures stated above.

This is joint work with Svante Janson.