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Four Descrepancies

Abstract: Paul selects $n$ vectors in $n$-space, all coordinates one or minus one. Carole is a balancer, assigning signs to each vector yielding a signed vector sum $\vec{P}$. The value $V$, which Carole attemps to minimize, is the maximal absolute value of the coordinates of $\vec{P}$.

We consider four variants of this problem. Paul may play randomly (then Carole minimizes the expectation of $V$) or adversarially. Carole may play On-Line, selecting each sign immediately upon seeing its vector; or Off-Line, waiting until Paul has given all the vectors before deciding on the signs.

All four variants are interesting and will be discussed. The order of $V$ is known in all variants, though the constants remain elusive. We emphasize new results, with Nikhil Bansal, for the random on-line variant.