K_r factors in graphs with low independence number

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Abstract

A classical result by Hajnal and Szemeredi states the minimal degree conditions necessary to guarantee for a graph to contain a K_r-factor. Namely any graph on n vertices, minimum degree $\delta(G) \geq (r-1)n/r$ and r dividing n has a K_r-factor. This result is tight but the extremal examples are unique in that they have large independent sets and very dense bipartite graphs between these sets. We show that as soon as we move away and forbid these extremal examples, the necessary minimum degree changes. If we require the independence number of G, $\alpha(G)$, to be at most a small linear fraction, then the K_r-factor we find has twice the size. More formally we show for every r and constant $\mu > 0$ there is a constant $\gamma(\mu)$ such that every graph G on n vertices with $\delta(G) \geq \left(1 - \frac{2}{r} + \mu\right) n$ and $\alpha(G) < \gamma n$ has a K_r factor.

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