

Subgraph statistics

Tuan Tran
ETH Zurich

Abstract

Consider integers k and ℓ such that $0 \leq \ell \leq \binom{k}{2}$. Given a large graph G , what is the fraction of k -vertex subsets of G which span exactly ℓ edges? When G is empty or complete, and ℓ is 0 or $\binom{k}{2}$, this fraction can be exactly 1. On the other hand if ℓ is not one of these extreme values, then by Ramsey's theorem, this fraction is strictly smaller than 1. The systematic study of the above question was recently initiated by Alon, Hefetz, Krivelevich and Tyomkyn who proposed several natural conjectures. In this talk we present solutions of two of them (one asymptotically) and make first steps towards analogous questions for hypergraphs. Our proofs involve some Ramsey-type arguments, and a number of different probabilistic tools, such as polynomial anti-concentration inequalities, hypercontractivity, and a coupling argument. Joint work with M. Kwan and B. Sudakov.