Rödl-Ruciński theorem for large graphs

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Abstract

A seminal result of Rödl and Ruciński from 1995 states that for every fixed graph H a random graph $G_{n,p}$ is w.h.p. such that every 2-colouring of its edges contains a monochromatic copy of H in one of the two colours, provided that $p \gg n^{-1/m_2(H)}$, where the value $m_2(H)$ is a certain (natural) density parameter of the graph H. We extend this theorem to all 'large' graphs H with maximum degree three. Namely, there is a $\beta > 0$ and a $C > 0$ for which w.h.p. a random graph $G_{n,p}$ is such that every 2-colouring of its edges contains a monochromatic copy of every graph H with βn vertices and maximum degree at most three, provided that $p \geq Cn^{-2/5}$. The value $2/5$ in the density p is optimal, which stems from the Rödl-Ruciński theorem by looking at $H = K_4$. The statement has immediate consequences for the size-Ramsey numbers of large graphs, improving a result of Kohayakawa, Rödl, Schacht, and Szemerédi. This is a joint work with David Conlon and Rajko Nenadov.