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The Brown-Erdős-Sós conjecture in groups

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(This talk is based on joint work with Rajko Nenadov and Benny Sudakov.)

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The conjecture of Brown, Erdős and Sós from 1973 states that, for any $k \geq 3$, if a 3-uniform hypergraph H with n vertices does not contain a set of $k+3$ vertices spanning at least k edges then it has $o(n^2)$ edges. The case $k = 3$ of this conjecture is the celebrated $(6,3)$-theorem of Ruzsa and Szemerédi which implies Roth's theorem on 3-term arithmetic progressions in dense sets of integers.

Solymosi observed that, in order to prove the conjecture, one can assume that H consists of triples (a, b, ab) of some finite quasigroup Γ. Since this problem remains open for all $k \geq 4$, he further proposed to study triple systems coming from finite groups. In this case he proved that the conjecture holds also for $k = 4$.

We completely resolve the Brown-Erdős-Sós conjecture in groups, for all values of k. Moreover, we prove that the hypergraphs coming from groups contain sets of size $\Theta(\sqrt{k})$ which span k edges, which is best possible.