An extremal problem concerning projective cubes

In the Boolean lattice, Sperner's, Erdos's, Kleitman's and Samotij's theorems state that families that do not contain many chains must have a very specific layered structure. We show that if instead of \mathbb{Z}_2^n we work in \mathbb{Z}_{2^n} , some analogous statements hold if one replaces the word k-chain by projective cube of dimension 2^{k-1} .

This is joint work with Jason Long.