Ramsey numbers of books and quasirandomness

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The *book graph* $B_n^{(k)}$ consists of n copies of K_{k+1} joined along a common K_k. The Ramsey numbers of $B_n^{(k)}$ are known to have strong connections to the classical Ramsey numbers of cliques. Recently, Conlon determined the asymptotic order of these Ramsey numbers for fixed k, showing that the lower bound coming from a random construction is asymptotically tight. This answered an old question of Erdős, Faudree, Rousseau, and Schelp.

In this work, we extend Conlon's result in several ways. First, we provide a new, simplified proof of Conlon's theorem. Next, answering a question of Conlon, we present a different proof that avoids the use of Szemerédi's regularity lemma and obtains a much better bound. Finally, we prove a conjecture of Nikiforov, Rousseau, and Schelp, showing that all extremal colorings for this Ramsey problem are quasirandom.