On the size Ramsey number of graphs with bounded degree and bounded treewidth

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A graph G is *Ramsey* for a graph H if every red/blue edge-colouring of the edges of G contains a monochromatic copy of H. We consider the following question: if H has bounded treewidth, is there a 'sparse' graph G that is Ramsey for H? We show that if the maximum degree and treewidth of H are bounded, then there is a graph G with $O(|V(H)|)$ edges that is Ramsey for H. This was previously known for the smaller class of graphs H with bounded bandwidth by the work of by Clemens, Jenssen, Kohayakawa, Morrison, Mota, Reding and Roberts. We actually prove a more general off-diagonal version of the above result: For graphs H_1 and H_2, the *size Ramsey number* $\hat{r}(H_1, H_2)$ is the minimum number of edges in a graph G such that every red/blue-colouring of the edges of G contains a red copy of H_1 or a blue copy of H_2. We prove that if H_1 and H_2 both have n vertices, bounded degree and bounded treewidth, then $\hat{r}(H_1, H_2) = O(n)$.

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