Yufei Zhao: A Reverse Sidorenko Inequality

We prove a number of tight graph homomorphism inequalities, where, for a fixed H, we wish to maximize the number of homomorphism from G to H (after exponentially normalizing by the size of G) under certain degree constraints on G (e.g., d-regular). Equivalently, we wish to maximize the partition function of a certain fixed spin model (e.g., Ising, Potts) among, say, d-regular graphs. A highlight of our results is that, among d-regular graphs of the same size, a disjoint complete bipartite graphs has the most number of proper q-colorings. Our results also extend to irregular graphs and list colorings. These results can be interpreted as a graphical analog of Brascamp–Lieb inequalities. These results settle a number of conjectures by Kahn, Galvin-Tetali, Galvin, and Cohen–Csikvri–Perkins–Tetali.

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