FIM Minicourse

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Universality of Random Matrices

June 2, 14.15 - 16.00, HG G 19.2
June 3, 14.15 - 16.00, HG F 26.1
June 8, 14.15 - 16.00, HG G 19.2
June 9, 14.15 - 16.00, HG G 19.2

ETH Zürich, Rämistrasse 101

Abstract

We study the universality of spectral statistics of large random matrices in the bulk. We consider N x N symmetric, hermitian or quaternion self-dual random matrices with independent, identically distributed entries (Wigner matrices), where the probability distribution for each matrix element is given by a measure with a subexponential decay tail as the only assumption. Our main result is that the correlation functions of the local eigenvalue statistics in the bulk of the spectrum coincide with those of the Gaussian Orthogonal Ensemble (GOE), the Gaussian Unitary Ensemble (GUE) and the Gaussian Symplectic Ensemble (GSE), respectively, in the limit $N \to \infty$. Similar results hold for sample covariance matrices and generalized Wigner matrices for which the variances of matrix elements can be different.

Our approach is based on the study of the Dyson Brownian motion via a related new dynamics, the local relaxation flow. As a main input, we establish that the density of eigenvalues converges to the Wigner semicircle law, and this holds even down to the smallest possible scale. As a corollary, we show also that eigenvectors are fully delocalized - a property critical to the study of random Schrödinger equations.

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