FIM Minicourse

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Holomorphic maps between bounded domains which preserve invariant forms

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Abstract

Consider a germ of holomorphic isometry $f: (X, ds_X^2; x_0) \to (Y, ds_Y^2; y_0)$ between Kähler manifolds (X, ds_X^2) and (Y, ds_Y^2) equipped with real-analytic Kähler metrics. When X is simply-connected and (Y, ds_V^2) is a complete Kähler manifold, Calabi proved in his seminal work on holomorphic isometries that f extends to a global holomorphic isometric immersion. He also proved rigidity results when Y is a space form, including the case of $\mathbb{P}^N, 1 \leq N \leq \infty$, equipped with the Fubini-Study metric. Motivated by differential-geometric questions raised by Clozel-Ullmo in connection to a problem in arithmetic dynamics concerning commutants of certain Hecke correspondences, we consider the analytic continuation of germs of holomorphic isometries between bounded domains equipped with multiples of the Bergman metric, and also of holomorphic measure-preserving maps from a bounded symmetric domain into its Cartesian products. Building upon the works of Calabi and results in CR-geometry by Webster-Huang, we have developed techniques of analytic continuation using Kähler geometry and several complex variables which in particular answer the afore-mentioned questions. This serves to illustrate that there is fertile soil in the interaction between complex geometry and problems of arithmetic and algebrogeometric origin, especially those concerning bounded symmetric domains and their finite-volume quotients including various modular varieties.

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