

# Portfolio Optimization and Risk Prediction with Non-Gaussian COMFORT Models

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# COMFORT ~~Road Map~~ Highway

Risk Modeling	Primary Model COMFORT	PSARM
<ul style="list-style-type: none"> <li>VaR Performance</li> <li>ES Performance</li> <li>MLE &gt; Ad-Hoc 2-Step</li> <li>JRSS-A 2016</li> </ul>	<ul style="list-style-type: none"> <li>MGHyp-CCC-GARCH Framework</li> <li>EM Algo. Estimation</li> <li>Common Market Factor</li> <li>Option Pricing</li> <li>JoE 2015</li> </ul>	<ul style="list-style-type: none"> <li>Analytic ES</li> <li>DCC Modeling</li> <li>Portfolio Optimization</li> <li>Risk Tracking Method</li> <li>Sharpe Ratios &gt; 1</li> <li>Submitted</li> </ul>
Industry Consulting	Academic	Large-Scale Port. Opt.
<ul style="list-style-type: none"> <li>Leverage Investing</li> <li>Risk Trackers</li> <li>Tactical Asset Allocation</li> <li>Large Swiss Banks and Pension Funds</li> </ul>	<p>Generalizing the MGHyp:</p> <ul style="list-style-type: none"> <li>Multi-Factors for Full Tail Heterogeneity</li> <li>Lévy-Stable / Tempered Stable</li> </ul>	<ul style="list-style-type: none"> <li>FREE-COMFORT</li> <li>Markov-Switching CCC</li> <li>PCA Factor Structure</li> <li>High-Freq. Strategy</li> <li>Black-Litterman</li> <li>Machine Learning</li> </ul>

PSARM - Portfolio Selection with Active Risk Monitoring.

# Normal Model vs. Stylized Facts of Financial Returns

## Normal World

VS.

## Real World

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Multivariate Normal (Gaussian)	Returns Distribution →	Multivariate Fat-Tailed
Volatility	Risk Measure →	VaR and Expected Shortfall
Correlations	Measure of Dependence →	Correlations & Tail Dependence
Constant Volatility & Correlations	Market Dynamics →	Time Varying Volatility & Correlation
All Returns Are Symmetric	Symmetry →	Asset Specific Skewness

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*... the biggest problems we now have with the whole evaluation of risk is the fat-tail problem, which is really creating very large conceptual difficulties. Because as we all know, the assumption of normality enables us to drop off a huge amount of complexity of our equations very much to the right of the equal sign. Because once you start putting in non-normality assumptions, which unfortunately is what characterizes the real world, then the issues become extremely difficult.*

Greenspan, A. (1997)

Our models capture the stylized facts of financial returns from the Real World.

# Pitfalls of Copula Models

## Copula Models

vs.

## COMFORT Models

All Taken From The Real World

Assumptions

→

All Taken From The Real World

Fast  
Two Step  
(I) Margins; (II) Dependency

Model Estimation

→

Fast  
Fixed Point Algorithm

No Closed Form - Needs Simulations  
(slow)

Portfolio Distribution

→

Closed Form  
(fast)

- Both methods allow for time-varying volatilities and non-ellipticity, and also are fast to estimate.
- Simulation from the copula is required to generate the portfolio predictive density, and as tail risk is often the concern, a very large number of replications will be required to achieve adequate accuracy, thus rendering the method, particularly for high dimensions, too slow for asset allocation purposes.

Our models maintain the flexibility of Copula models (**real world assumptions, estimation speed**), but deliver a closed form expression for the portfolio distribution.

## Markowitz Portfolio

vs.

## COMFORT Portfolio

All Taken From The Normal World

Assumptions

→

All Taken From The Real World

Huge Modeling Error

Errors

→

Small Modeling Error

Closed Form - Univariate Normal

Portfolio Distribution

→

Closed Form

Fast  
Quadratic Programming Problem

Portfolio Optimization

→

Fast  
Convex Optimization Problem

Our portfolio methods maintain the benefits of the iid Multivariate Normal framework (**estimation speed**, **tractability of the portfolio distribution**), but have much smaller modeling error, and deliver a superior forecast.

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# Multivariate Normal Mean-Variance Mixture Distribution

The random vector  $\mathbf{Y}$  is said to have a multivariate normal mean-variance mixture distribution (MNMVM) if

$$\mathbf{Y} = \mathbf{m}(G) + \mathbf{H}^{1/2} \sqrt{G} \mathbf{Z},$$

where

- $\mathbf{Z} \sim N(\mathbf{0}, \mathbf{I}_K)$ ;
- $G \geq 0$  is a non-negative, univariate random variable which is independent of  $\mathbf{Z}$ ;
- $\mathbf{H}$  is a  $K \times K$  symmetric and positive definite matrix; and
- $\mathbf{m} : [0, \infty) \rightarrow \mathbb{R}^d$  is a measurable function.

The name MNMVM comes from the fact that:

$$\mathbf{Y} \mid (G = g) \sim N(\mathbf{m}(g), g\mathbf{H}).$$

The multivariate generalized hyperbolic (MGHyp) distribution is a special case of MNMVM with:

$$\mathbf{m}(G) = \boldsymbol{\mu} + \boldsymbol{\gamma}G \text{ and } G \sim \text{GIG}(\lambda, \chi, \psi).$$

$$\mathbf{Y} = \boldsymbol{\mu} + \boldsymbol{\gamma}\mathbf{G} + \mathbf{H}^{1/2}\sqrt{\mathbf{G}}\mathbf{Z}, \text{ and } G \sim \text{GIG}(\lambda, \chi, \psi).$$

- introduced by Barndorff-Nielsen (1977);
- an inf. div. dist. used to construct Lévy process (e.g. Ornstein-Uhlenbeck model driven by Lévy & superpositions of them);
- tractable portfolio dist.:  $\mathbf{w}'\mathbf{Y} \sim \text{GHyp}(\mathbf{w}'\boldsymbol{\mu}, \mathbf{w}'\boldsymbol{\gamma}, \mathbf{w}'\mathbf{H}\mathbf{w}, \lambda, \chi, \psi)$ ;
- used in: (i) portfolio, (ii) risk management, and (iii) option pricing;
- special and limiting cases include:
  - multivariate normal
  - multivariate Laplace
  - multivariate normal inverse Gaussian (NIG)
  - multivariate t-distribution
  - variance-gamma, Madan and Seneta (1990), Seneta (2004);
- the tail behavior of the GHyp distribution spans a range from exponential, to power tail.



# New Class of MGHyP-CC and hybrid GARCH-SV Models

unconditional models

$$\mathbf{Y}_t \stackrel{\text{i.i.d.}}{\sim} \mathbf{N}(\boldsymbol{\mu}, \mathbf{H}) \quad \rightarrow$$

↓

$$\mathbf{Y}_t \stackrel{\text{i.i.d.}}{\sim} \text{MGHyp}(\boldsymbol{\mu}, \boldsymbol{\gamma}, \mathbf{H}, \lambda, \chi, \psi) \quad \rightarrow$$

conditional models

univariate:

$$(\text{t-GARCH}) \quad Y_t | \mathcal{F}_{t-1} \sim t(\mu_t, \sigma_t, \nu)$$

$$Y_t | \mathcal{F}_{t-1} \sim \text{GHyp}(\mu, \gamma, \sigma_t, \lambda, \chi, \psi)$$

multivariate (Gaussian):

$$\mathbf{Y}_t | \mathcal{F}_{t-1} \sim \mathbf{N}(\boldsymbol{\mu}_t, \mathbf{H}_t); \quad \boldsymbol{\mu}_t, \mathbf{H}_t \in \mathcal{F}_{t-1}$$

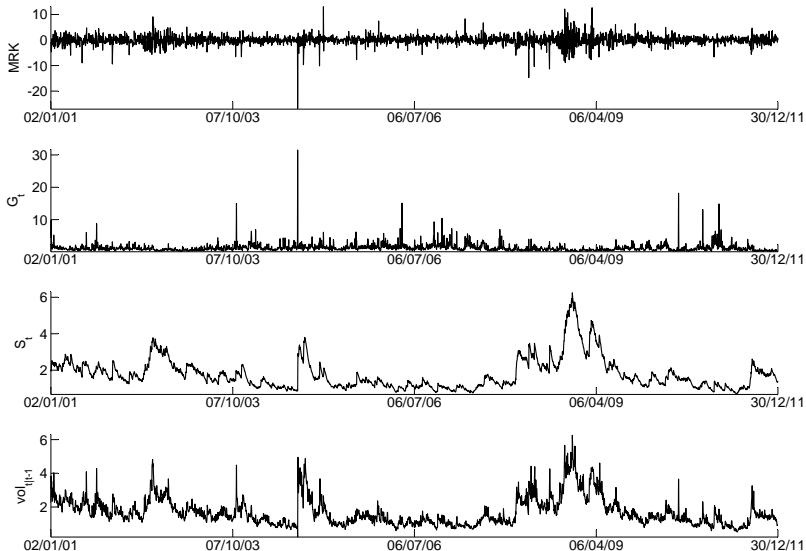
BEKK, CCC, VC, DCC, ADCC, RSDC

↓

$$\text{MGHyp}(\boldsymbol{\mu}, \boldsymbol{\gamma}, \mathbf{H}_t, \lambda, \chi, \psi)\text{-CC models}$$
$$\mathbf{H}_t = \mathbf{S}_t \boldsymbol{\Gamma}_t \mathbf{S}_t; \quad \mathbf{S}_t \in \mathcal{F}_{t-1}$$
$$\boldsymbol{\Gamma}_t \text{ from CCC, VC, DCC, cDCC, RSDC, } \dots$$

Notation:  $\mathcal{F}_t = \sigma(\boldsymbol{\varepsilon}_s : s \leq t)$ ;  $\xi \in \mathcal{F}_{t-1}$  means that  $\xi$  is measurable with respect to  $\mathcal{F}_{t-1}$ .

# One Interesting Feature: The Common Market Factor $G$



The MGHyP class, including all the limiting cases, is closed under linear operations, so, the conditional distribution of the portfolio  $P_t = \mathbf{w}'\mathbf{Y}_t$  is directly available and it is given by

$$P_t \mid \Phi_{t-1} \sim \text{GHyp}(\mathbf{w}'\boldsymbol{\mu}, \mathbf{w}'\boldsymbol{\gamma}, \mathbf{w}'\mathbf{H}_t\mathbf{w}, \lambda, \chi, \psi).$$

Given the density of the portfolio, VaR and ES can be numerically computed.

Standard methods to compute VaR and ES are time consuming, here we propose faster solutions.

From the predictive portfolio distribution, one can do mean–variance or mean–ES portfolio optimization.

The corresponding portfolio cdf can be written as

$$\Pr(P_t \leq r) = \int_0^\infty \Phi\left(\frac{r - \mu_{\mathbf{w}} - \gamma_{\mathbf{w}}g}{\sigma_{t,\mathbf{w}}}\right) f_{G_t}(g; \lambda, \chi, \psi) dg,$$

where  $\Phi$  is the cdf of standard normal random variable.

Use of this representation reduces the computation time required for the numerical inversion in the computation of the VaR values by a factor of four.

## Proposition

If  $P \sim \text{GHyp}(\mu, \gamma, h, \lambda, \chi, \psi)$ , then

$$\text{ES}_\alpha(P) = -\mu - \frac{\gamma}{\alpha} \mathbb{E}[G] F_{P^*}(-\text{VaR}_\alpha(P)) + \frac{h}{\alpha} \frac{C}{\sqrt{2\pi}},$$

where  $F_{P^*}$  is the cdf of  $P^* \sim \text{GHyp}(\mu, \gamma, h, \lambda + 1, \chi, \psi)$ ; and the constant  $C$  is given by

$$C = \frac{\chi^{-\lambda} (\sqrt{\chi\psi})^\lambda}{2K_\lambda(\sqrt{\chi\psi})} \frac{2K_{\tilde{\lambda}}(\sqrt{\tilde{\chi}\tilde{\psi}})}{\tilde{\chi}^{-\tilde{\lambda}} (\sqrt{\tilde{\chi}\tilde{\psi}})^{\tilde{\lambda}}} \exp\left(-\frac{(\text{VaR}_\alpha(P) + \mu)\gamma}{h^2}\right),$$

with  $\tilde{\lambda} = \lambda + 1/2$ ,  $\tilde{\chi} = \chi + \frac{(\text{VaR}_\alpha(P) + \mu)^2}{h^2}$ , and  $\tilde{\psi} = \psi + \frac{\gamma^2}{h^2}$ .

Fast and accurate computation of ES for GHyp portfolio, e.g., in the elliptical case this is 100 times faster than standard ES computation!

The minimum-variance portfolio and the mean-variance portfolio are the solutions of

$$\min_{\mathbf{w} \in \mathfrak{W}} \sigma_{t, \mathbf{w}}^2, \quad (1)$$

where

- $\mathfrak{W} = \{\mathbf{w} \in \mathbb{R}^K : \sum_{k=1}^K w_k = 1, w_k \geq 0, k = 1, \dots, K\}$ , and,
- in case of the classical Markowitz (1952) mean-variance portfolio,  $\mu_{\mathbf{w}} + \gamma_{\mathbf{w}} \mathbb{E}[G_t | \Phi_{t-1}] \geq \bar{p}$ , where  $\bar{p}$  is a constant for the target minimum return of the portfolio,
- the minimum-ES (min-ES) portfolio and mean-ES portfolio replace the conditional variance  $\sigma_{t, \mathbf{w}}^2$  by the conditional ES in (1),
- the computational efficiency of min-ES portfolio optimization is greatly increased by using the results from Rockafeller and Uryasev (2000).

# mean-ES and min-ES Portfolio Optimization

Rockafeller and Uryasev (2000) introduce an auxiliary function

$$F_{\alpha}(x, \mathbf{w}) = x + \frac{1}{\alpha} \int_{-\infty}^{\infty} [p + x]^{-} f_{P_t}(p \mid \mu_{\mathbf{w}}, \gamma_{\mathbf{w}}, \sigma_{t, \mathbf{w}}, \lambda, \chi, \psi) dp,$$

where  $[z]^{-} = -z$  if  $z \leq 0$ , and  $[z]^{-} = 0$  if  $z > 0$ , and

$$\min_{(x, \mathbf{w}) \in \mathbb{R} \times \mathfrak{W}} F_{\alpha}(x, \mathbf{w}) = \min_{\mathbf{w} \in \mathfrak{W}} \text{ES}_{\alpha}^{t|t-1}(P_t), \quad (2)$$

where the pair  $(x^*, \mathbf{w}^*)$  achieves the first minimum if and only if  $\mathbf{w}^*$  achieves the second minimum and  $x^* = \text{VaR}_{\alpha}^{t|t-1}(\mathbf{w}^{*'} \mathbf{Y}_t)$ .

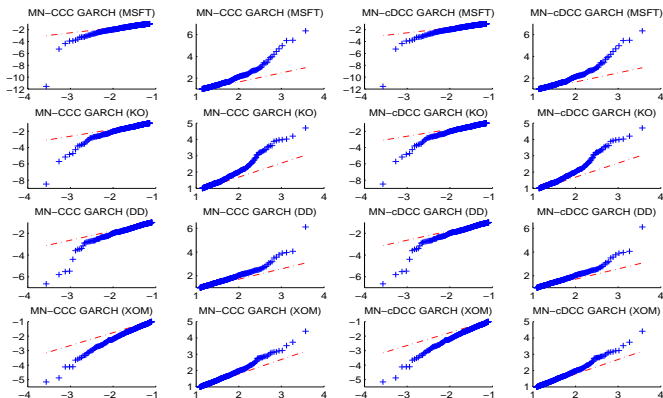
The first minimization in (2) is more efficient and provides global optimum because :

- it completely avoids the calculation of the ES (and VaR) of the candidate portfolios,
- it is a convex programming problem.

- Data: 2,767 daily vector returns of  $K = 30$  components of the Dow Jones Industrial Index (DJ-30) from January 2nd, 2001, to December 30th, 2011 (from Wharton Data Base).
- The returns for each asset are computed as  $y_{k,t} = 100 \log(p_{k,t}/p_{k,t-1})$ , where  $p_{k,t}$  is the price of a share of asset  $k$  at time  $t$ .
- Models compared:
  - Bollerslev (1990) MN-CCC; Tse and Tsui (2002) MN-VC; Engle (2002) MN-DCC; Aielli (2011) MN-cDCC;
  - COMFORT models, non-elliptical (estimated via EM):
    - MALap, MNIG, MA $t$ ;
    - CCC, VC, DCC, cDCC;
    - GARCH(1,1), GJR-GARCH(1,1);
  - some elliptical models (estimated in 3-steps and via EM):
    - MLap, MNIG, M $t$

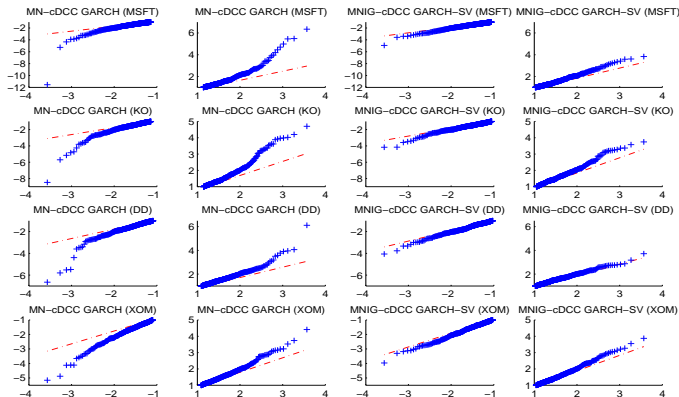


# Tails of Q-Q Plots for Residuals from MN-CCC vs. MN-cDCC GARCH(1,1)



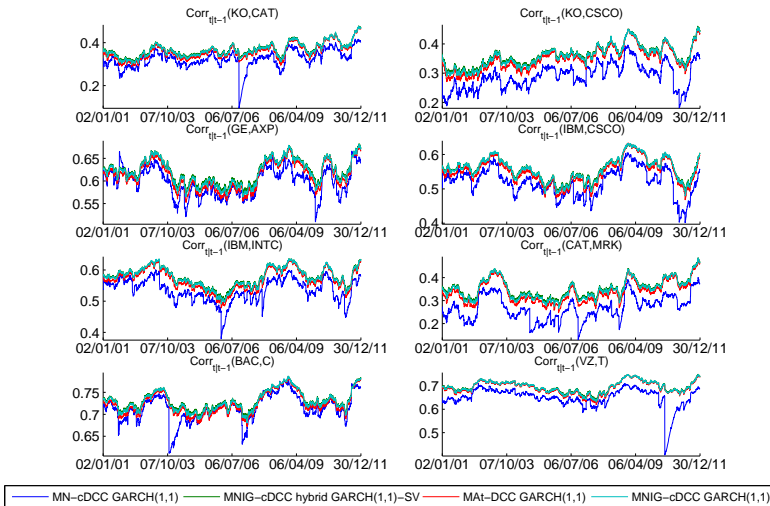
$$\hat{H}_t^{-1/2}(\mathbf{y}_t - \hat{\boldsymbol{\mu}})$$

# Tails of Q-Q Plots for Residuals from MN-cDCC vs. MNIG-cDCC GARCH(1,1)-SV



$$\hat{H}_t^{-1/2}(\mathbf{Y}_t - \hat{\boldsymbol{\mu}}) \quad \text{and} \quad \hat{G}_t^{-1/2} \hat{H}_t^{-1/2}(\mathbf{Y}_t - \hat{\boldsymbol{\mu}} - \hat{\boldsymbol{\gamma}} \hat{G}_t)$$

# Correlation Dynamics



KO - Coca-Cola; CSCO - Cisco; CAT - Caterpillar; IBM - Intl. Business Machines; INTC - Intel; AXP - American Express; GE - General

Electric; BAC - Bank of America; MRK - Merck & Co; C - Citygroup

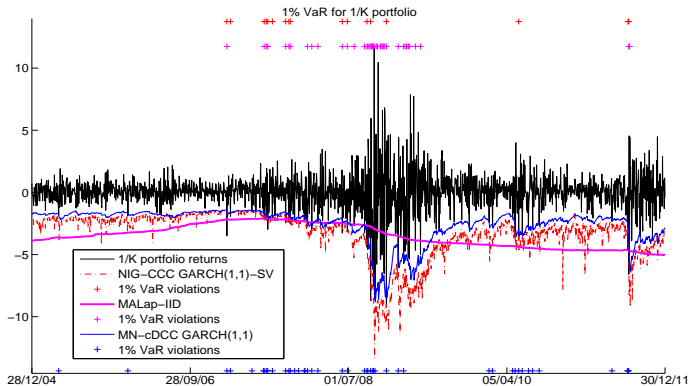
Marc S. Paoletta & Paweł Polak

COMFORT: Portfolio Optimization, Risk Management

# VaR Backtesting - Forecast Failure Rates

$\mathcal{M}$	VaR 1%	VaR 5%	VaR 10%
MNIG-CCC GARCH(1, 1)-SV	<b>0.0096</b>	<b>0.0526</b>	0.0905
MNIG-CCC GJR-GARCH(1, 1)-SV	<b>0.0084</b>	<b>0.0469</b>	0.0837
MNIG-CCC GJR-GARCH(1, 1)	<b>0.0118</b>	<b>0.0481</b>	0.0888
MA <sub>t</sub> -CCC GJR-GARCH(1, 1)	0.0118	0.0605	0.0984
MNIG-CCC GARCH(1, 1)	0.0130	0.0571	0.0962
MNIG (EM-elliptical)-CCC GARCH(1, 1)	0.0130	0.0605	0.1018
MALap-CCC GJR-GARCH(1, 1)	0.0135	0.0565	0.0945
Mlap (EM-elliptical)-CCC GARCH(1, 1)	0.0135	0.0611	0.1013
Mt (EM-elliptical)-CCC GARCH(1, 1)	0.0135	0.0639	0.1052
MA <sub>t</sub> -CCC GARCH(1, 1)	0.0147	0.0633	0.1058
MALap-CCC GARCH(1, 1)	0.0152	0.0616	<b>0.1001</b>
MLap (3-step elliptical)-CCC-GARCH(1, 1)	0.0045	0.0255	0.0617
MALap-CCC GJR-GARCH(1, 1)-SV	0.0158	0.0656	0.1041
Mt (3-step elliptical)-CCC-GARCH(1, 1)	0.0040	0.0357	0.0775
MNIG (3-step elliptical)-CCC-GARCH(1, 1)	0.0040	0.0272	0.0623
MALap-CCC GARCH(1, 1)-SV	0.0175	0.0696	0.1052
MALap-IID	0.0237	0.0628	<b>0.1007</b>
MN-cDCC GARCH(1, 1)	0.0254	0.0718	0.1041
MN-DCC GARCH(1, 1)	0.0265	0.0718	0.1041
MN-VC GJR-GARCH(1, 1)	0.0271	0.0696	0.1013
MN-CCC GJR-GARCH(1, 1)	0.0271	0.0701	0.1013
MN-DCC GJR-GARCH(1, 1)	0.0271	0.0701	<b>0.1007</b>
MN-cDCC GJR-GARCH(1, 1)	0.0271	0.0701	0.1007
MN-VC GARCH(1, 1)	0.0271	0.0724	0.1041
MN-CCC GARCH(1, 1)	0.0277	0.0730	0.1041

# VaR Backtesting MN-GARCH vs. MALap-IID vs. COMFORT



Failure rates: NIG-CCC GARCH(1, 1)-SV 0.0102; MALap-IID 0.0238; MN-cDCC GARCH(1, 1) 0.0255.

# The Risk Fear Portfolio: “Run For The Exit”

Define, respectively,  $\sigma^\#$  and  $ES^\#$  as a portfolio volatility and expected shortfall thresholds above which the investor exits the market. Then the min-Var and min-ES optimal portfolios with risk fear are defined by

$$\mathbf{w}_{VF}^* (\sigma^\#) = \mathbf{w}^* \mathbf{1}_{(0, \sigma^\#)} (\sigma_{\mathbf{w}^*}), \quad \mathbf{w}_{ESF}^* (ES^\#) = \mathbf{w}^* \mathbf{1}_{(0, ES^\#)} (ES_{\mathbf{w}^*}),$$

where  $\sigma_{\mathbf{w}^*}$  is the predicted conditional volatility for the min-Var optimal portfolio  $\mathbf{w}^*$ , and  $ES_{\mathbf{w}^*}$  is the predicted conditional expected shortfall for the optimal portfolio  $\mathbf{w}^*$ .

The strategy agrees with the stylized fact of “Runs for the Exit”, i.e., stock market major sell-offs during the initial stage of market downturns.

# The Risk Fear Portfolio: Risk Thresholds

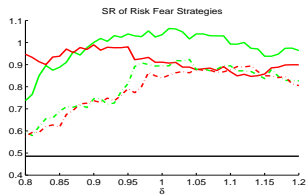
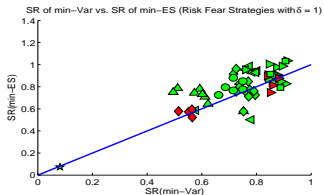
We assume that the investor uses the following simple rules to set the thresholds

$$\sigma^\#(\delta) = \bar{\sigma}_{\mathbf{w}^*} + \delta \text{std}(\sigma_{\mathbf{w}^*}), \quad \text{ES}^\#(\delta) = \overline{\text{ES}}_{\mathbf{w}^*} + \delta \text{std}(\text{ES}_{\mathbf{w}^*}),$$

where

- $\bar{\sigma}_{\mathbf{w}^*}$  and  $\overline{\text{ES}}_{\mathbf{w}^*}$  are sample moving averages, based on 1,000 observations, of the optimal portfolio volatility and expected shortfall, respectively;
- $\text{std}(\sigma_{\mathbf{w}^*})$  and  $\text{std}(\text{ES}_{\mathbf{w}^*})$  are the associated sample standard deviations; and
- $\delta$  is a parameter to be selected by the investor, e.g.,  $\delta = 1$ .

# The Risk Fear Portfolio: Performance



The minimum ES portfolios tend to perform better than the minimum variance portfolios.

In result, the SR increases from 0.49 up to 1.1.



Equities + Fixed Income Example: Three Different Portfolio Models with  
Portfolio Weights Constraints  
(Case study done for a large Swiss Bank)

# Prices, Returns and PSARM Forecasted Volatilities



# Portfolio Models, Constraints & Transaction Costs

Model 1						
Asset Class	Lower Limit	BM	Upper Limit	TC - Buy	TC - Sell	
CASH	0.00%	0.00%	0.10%	0.00%	0.00%	
Fixed Income Index	13.00%	15.00%	17.00%	0.12%	0.12%	
World Government Bond Index	9.00%	10.00%	11.00%	0.09%	0.02%	
MSCI World Index	24.00%	27.00%	30.00%	0.09%	0.07%	
Bond Index Rating AAA - BBB	22.00%	25.00%	28.00%	0.65%	0.00%	
Swiss Performance Index	16.00%	18.00%	20.00%	0.02%	0.02%	
Real Estate Funds	4.00%	5.00%	6.00%	0.05%	0.05%	
Model 2						
Asset Class	Lower Limit	BM	Upper Limit	TC - Buy	TC - Sell	
CASH	0.00%	0.00%	10.00%	0.00%	0.00%	
Fixed Income Index	5.00%	15.00%	25.00%	0.12%	0.12%	
World Government Bond Index	0.00%	10.00%	20.00%	0.09%	0.02%	
MSCI World Index	17.00%	27.00%	37.00%	0.09%	0.07%	
Bond Index Rating AAA - BBB	15.00%	25.00%	35.00%	0.65%	0.00%	
Swiss Performance Index	8.00%	18.00%	28.00%	0.02%	0.02%	
Real Estate Funds	0.00%	5.00%	15.00%	0.05%	0.05%	
Model 3						
Asset Class	Lower Limit	BM	Upper Limit	TC - Buy	TC - Sell	
CASH	0.00%	0.00%	100.00%	0.00%	0.00%	
Fixed Income Index	0.00%	15.00%	100.00%	0.12%	0.12%	
World Government Bond Index	0.00%	10.00%	100.00%	0.09%	0.02%	
MSCI World Index	0.00%	27.00%	100.00%	0.09%	0.07%	
Bond Index Rating AAA - BBB	0.00%	25.00%	100.00%	0.65%	0.00%	
Swiss Performance Index	0.00%	18.00%	100.00%	0.02%	0.02%	
Real Estate Funds	0.00%	5.00%	100.00%	0.05%	0.05%	

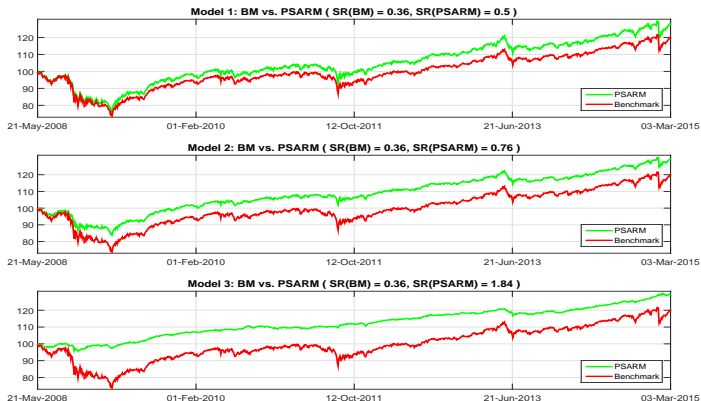
# Out-Of-Sample Performance Comparison

Asset Class \ Portfolio Strategy	Whole Sample 21/05/2008-03/03/2015			Subperiod 1 21/05/2008-31/01/2010			Subperiod 2 01/02/2010-11/10/2011			Subperiod 3 12/10/2011-20/06/2013			Subperiod 4 21/06/2013-03/03/2015		
	Exp. Ret.	Vol.	SR	Exp. Ret.	Vol.	SR	Exp. Ret.	Vol.	SR	Exp. Ret.	Vol.	SR	Exp. Ret.	Vol.	SR
Fixed Income Index	4.03%	5.46%	0.74	2.53%	6.92%	0.37	7.28%	5.19%	1.40	1.82%	4.92%	0.37	4.51%	4.48%	1.01
World Government Bond Index	3.89%	2.56%	1.52	4.55%	3.36%	1.36	3.46%	2.53%	1.36	2.35%	2.15%	1.09	5.22%	1.96%	2.66
MSCI World Index	-0.28%	20.51%	-0.01	-13.77%	30.12%	-0.46	-7.39%	20.15%	-0.37	9.90%	11.79%	0.84	10.23%	15.15%	0.67
Bond Index Rating AAA - BBB	4.04%	2.31%	1.75	6.60%	2.71%	2.44	3.39%	2.44%	1.39	1.08%	1.98%	0.55	5.09%	2.04%	2.49
Swiss Performance Index	3.58%	18.17%	0.20	-12.04%	26.01%	-0.46	-0.01%	17.20%	0.00	14.47%	12.65%	1.14	12.02%	13.69%	0.88
Real Estate Funds	6.06%	6.85%	0.88	6.68%	6.65%	1.00	4.76%	6.76%	0.70	1.87%	6.40%	0.29	10.98%	7.57%	1.45
Benchmark	2.88%	7.98%	0.36	-3.07%	11.76%	-0.26	0.53%	7.44%	0.07	6.15%	4.93%	1.25	7.95%	6.07%	1.31
PSARM Model 1	3.80%	7.67%	0.50	-0.97%	11.13%	-0.09	1.71%	7.26%	0.23	6.46%	4.91%	1.32	8.02%	5.86%	1.37
PSARM Model 2	3.76%	4.94%	0.76	0.91%	7.22%	0.13	2.97%	4.49%	0.66	4.42%	3.35%	1.32	6.76%	3.78%	1.79
PSARM Model 3	3.69%	2.01%	1.84	3.94%	2.71%	1.46	2.43%	1.72%	1.42	2.60%	1.79%	1.45	5.80%	1.63%	3.55

Asset Class	Correlations					
Fixed Income Index	1.00	0.74	-0.28	0.46	-0.23	-0.05
World Government Bond Index	0.74	1.00	-0.38	0.55	-0.33	-0.09
MSCI World Index	-0.28	-0.38	1.00	-0.22	0.74	0.17
Bond Index Rating AAA - BBB	0.46	0.55	-0.22	1.00	-0.23	-0.07
Swiss Performance Index	-0.23	-0.33	0.74	-0.23	1.00	0.19
Real Estate Funds	-0.05	-0.09	0.17	-0.07	0.19	1.00

- PSARM outperforms the Benchmark in the whole sample, in every subperiod, and for every model.
- Relaxing the constraints on the portfolio weights improves the performance of PSARM.
- There is no need to invest into cash because there are highly negatively correlated assets in the portfolio.

# PSARM Cumulative Returns After Transaction Costs



- PSARM increases Sharpe ratios, from **0.36** for the Benchmark, to **0.5**, **0.76**, and **1.84**, for Model 1, Model 2, and Model 3, respectively (after transaction costs).
- Very tight constraints on the portfolio weights do not allow for large improvements in Model 1.

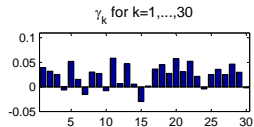
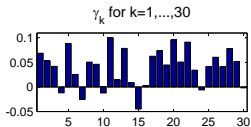
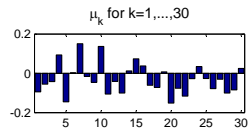
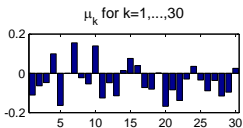
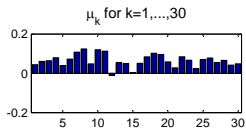
We propose a unified framework for large-scale portfolio optimization with:

1. A new class of models for multivariate asset returns which:
  - a hybrid of GARCH and SV (but without the estimation problems associated with the latter);
  - can be applied to a portfolio of hundreds of assets;
  - are simple and fast to estimate (allow for parallel estimation);
  - support various forms of the dynamics in the dependency structure (VC, DCC, cDCC, Markov-switching);
  - are based on a non-elliptical distribution with a tractable portfolio distribution;
  - nest several models previously proposed in the literature and massively outperform them in terms of in-sample fit, out-of-sample density forecasting, risk prediction, and portfolio performance.
2. A new risk fear portfolio strategy, which combines portfolio optimization with active risk monitoring. It:
  - outperforms all types of optimal portfolios, even in the presence of conservative transaction costs and frequent rebalancing;
  - avoids all the losses during the 2008 financial crisis, and;
  - profits from the subsequent market recovery.

## FREE-COMFORT: Fast Reduced Estimation COMFORT (Ongoing Research, joint work with Joris van der Aa from ETH)

(High Frequency Portfolio Optimization On the Components of the Dow Jones Industrial Index (DJ-30) from February 1st, 2010 to February 5th, 2016.)

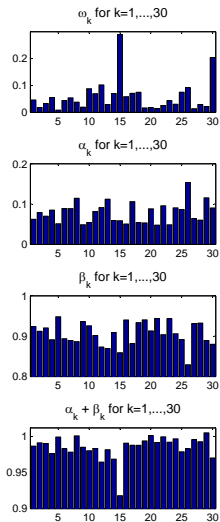
# GARCH(1, 1) Estimates Across Models



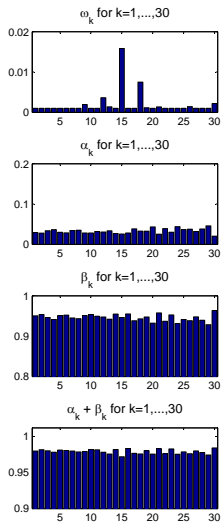


# GARCH(1, 1) Estimates Across Models

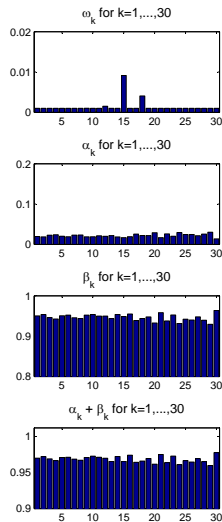
MN-CCC GARCH(1, 1)



MALap-CCC GARCH(1, 1)



MALap-CCC GARCH(1, 1)-SV



- To demonstrate the performance of the HF min fear portfolio, we use a data set consisting of 115'654 intra-day returns on the 5-minute level and 1501 overnight returns. Our combined data set consists of 117'155 returns for  $K = 30$  components of the Dow Jones Industrial Index (DJ-30) from February 1st, 2010 to February 5th, 2016.
- We take all DJ-30 components that are part of the composition as of 01.12.2015.
- Each trading day consists of 77 equally spaced time points on which it is possible to trade.
- The first possible trading moment of the day is at 9:31AM and the last trading moment is at 3:54PM.
- At 3:54PM, we take on the optimal portfolio  $\mathbf{w}^*$  for 3:59PM and hold this position overnight.
- For our trading strategy, it is not possible to rebalance, close or open any positions before 9.31AM or after 15.54PM.

# Entry & Exit Thresholds

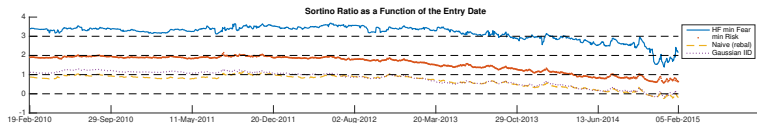
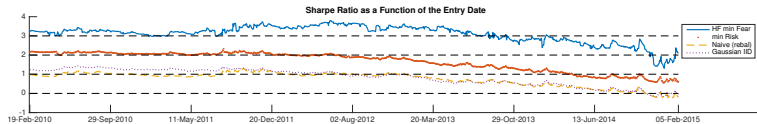
We set the thresholds for entering and exiting the market using the following two rules

$$ES^{\delta_{\text{exit}}} = \overline{ES}_{\mathbf{w}^*} + \delta_{\text{exit}} \text{std}(ES_{\mathbf{w}^*}), \quad (3)$$

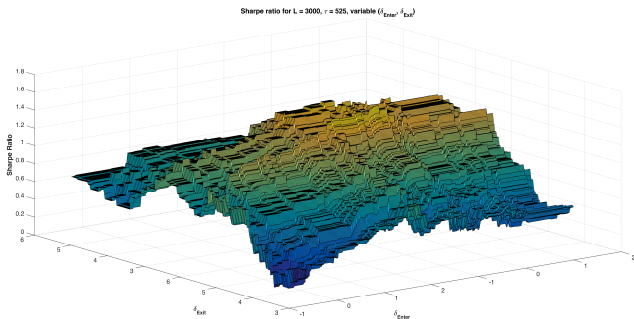
$$ES^{\delta_{\text{enter}}} = \overline{ES}_{\mathbf{w}^*} + \delta_{\text{enter}} \text{std}(ES_{\mathbf{w}^*}), \quad (4)$$

where  $\overline{ES}_{\mathbf{w}^*}$  is the sample moving average, based on the last 1000 observations of the optimal portfolio expected shortfall;  $\text{std}(ES_{\mathbf{w}^*})$  is the associated sample standard deviation; and  $\delta_{\text{enter}}$  and  $\delta_{\text{exit}}$  are parameters to be selected by the investor with  $\delta_{\text{exit}} > \delta_{\text{enter}}$ . We term our trading strategy the *High Frequency Risk Fear Portfolio Strategy* (hereafter HF min fear).

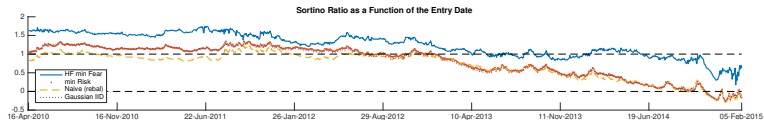
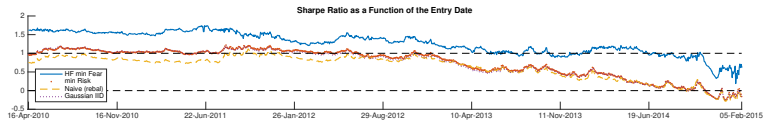
# Gross Performance



# Parameters Calibration



# Analysis of Net Performance for Optimal Parameters



# COMFORT & Machine Learning for Improved Portfolio Performance

# Trend Chasing Product For Improved Portfolio Performance

- We combine statistical and machine learning techniques to predict future returns of the components of a large portfolio of assets.
  - Advanced Multivariate Times Series Modeling Captures The Dynamics of Asset Returns.
  - Machine Learning Techniques Allow to Predict Future Returns.



# Portfolio Optimization with Trend Prediction

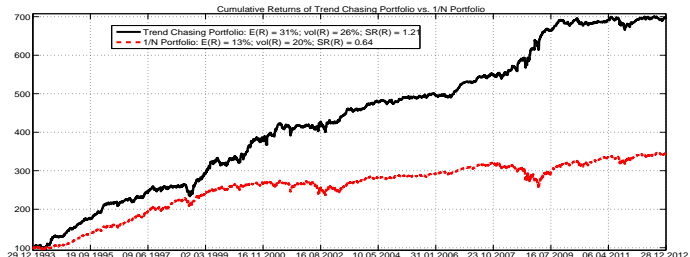
- We backtest our Portfolio Strategy with Trend Prediction on 4 sets of stock returns:
  - Long History of the returns: daily returns of the 30 components of the Dow Jones Index, from 04.01.1993 until 31.12.2012.
  - The most recent data: daily returns of the 30 components of the Dow Jones Index, more recent data and components from 01.06.1999 until 31.12.2014.
  - Large Set of Assets: daily returns of the 459 components of the S&P 500 Index, from 04.01.2009 until 15.05.2014.
  - High Frequency: 5 minutes returns of the 30 components of the Dow Jones Index, from 02.01.2009, 10.00.00 until 31.12.2009, 15.25.00.
- In the backtesting procedure we used a rolling window of 250 observations. First out-of-sample date is 29.12.1993, 24.05.2000, 05.01.2010, and 07.01.2009, 14.25.00, respectively.

# Portfolio Optimization with Trend Prediction

- The strategy uses only information on past returns, does not use future returns, nor any exogenous variables.
- It is a long-only portfolio (for negative predicted means, the portfolio weights are set to zero).
- The Trend Prediction Portfolio Strategy can be integrated with the existing portfolio strategy in a form of *views* on the market analogously to the views of the investor in the *Black-Litterman model*.

# Equities Example: Components of DJ-30 Index (1993-2012)

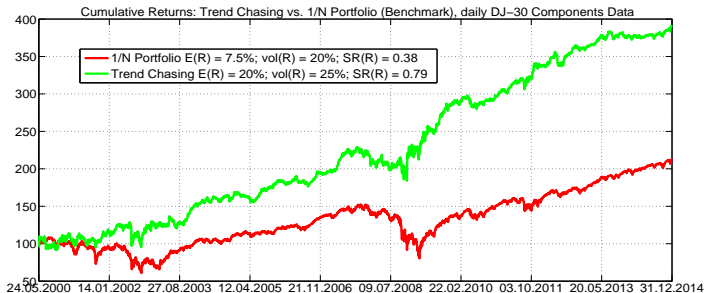
# Trend Prediction: DJ-30 Components (1993-2012)



- The cumulative returns of the portfolio are plotted over time, and compared with the equally weighted portfolio (1/N portfolio).
- For the Trend Chasing Portfolio the average annual return, volatility, and Sharpe ratio are given by: 31%, 26%, 1.21, respectively.
- While the equally weighted portfolio (proxy for the market) has the average annual return, volatility, and Sharpe ratio equal to 13%, 20%, 0.64, respectively.

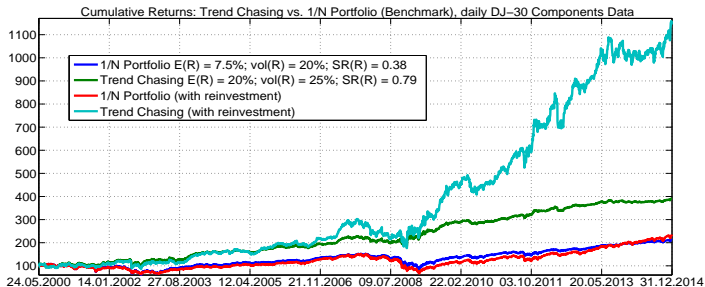
# Equities Example: Components of DJ-30 Index (2000-2014)

# Trend Prediction: DJ-30 Components (2000-2014)



- The cumulative returns of the portfolio are plotted over time, and compared with the equally weighted portfolio (1/N portfolio).
- For the Trend Chasing Portfolio the average annual return, volatility, and Sharpe ratio are given by: 20%, 25%, 0.79, respectively.
- While the equally weighted portfolio (proxy for the market) has the average annual return, volatility, and Sharpe ratio equal to 7.5%, 20%, 0.38, respectively.

# DJ-30 Components (2000-2014) With Reinvestment

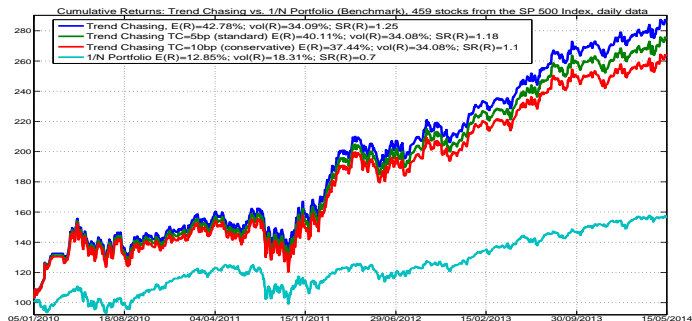


- The cumulative returns of the portfolio are plotted over time, and compared with the equally weighted portfolio (1/N portfolio).
- Two types of portfolio with and without reinvestment.
- For the Trend Chasing Portfolio the average annual return, volatility, and Sharpe ratio are given by: 20%, 25%, 0.79, respectively.
- While the equally weighted portfolio (proxy for the market) has the average annual return, volatility, and Sharpe ratio equal to 7.5%, 20%, 0.38, respectively.

# Equities Example: Components of SP-500 Index (2010-2014)



# Portfolio with Trend Prediction & Proportional Transaction Costs

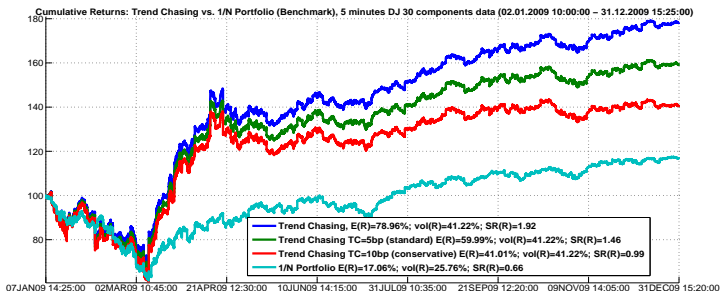


- The cumulative returns of the portfolio are plotted over time, and compared with the equally weighted portfolio (1/N portfolio).
- The average annual return, volatility, and Sharpe ratio for the Trend Chasing Portfolio:
  - (i)  $TC = 0$ : 42.78%, 34.09%, 1.25;
  - (ii)  $TC = 5\text{bp}$  (standard): 40.11%, 34.08%, 1.18; and
  - (iii)  $TC = 10\text{bp}$  (conservative): 37.44%, 34.08%, 1.1, respectively.
- While the equally weighted portfolio (proxy for the market) has the average annual return,

## Equities Example: Components of DJ-30 Index

High Frequency Portfolio Strategy: 5 Minutes Data  
(07.01.2009 - 31.12.2009)

# Portfolio with Trend Prediction & Proportional Transaction Costs: HF



- The cumulative returns of the portfolio are plotted over time, and compared with the equally weighted portfolio (1/N portfolio).
- The average annual return, volatility, and Sharpe ratio for the Trend Chasing Portfolio:
  - (i)  $TC = 0$ : 78.96%, 41.22%, 1.92;
  - (ii)  $TC = 5\text{bp}$  (standard): 59.99%, 41.22%, 1.46; and
  - (iii)  $TC = 10\text{bp}$  (conservative): 41.01%, 41.22%, 0.99, respectively.
- While the equally weighted portfolio (proxy for the market) has the average annual return, volatility, and Sharpe ratio equal to 17.06%, 25.76%, 0.66, respectively.

# COMFORT ~~Road Map~~ Highway

Risk Modeling	Primary Model COMFORT	PSARM
<ul style="list-style-type: none"> <li>• VaR Performance</li> <li>• ES Performance</li> <li>• MLE &gt; Ad-Hoc 2-Step</li> <li>• JRSS-A 2016</li> </ul>	<ul style="list-style-type: none"> <li>• MGHyP-CCC-GARCH Framework</li> <li>• EM Algo. Estimation</li> <li>• Common Market Factor</li> <li>• Option Pricing</li> <li>• JoE 2015</li> </ul>	<ul style="list-style-type: none"> <li>• Analytic ES</li> <li>• DCC Modeling</li> <li>• Portfolio Optimization</li> <li>• Risk Tracking Method</li> <li>• Sharpe Ratios &gt; 1</li> <li>• Submitted</li> </ul>
<b>Industry Consulting</b> <ul style="list-style-type: none"> <li>• Leverage Investing</li> <li>• Risk Trackers</li> <li>• Tactical Asset Allocation</li> <li>• Large Swiss Banks and Pension Funds</li> </ul>	<b>Academic</b> <p>Generalizing the MGHyP:</p> <ul style="list-style-type: none"> <li>• Multi-Factors for Full Tail Heterogeneity</li> <li>• Lévy-Stable / Tempered Stable</li> </ul>	<b>Large-Scale Port. Opt.</b> <ul style="list-style-type: none"> <li>• FREE-COMFORT</li> <li>• Markov-Switching CCC</li> <li>• PCA Factor Structure</li> <li>• High-Freq. Strategy</li> <li>• Black-Litterman</li> <li>• Machine Learning</li> </ul>

**Thank you for your attention!**

Paweł Polak