Skewness Preferences in Choice Under Risk

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Summary

We analyze and compare the skewness preferences

—the attitudes toward rare, high-impact risks—

implied by important theories of decision-making under risk.

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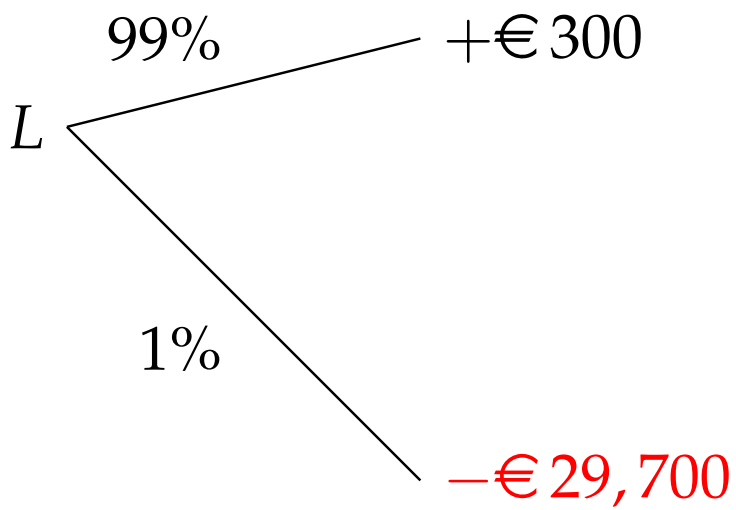
Research question: Which theories of decision-making under risk get skewness preferences "right?"

Preview: Skewness preference in expected utility (I/III)

Say wealth is 30.000. Consider the choice between the zero-mean, same-variance risks:



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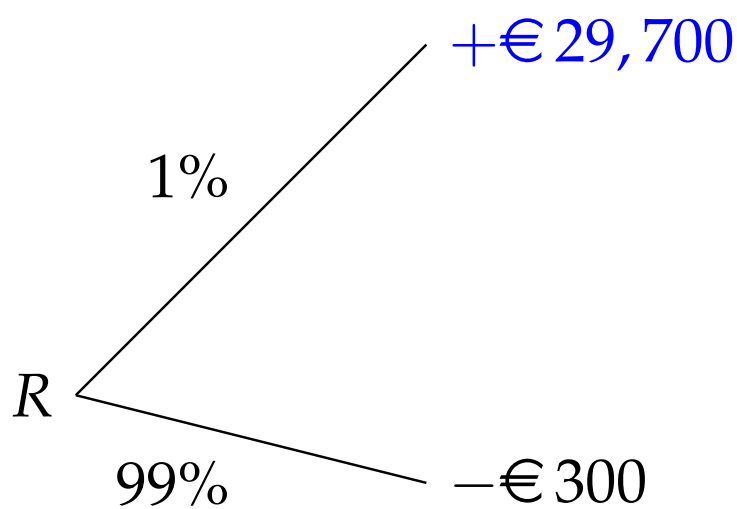

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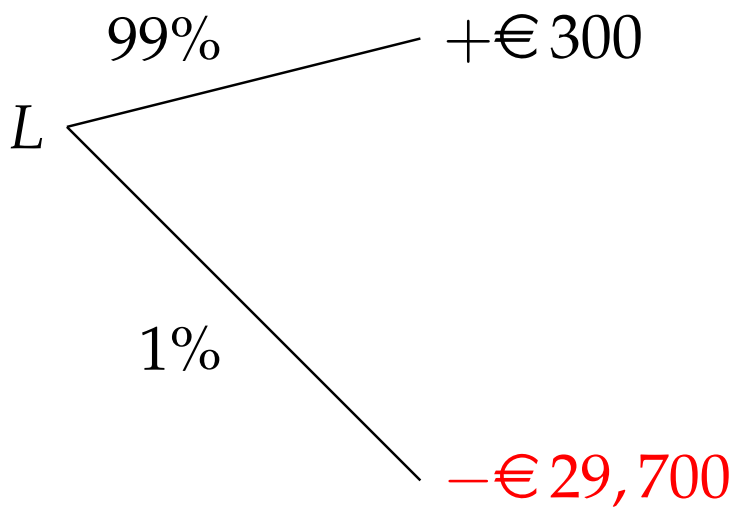

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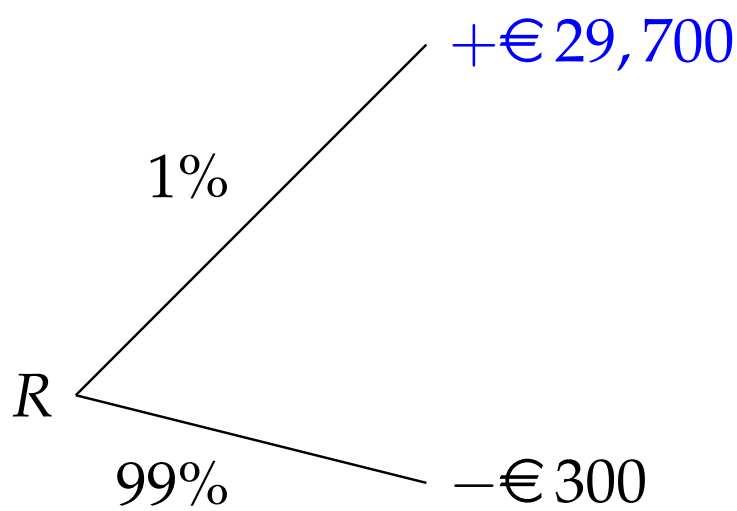

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QUIZ: What does an expected utility (EU) maximizer prefer if

(a) $u(x) = x^{0.5}$?

(b) $u(x) = x^{1.5}$?

Reminder: $EU[L] = 99\% \cdot u(30.300) + 1\% \cdot u(300)$.

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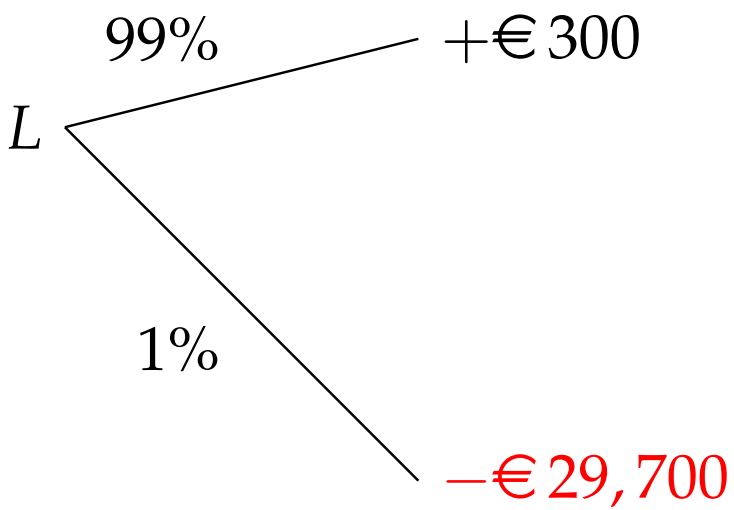

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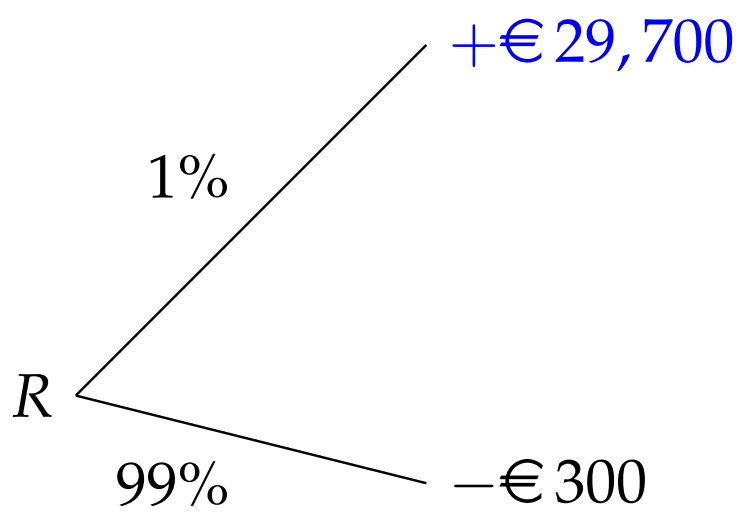

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ANSWER: If

(a) $u(x) = x^{0.5}$, then $R \succ L$ (skewness-seeking)

(b) $u(x) = x^{1.5}$, then $L \succ R$ (skewness-averse)

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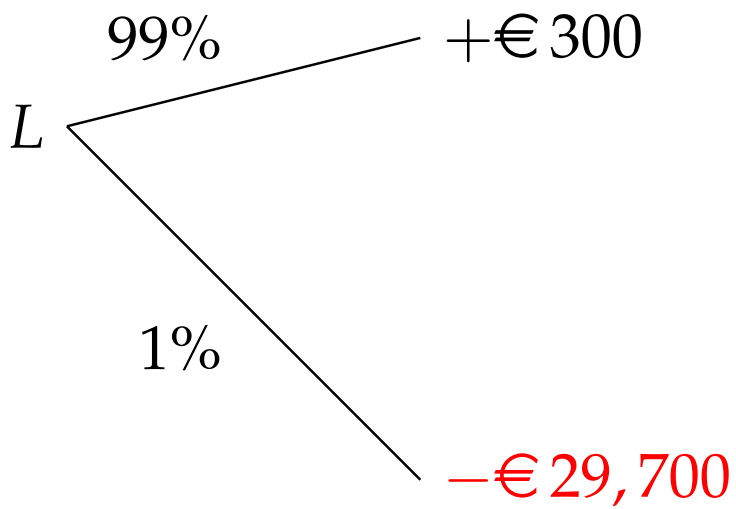

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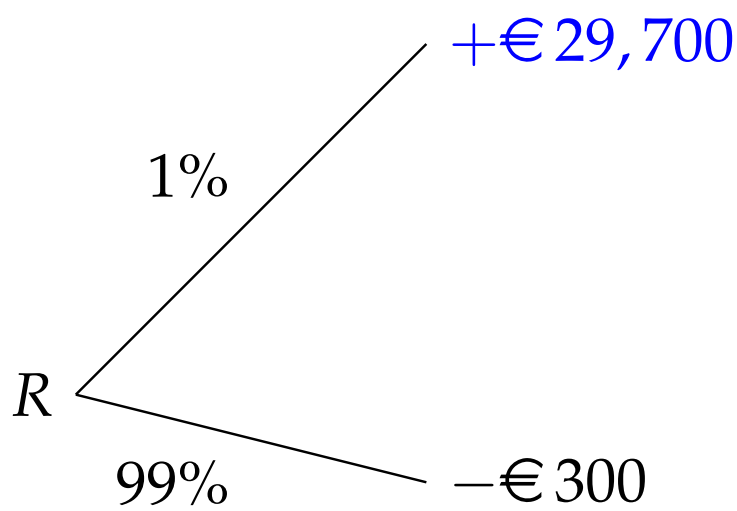

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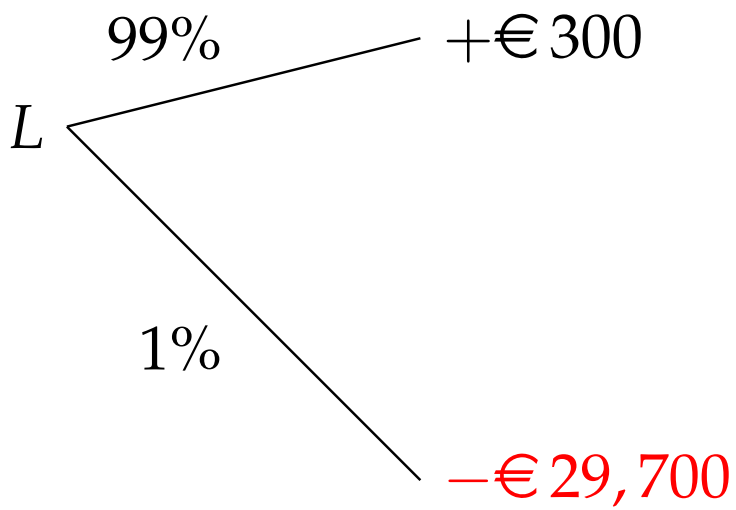

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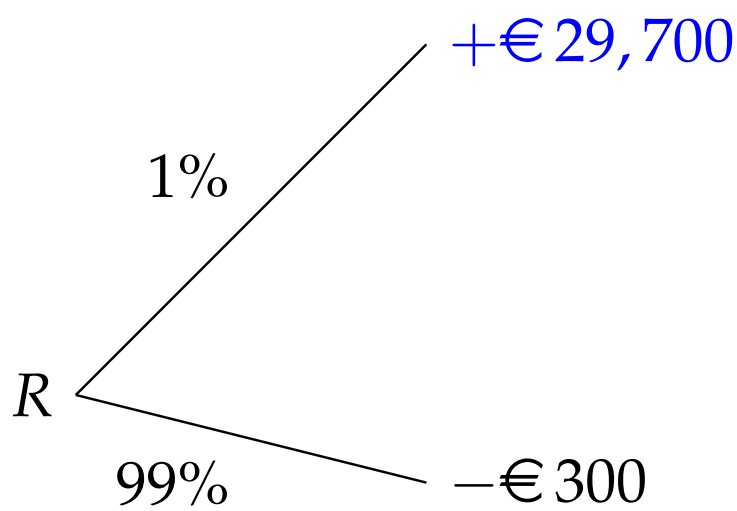

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Intuition for the Quiz result: A Taylor expansion at wealth level z_0 yields

$$E[u(z_0+R)] \approx u(z_0) + u'(z_0)E[R] + \frac{1}{2}u''(z_0)Var(R) + \frac{1}{6}u'''(z_0)Skew(R)$$

Note that $(x^\alpha)''' < 0 \iff \alpha \in (1,2)$.

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Remark III: EU implies "third-order" skewness preferences (3SP).

Empirically, skewness-seeking is "first-order"

Examples include

- The classics: insurance and lottery gambling
- Binary choice experiments
- Asset pricing: skewed assets are overpriced (e.g., out-of-the money options, growth stocks), variance premium puzzle
- Corporate finance: capital budgeting, window dressing
- Household finance: underdiversification
- Labor economics: career choices
- ...

Contribution of this paper

Contribution I/II: What we do

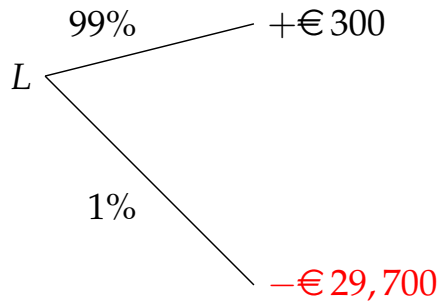
We present

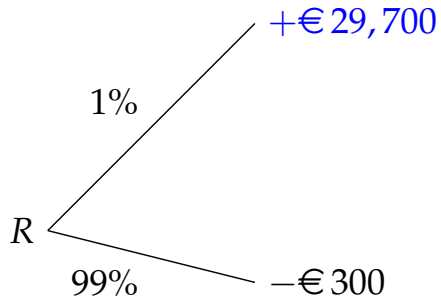
a definition of skewness preference and its order

and characterize it in important theories of choice under risk.

We reveal and make explicit the skewness preferences they imply.

Contribution II/II: What we find





The bad news:

EU cannot feature first-order ("strong") skewness preference. (Impossibility Theorem).

The good news:

"Behavioral/psychology-based" theories get the high importance of skewness right.

Takeaway: When risks are skewed, EU predictions are unrealistic; work with a (i.e., any) behavioral model.

Overview of formal results

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Cumulative Prospect Theory (CPT; Tversky and Kahneman 1992)		
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Power-S-shaped utility (at the reference point)	1SS* or 1SA*	Prop. 11 & Corr. 3
Disappointment Aversion (DA; Gul 1991)		
Always (unless DA = EUT)	1SS	Prop. 9
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Most specifications	2SS	Prop.'s 8 & 9
Regret Theory (RT; Bell 1982, Loomes and Sugden 1982)		
Most specifications	2SS	Prop. 12
Salience Theory (ST; Bordalo et al. 2012)		
Rank-dependent ST (always, unless ST = EUT)	1SS	Prop. 13
Continuous ST (always, unless ST = EUT)	2SS	Prop. 13
Optimal Expectation Theory (OET; Brunnermeier and Parker 2005)		
Always (unless OET = EUT)	1SS	Prop. 14
Optimal Anticipation with Savoring and Disappointment (OASD; Gollier and Mürmann 2010)		
Always (unless OASD = EUT)	1SS	Prop. 15

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Skewness preference in EU

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Verbal definition of skewness-seeking

A preference functional U features skewness-seeking at wealth level z_0 if

- for every pair of two-outcome risks
- with equal mean and variance
- and skewness opposite in sign

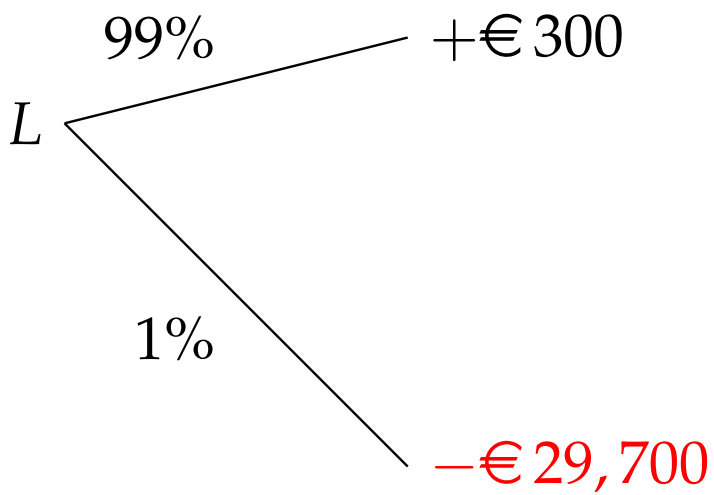
a small amount of the *right-skewed risk*

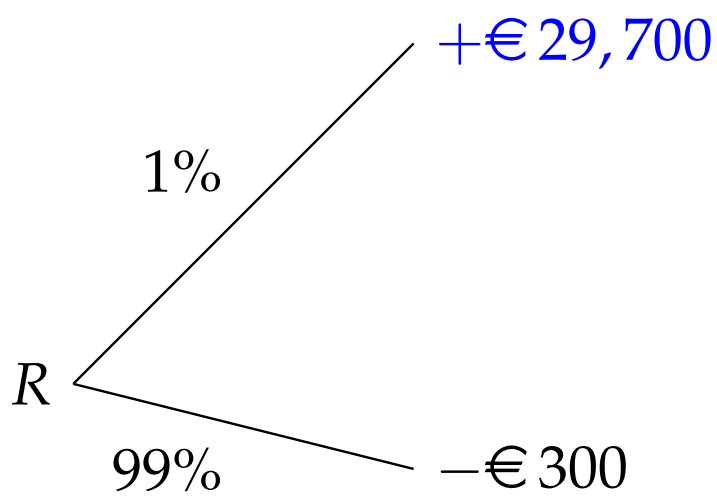
is preferred over

a small amount of the *left-skewed risk.*

... a bit more formal

Skewness-seeking means that, for $t > 0$ small enough, $tR \succ tL$





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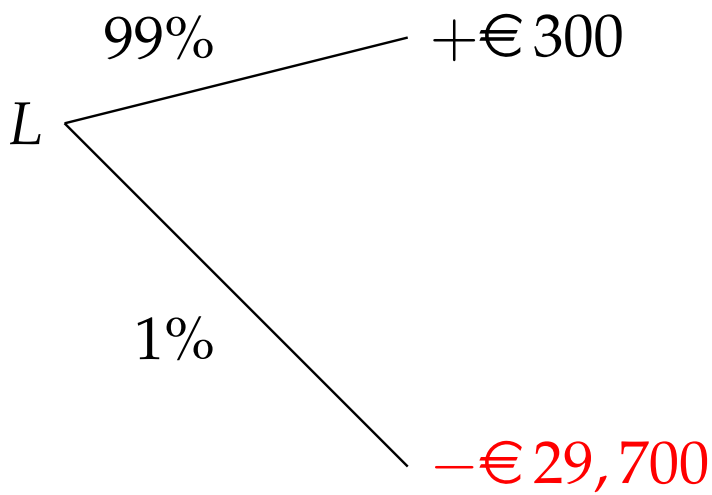

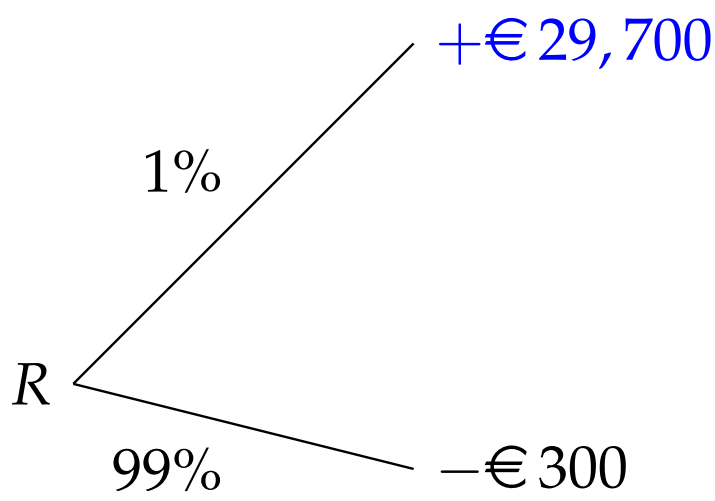

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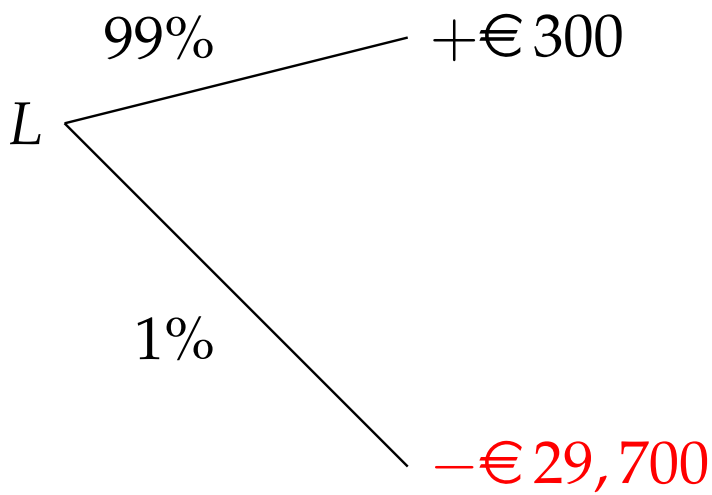

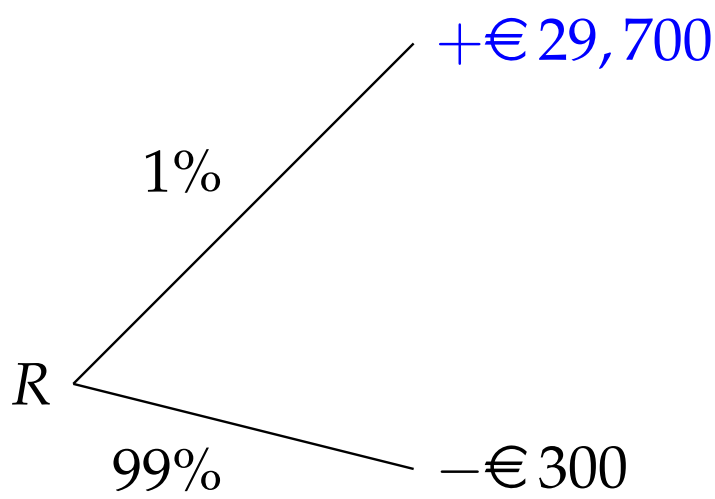

... and also "for all other parameters;" that is, all payoffs and probabilities for which

$$\mathbb{E}[R] = \mathbb{E}[L], \mathbb{V}[R] = \mathbb{V}[L], \text{ and } Skew[R] = -Skew[L].$$

Skewness-seeking: Formal definition

U implies skewness-seeking (SS) if, for all these risks R and L,

$$f(t) = \overbrace{U[z_0 + tR] - U[z_0 + tL]}^{\text{skewness utility premium}} > 0$$

when t small.

Example: Skewness-seeking in smooth EU

For EU with smooth utility function u it can be shown that

$$f(t) = \frac{\sigma^3}{3} \underbrace{\frac{1-2p}{\sqrt{p(1-p)}}}_{>0} u'''(z_0)t^3.$$

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Remark:

- $SS(z_0)$ is a new definition of prudence ...
- ... that, within EU, coincides with Eeckhoudt-Schlesinger's definition, but ...
- ... is simpler so that it can be characterized outside EU.

Orders of skewness-seeking: Intuition

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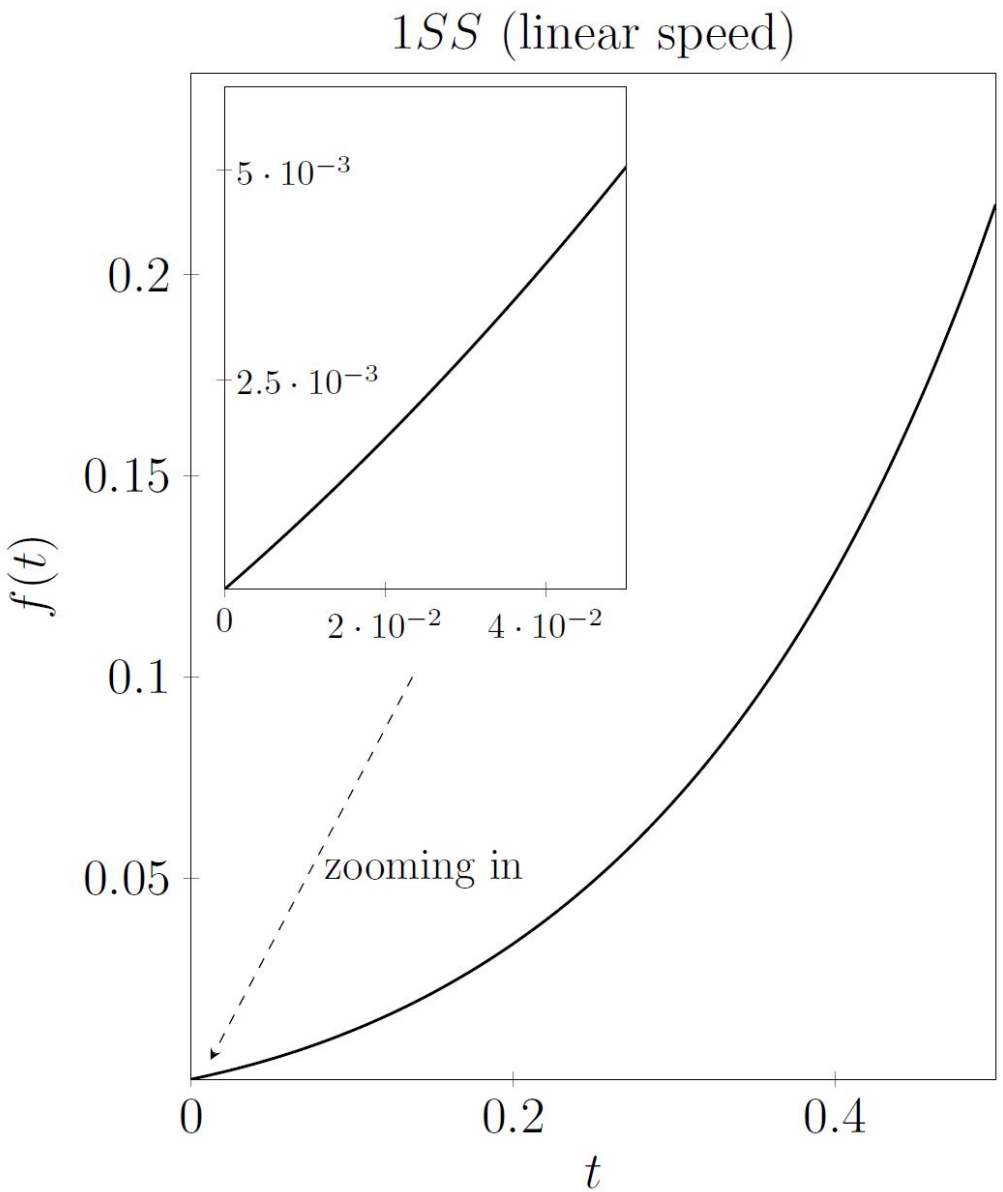

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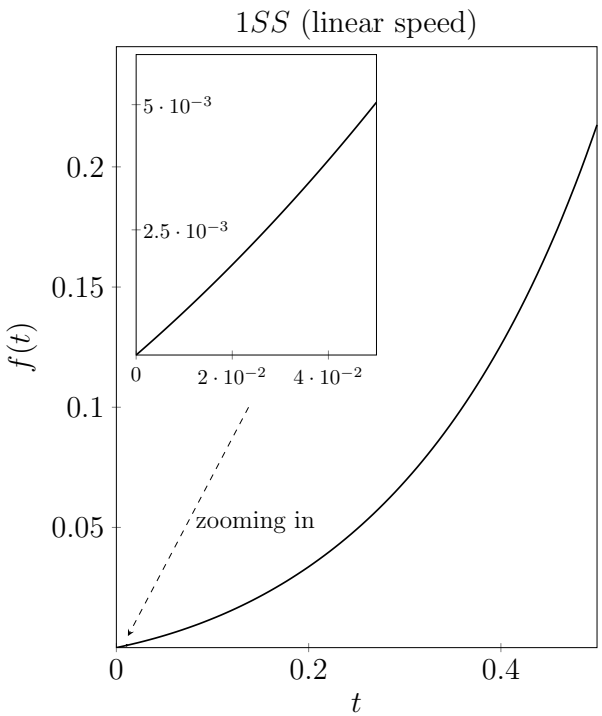

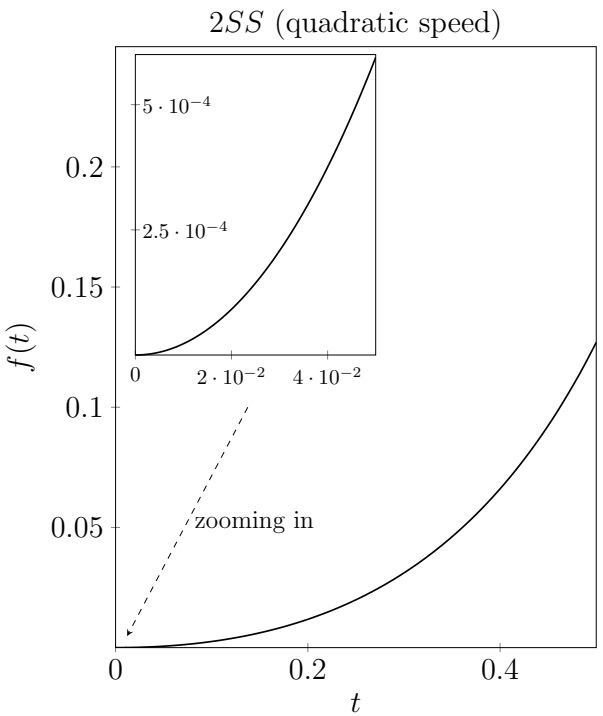

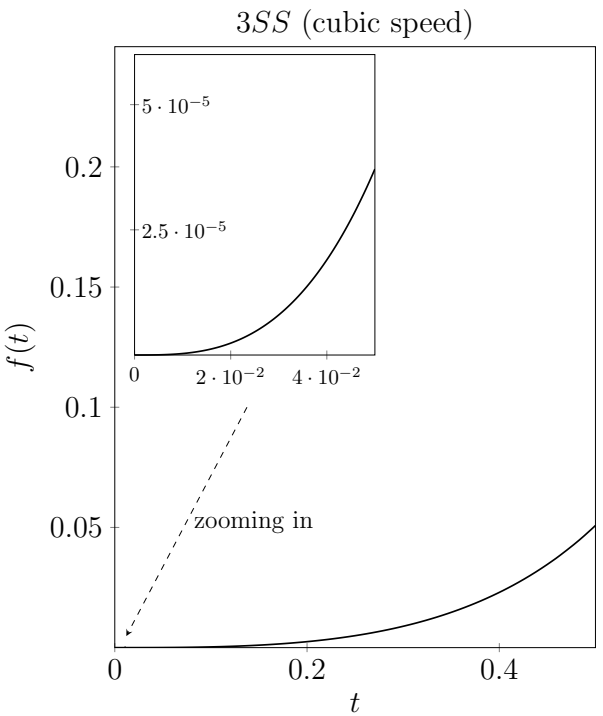

(Generalized) Proposition. In smooth EU:

$$3SS(z_0) \iff u'''(z_0) > 0.$$

Full Definition

Definition (Skewness-seeking and its orders).

1. U exhibits skewness-seeking at z_0, denoted by $SS(z_0)$, if there exists $t^* > 0$ such that $f(t) > 0$ on $(0, t^*)$.

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2. U exhibits skewness-seeking of order N $(N = 1, 2, 3)$ at z_0, denoted by $N\text{SS}(z_0)$, if

   (i) $f^{(n)}(0) = 0$ for $n = 1, \ldots, N-1$ and

   (ii) $f^{(N)}(0) > 0$.

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Impossibility Theorem

Theorem. EU cannot induce first-order skewness-seeking.

Two remarks:

- This holds even if one would invent "new & crazy" utility functions.
- To accommodate the empirical evidence for strong skewness-seeking, we must depart from EU.

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Prospect Theory

Postulated by Daniel Kahneman and Amos Tversky (1979)

2002 Nobel Prize for Daniel Kahneman:

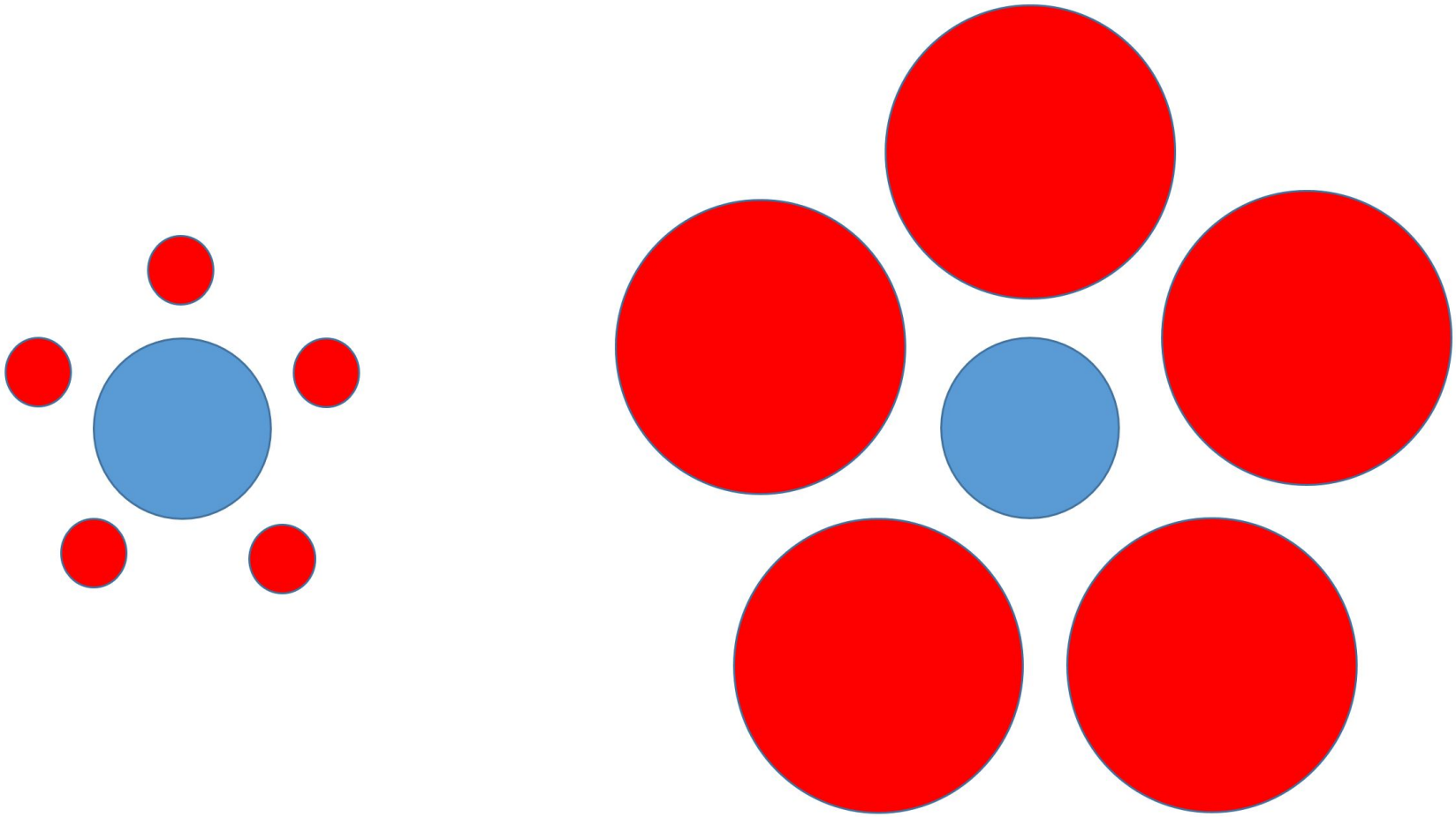
> *"...for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty."*

Prospect Theory

Four main ideas:

1. Reference point: Utility is defined over changes, not absolutes

Humans are sensitive to the relative, not the absolute



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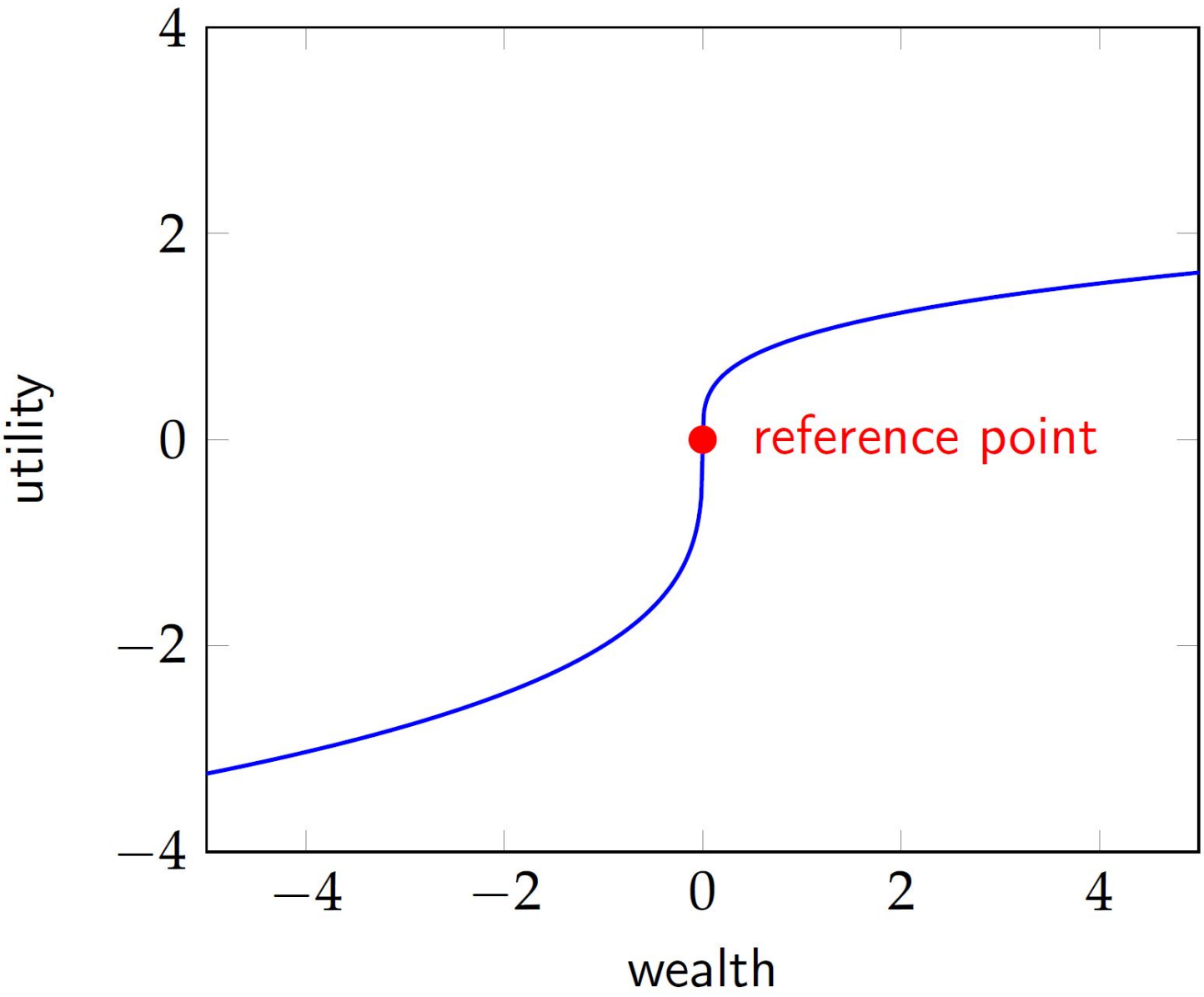
S-shaped utility captures 1 to 3



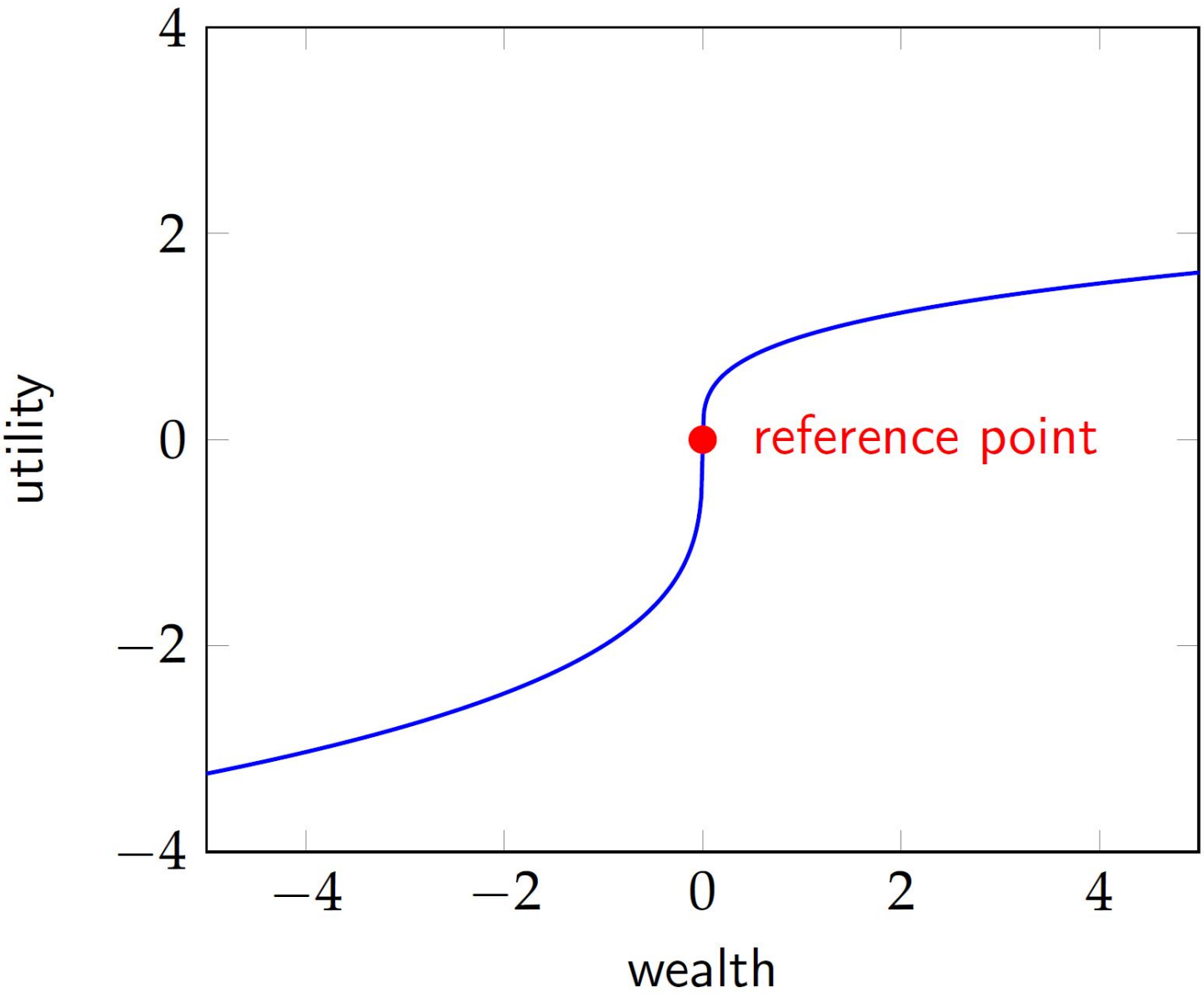
Diminishing Sensitivity over gains and losses

Diminishing sensitivity: Machiavelli already knew

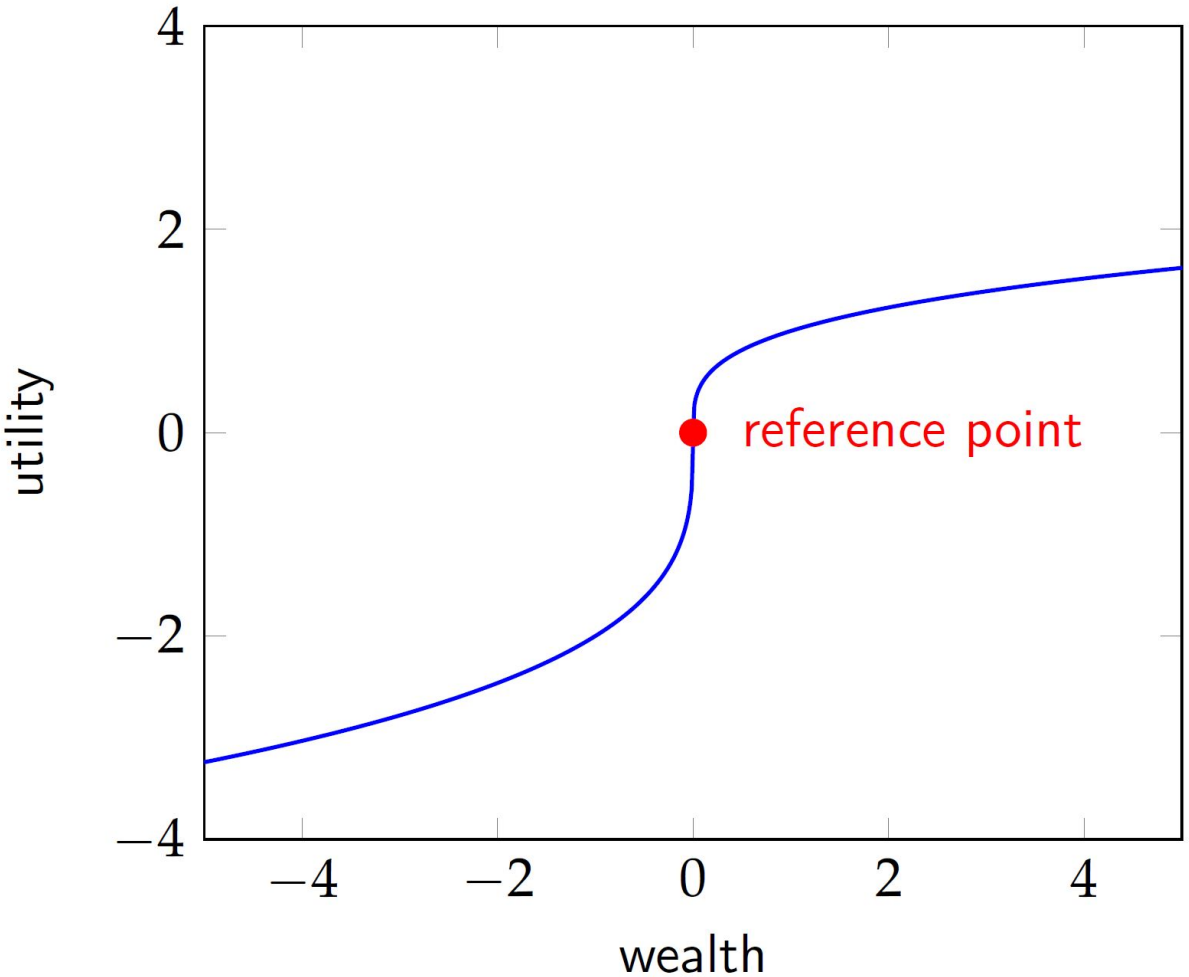
"[I]njuries ought to be done all at one time, so that, being tasted less, they offend less; benefits ought to be given little by little, so that the flavor of them may last longer."

Nicolo Machiavelli, Il Principe ($\approx$ anno 1513; Chapter 8).

The famous prospect theory S-shape again



Proposition:
Loss-averse S-shaped utility $\Longrightarrow$ 2SA



The famous S-shape gets it wrong: skewness aversion!

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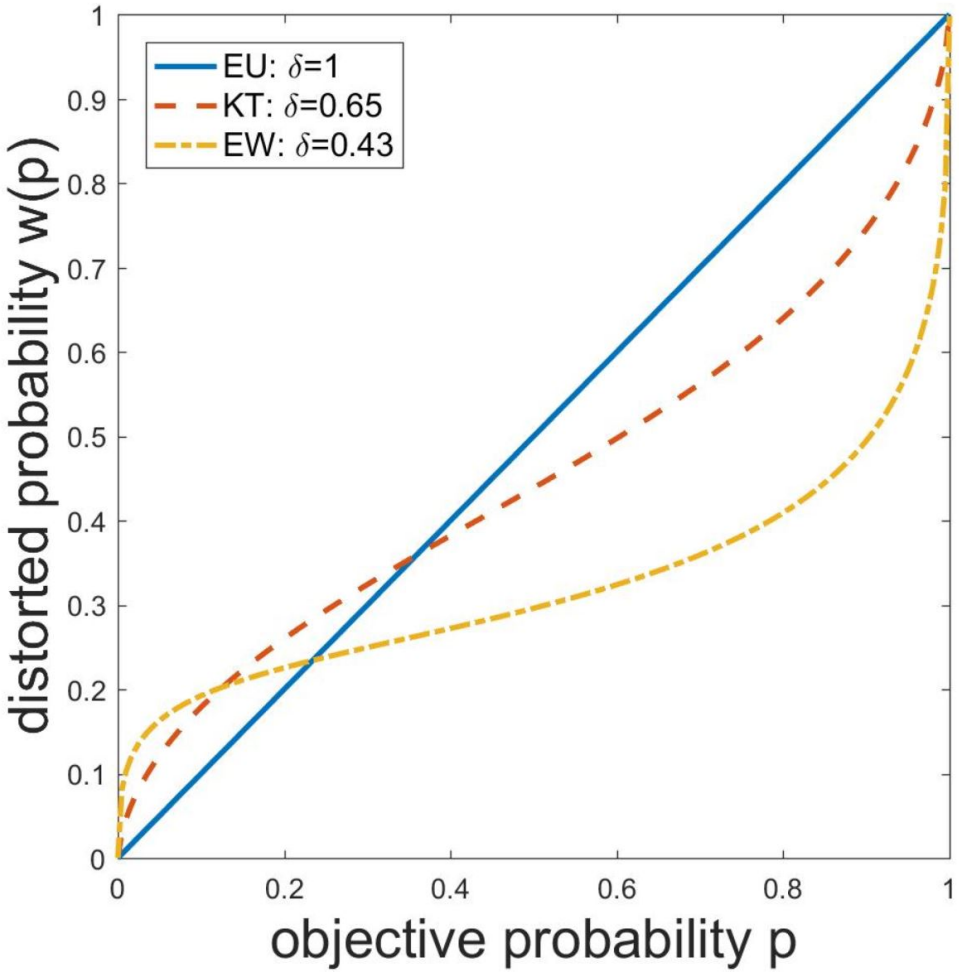
3. Diminishing sensitivity: The utility function is concave over gains and convex over losses
4. Probability weighting: Overweighting of small probabilities

Now we add prospect theory's idea 4: probability weighting

Probability weighting function

Probabilities p are distorted through a weighting function $w(p)$ such that small probabilities are overweighted:



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Proposition (Prospect theory preference flip).

1. Prospect theory without probability weighting implies SA.
2. Prospect theory with probability weighting implies SS.

If we had more time...

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Rank-dependent ST (always, unless ST = EUT)	1SS	Prop. 13
Continuous ST (always, unless ST = EUT)	2SS	Prop. 13
Optimal Expectation Theory (OET; Brunnermeier and Parker 2005)		
Always (unless OET = EUT)	1SS	Prop. 14
Optimal Anticipation with Savoring and Disappointment (OASD; Gollier and Mürmann 2010)		
Always (unless OASD = EUT)	1SS	Prop. 15

Agenda

Introduction

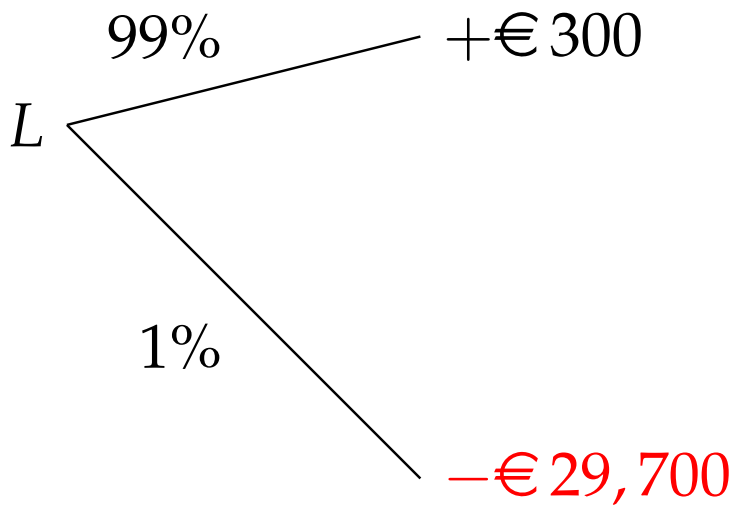
Skewness preference: A definition

Skewness preference in EU

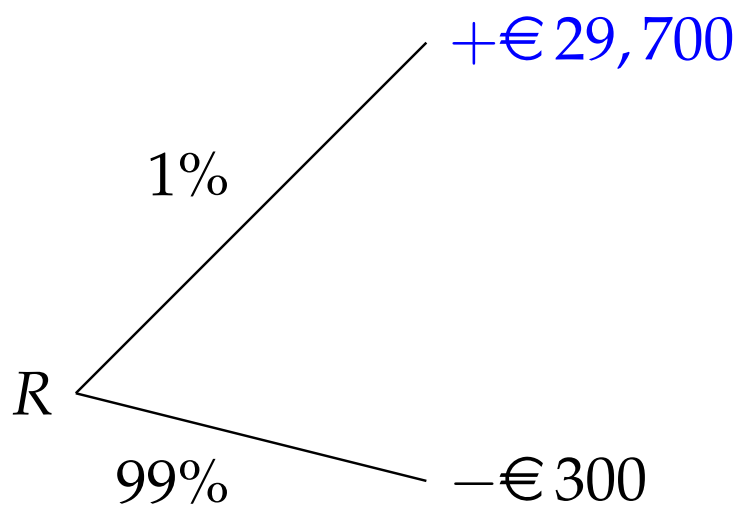
The skewness preferences induced by prospect theory

Conclusion

Skewness preferences determine choices over rare, high-impact risks:



(left- or negatively skewed)



(right- or positively skewed)

Conclusion

We propose

a definition of skewness preference and its order

and characterize it in important theories of choice under risk.

- EU is unable to induce strong skewness-seeking (Impossibility result).
- All successful behavioral theories do.

Conclusion: We need behavioral models to get skewness preferences right.