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Essays of Discrete Structures:

Purposeful Design of Grammatical Structures by Directed Stochastic Search

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ABSTRACT

This work presents a computational approach to the layout of discrete structures that incorporate practical design goals for routine and challenging design problems. The number of alternatives for the configuration of discrete structures that satisfy multiple design goals is quite large and the competition among design goals can make the relation between form and function unclear. Therefore, the objective of this work is a computational method capable of searching this ill-defined design space to generate innovative design alternatives that enhance creativity and provide insight into form-function relations for multiobjective structural design.

A grammatical approach to structural design is enabled by applying shape annealing, a computational design technique that combines a grammatical formalism (shape grammars) with directed stochastic search (simulated annealing), to the layout of discrete structures. A shape grammar is used to define a language of discrete structures through the specification of spatial design transformations that implicitly represent the relation between form and function in trusses. Two shape grammars, a planar truss grammar and a single-layer space truss grammar, will be presented. In order to generate purposeful designs from this language, an optimization model is presented that incorporates the structural design goals of efficiency, economy, utility and elegance.

Applying a grammatical approach to the design of discrete structures has resulted in the generation of structural essays where an essay of designs explores relations among design criteria and spatial forms in a particular structural design domain. Structural essays are presented for planar trusses, towers, pseudo-tensegrities, and single-layer space trusses that form domes and complex roof shapes. The purpose in generating structural essays is to provide the designer with a set of functionally feasible and optimally directed designs that explore the relation between form and function in the context of the design application. Additionally, presenting alternative design styles could enhance the creativity of a designer in conceiving a novel solution. Four main contributions result from this work: (1) a grammatical approach to structural design, (2) a computational design method for the generation of essays of innovative, discrete structures that reflect practical design goals, (3) a stochastic, discrete method for structural topology, shape and sizing optimization, and (4) a proof-of-concept of the shape annealing method as an effective method for design configuration problems.

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1. INTRODUCTION

The goal of this work is a computational design method for the generation of essays of discrete structures for both standard and difficult problems that reflect practical design goals. The purpose of generating structural essays is to enhance the creativity and insight of the designer by providing multiple, spatially innovative, yet functional solutions to structural design problems. The number of alternatives for the configuration of discrete structures that satisfy multiple design goals is quite large and the competition among design goals can make the relation between form and function unclear. Thus, the objective of this work is a computational method capable of searching this complex design space and providing insight into form-function relations for multiobjective structural design.

The shape annealing method, a design technique that combines a shape grammar (Stiny, 1980) with simulated annealing (Kirkpatrick, et al., 1983) to generate optimally directed designs, is applied to the configuration of discrete structures considering multiple design goals. A shape grammar is a means of defining a language of discrete structures through the specification of spatial design transformations that implicitly represent the relation between form and function in discrete structures. Considering that the language defined by the shape grammar is infinite, stochastic optimization is used to guide the generation of purposeful designs. This merger of design language specification and directed generation leads to the following thesis:

The combination of a grammatical formalism and directed stochastic search enables the generation of structural essays from a language of discrete structures.

Since multiple styles exist for solving any one design problem, and designer preferences are often unique, the design goals or the design language, which is defined by the shape grammar, can be varied to generate different styles of designs within a structural essay. Additionally, shape annealing utilizes lateral exploration to generate multiple design styles from the same language that are of similar quality based on the design goals. Structural essays for classes of structures including planar trusses, pseudo-tensegrity structures, and single-layer space trusses will be presented to demonstrate the capabilities of the shape annealing method in the design of purposeful, innovative discrete structures. The philosophy behind this work will be discussed in the context of the individual methodologies upon which it was based, configuration optimization, grammatical design and structural design to provide a foundation.

1.1 Methodology

Multiple design methodologies, configuration optimization, structural design, and grammatical design, have been united in this work (Figure 1.1). Bringing together configuration optimization and grammatical design results in shape annealing, a computational method for general network flow configuration problems that is capable of parametric and topology design optimization (Cagan and Mitchell, 1994). Joining configuration optimization with structural design results in structural optimization for discrete structures that reflect a broad range of practical design goals and constraints. Finally, combining structural design with grammatical design creates structural design languages for both architectural forms (Mitchell, 1991) and structural forms (Fenves and Baker, 1987). Mixing all three methodologies provides the means for the generation of essays of discrete structures where an essay is a set of designs within a structural language that explore solutions for a structural design application.[1] For example, an essay on transmission tower design would explore varying styles of designs that meet the design requirements and satisfy tradeoffs of different sets of design objectives. A structural language defines the space of design alternatives, or the design state space, whereas a model of structural design goals defines the design interpretation, or the semantic space that determines appropriate structures. Optimization is then used to search this vast space defined by the structural language to find not just feasible solutions but optimally directed, purposeful solutions.

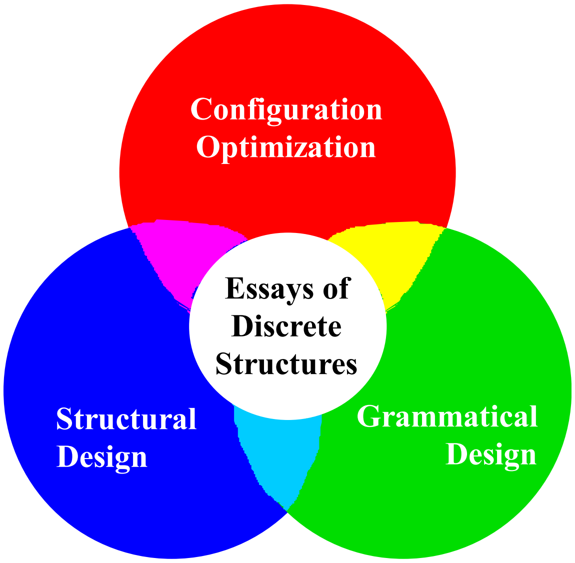

Figure 1.1: Mixing multiple design domain methodologies

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[1] An analogy is made to an essay in natural language which can be defined as a discussion of a restricted topic (Holman,1981).

1.1.1 Configuration Optimization

Configuration design (synthesis) is a search and discovery process for combinations of design objects that produce new effects to achieve global design goals, such as cost, reliability and quality (Pahl and Beitz, 1996). Based on this definition, keys to conceptual design tools are the combination of design objects in novel manners, the introduction of new design objects into the design, and the ability to evaluate the global design goals. Configuration design is a difficult portion of the design process since it requires a designer to develop potential alternative solutions to the design problem. Designers can often get trapped into alternatives based on previous experience and intuition that are not necessarily beneficial. The first motivation for this work is a computational method that provides the designer with a tool for the generation of conceptual configurations of structures such that they enhance problem insight and the range of solution possibilities considered by the designer.

The shape annealing method (Cagan and Mitchell, 1993 and 1994), a computational approach to the configuration design of network flow problems, is capable of meeting these objectives. Shape annealing combines a grammatical formalism to define the language of design alternatives and stochastic optimization to search this language for optimally directed designs. In this work a combination of shape grammars and simulated annealing is used. Optimally directed design is an approach to design optimization that directs the design generation towards the numeric range of a global optimum.[2] While known solutions in the language of designs may be simple to find, it is the possibility of generating innovative designs that is most appealing about this method.

The design representation used in the shape annealing method is based on an analogy to a network. A network is a means of representing design objects and the coupled relations between these objects, where design objects and their relations can have both functional and spatial attributes. In this work, a network applied to discrete structures represents structural shape, where the design objects are joints and the relations between joints are the flow of force through structural members. A network, and thus a structure, can be evaluated and optimized based on global and local design goals of design object and relation attributes and acceptable levels of system flow, that is, behavioral constraints. The application of shape annealing to structural design provides a demonstration of the capabilities of a grammatical approach to the configuration optimization of network flow problems.

Applied to structural configuration, the shape annealing method is quite different from most methods since it is capable of both discrete topology and geometry configuration. An important aspect of the shape annealing method is the introduction of new components in the design through the use of shape grammar rules. While structural configuration methods have been presented that lay out structures on a predefined grid, the topology and often the shape is limited to a design universe defined by the grid. Additionally, the applicability of grid-based topology design outside the domain of structural design is small since there are few design problems that can be broken down in this fashion. The shape annealing approach gains its strength in generality.

Considering that design configuration is a series of transformations, the combination of topology with shape and sizing optimization makes the design state space, at the transformation level, three-dimensional. A pictorial description of the design transformation state space is shown in Figure 1.2. The axes represent the possible design transformations and are not indexed so as not to imply any sort of ordering of designs. If a given topology, topology 1, at design state 1, is transformed by sizing, the design moves along the topology plane to state 2. Likewise, if design state 1 is transformed by shape transformation, the design moves to state 3. But, if the topology of design state 1 is transformed, the new design, state 4, exists on a new plane of shape and sizing transformations.

The configuration design process could be considered hierarchical such that topology layout occurred on one level and parametric design of a fixed topology on another. It seems though that due to the complexity of the design space it is necessary to combine the topology, shape and sizing design transformations in order to enable the generation of a global optimum. Applied to structural topology optimization, a change in topology may change the design space, that is the design space in terms of shape and sizing, such that the optimal solution may be a singular point, which may be impossible to reach by numerical search algorithms (Kirsch, 1993, p.267). One goal in applying the shape annealing method to structural configuration optimization is to provide a means of searching for such optimal topologies through the combination of topology, shape and sizing transformation rules and stochastic optimization.

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[2] This definition of optimally directed design is different than that presented by Cagan (1990) who used the term to mean the region around the optimum.

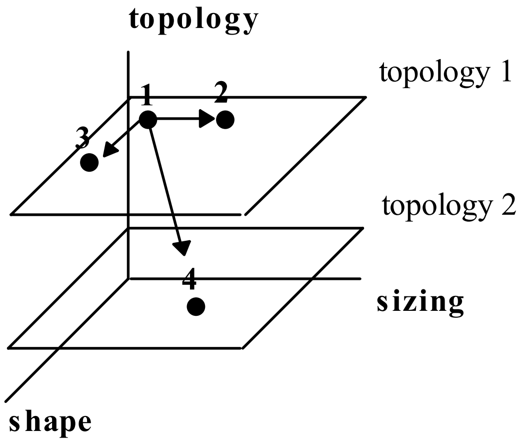

Figure 1.2: Pictorial description of the design state space for discrete structures

1.1.2 Structural Design

Structural design of discrete structures is the configuration of structural elements into forms that both control forces and provide visual impact. A discrete model for structures considers a design as the composition of structural elements that can be straight-line elements (truss or frame), surface elements (plate or shell), or solid elements (Schodek, 1980). The class of discrete structures considered in this work is truss elements that can be broken down into categories of planar or curved structures. The purpose of a truss structure can range from a purely utilitarian structure to a sculpture and, due to this latitude of design objectives, are products of both architects and engineers. Since all structures are both physical objects as well as visual objects, behavioral and aesthetic design goals must be considered. The second motivation for this work is a computational method that supports the design of structures considering a broad range of design goals in order to support the design intentions of both civil engineers and architects.

Structural design can be described as the ability to conceive innovative and appropriate structural solutions to both familiar and new problems that balance efficiency, economy, utility and elegance (Billington, 1983; Addis, 1990). The motivation for the formulation of the optimization model to be used within shape annealing is one that embodies this definition of structural design. Largely, the development of computational structural design tools has focused on deterministic structural optimization methods that provide one maximally efficient design solution to a specified problem based on behavioral and geometric constraints. This limited scope of structural design goals does not consider economic factors involved in realizing the design, design usage, or a design's visual elegance. Since structural design is more than iterative analysis, an effective computer tool to aid the designer must take into account the design goals considered in the conceptual design stage. Through the creation and incorporation of computational models for a robust set of structural design considerations, the structures generated will be more appropriate to the design problem. The motivation behind each structural design goal will be addressed in Chapter 3.

The optimization model defines the semantic space that describes a set of purposeful designs for a structural design application. The semantic space provides for the interpretation of designs existing in the design state space in terms of the individual goals of the designer (objectives) and the requirements of the design problem (constraints). The structuring of the semantic space is determined by the relative importance among the design objectives and constraints and is used to rank designs. By modifying either the set of design objectives and constraints or the relative importance among them, the interpretation of a designs is changed such that the optimization will be guided towards the generation of different designs. Since multiple design states can satisfy the same semantic model, there is a one to many mapping between points in the semantic space and points in the design state space. It is this relation between design states and semantic states that provides for multiple similar quality designs to be generated for the same design problem. A semantic model provides a means of interpreting and comparing numerous design alternatives within a design language.

1.1.3 Grammatical Design

A computational method for structural configuration design must also incorporate a powerful parametric representation of structural form and function and the relation between the two. This can be achieved through the use of a shape grammar to define a set of shape transformations that embody the relationship between form and function for a defined set of structural elements. Grammars lend themselves to configuration design problems since they provide a concise representation of design transformation knowledge, yet when transformations are applied iteratively, can provide an infinite number of design alternatives that compose a design language. The third motivation for this work is a grammatical approach to structural design.

A language can be described as a system of symbols used in a uniform manner that enables intelligent communication (Barnhart, 1960). Just as a natural language enables the communication between two people, a structural language can enable the configuration and understanding of structural designs. Structural languages have been used in the past to describe the composition and style of architectural form (Mitchell, 1990) and here will be used to describe a language of discrete structures.

A grammatical approach to design views the design process as a series of transformations where these design transformations model the underlying principles needed to produce a certain style or type of design. The enumeration of all allowable transformations results in the definition of a design language. A formal language is composed of four parts: alphabet, vocabulary, grammar and word phrase. Following this structure, a design language is defined by a set of design objects (alphabet) that are transformed according to a defined set of grammar rules to generate designs (word phrase) in the language. A vocabulary is a useful mechanism to classify sub-designs according to design attributes. For example, in the language of architectural design that Mitchell (1990) describes, lines (alphabet) are transformed to produce compositions of shapes that describe classes of architectural elements, such as a vocabulary of columns or windows. Elements from these vocabularies can then be combined to configure architectural designs.

In the design of discrete truss structures, although as in an architectural language the alphabet consists only of lines, the vocabulary is much simpler and consists only of a parametric triangle. In a parametric vocabulary two configurations of structural members can have the same topology but a different functional and visual interpretation based on the parametric properties of the configuration. This is similar to homophones found in natural languages, which are words with different meanings but the same sound. On the auditory level, homophones are equivalent, but on the functional level the words are distinct. Thus, the parametric properties (relative positions of the members, dimensions of individual members and material properties) of structural configurations provide distinctions between vocabulary elements that can describe an infinite vocabulary of shapes. A language of discrete truss structures is quite like the artificial languages of binary or Morse code both of which are comprised of a limited number of symbols that rely on the relations between symbols to express meaning. Although at a symbolic level only one symbol is used in the language of discrete truss structures, it is the infinite vocabulary of shapes and the combinations of these shapes that result in an infinite language of designs.

1.2 Essays of Discrete Structures

With the shape annealing method, essays of designs are created through the combination of grammatical generation of structural designs and recognition of functional and aesthetically pleasing forms. While a design language defines an infinite set of designs for a class of structures, an essay of designs is used to present a set of designs appropriate for a particular problem domain within that class. Essays of structures explore alternative design styles that satisfy functional and visual design goals to provide insight into the form-function relations of discrete structures. The essays that will be presented in this work are classified as either formal or informal. Formal essays will be used to present design problems from the perspective of structural design as an optimization problem while an informal essay will be used to explore the generation of novel solutions.

Style plays a large part in the design of structures just as it does in the use of a language. In natural language, the same language can be used to express poetry and to communicate laws;

while one is artistic the other is functional. Similarly in structural design, there are structures designed primarily for technical merit such as bridges and towers, where their beauty relates to their functional prowess, and there are structures designed for their expressive powers and can be considered poetic structures. Style can be incorporated in an essay of designs either by limiting the design language through syntactic constraints added to the grammar rules or as semantics used in the interpretation of designs to guide the optimization.

To explore the tradeoffs among design objectives, the analogy between languages and structures is useful to express the idea that structural design is a balance among often competing ideals. While a piece of technical writing can be written in the most functional manner such that not a word is used in a superfluous fashion, technical writing with a hint of humor and wit can be a joy to read without loosing sight of the technical contribution. The converse is also true, poetry can be very free flowing and expressive but certain rules must be adhered to in formulating the poetry such that it can be understood and appreciated. Just as in writing, in structural design the idea is to seek a balance between functional communication and visual expression such that the message to be conveyed is not only understood but can also be appreciated. An essay of structures can be used to explore the balance among design goals in the generation of purposeful structures.

An essay of designs presents alternative ways of satisfying design requirements and objectives and the resulting spatial forms to the designer for further evaluation. Similar to linguistic understanding, the interpretation and understanding of an essay of designs varies from designer to designer depending on their background. For a structural designer, generally functional efficiency and clarity are primary design criteria and a structure will be appreciated if its form intuitively implies its function. On the other hand, an architect, whose eye is generally looking for visual design effects, can interpret the same design differently based on the impact of the structure on the surrounding space. Incorporating multiple perspectives into an essay of structures, the use of different optimization models for the same problem will allow an essay of structures to explore a design problem from different viewpoints.

1.3 Method Application

In this work, a method for the generation of structural essays that incorporate practical design goals will be presented to provide a foundation for a conceptual design tool for the design of discrete structures. While the number of computer tools that support the designer in the embodiment design stage are abundant, methods and tools to support the designer in the conceptual design stage lag behind. This gap can partly be attributed to the fact that the integration of conceptual design tools into the design process requires designers to make a paradigm shift in the way they approach design. In the past we have seen this shift slowly take place, for instance with the integration of analysis tools into the design process, and we will continue to progress in due time. In order to provide computer tools for conceptual design, the proper design methodology and computational methods must be developed to provide the foundation for computer tools that are capable of not only meeting the designer's needs but also expanding on their capabilities. Potentially, the greatest impact that computer tools can have on a designer is in the conceptual stage of the design process since at this point the designer is most open to new ideas while searching an often limitless space of design alternatives for novel and purposeful solutions. In structural design, while a CAD tool provides a designer with a means of expressing design concepts through drawings and an analysis tool can pinpoint behavioral flaws for modification, a conceptual design tool could provide whole new avenues for the exploration of innovative structures. The primary goal of a computational method for the conceptual design of structures is to enhance the creativity and insight of the designer that in turn could impact the quality and ingeniousness of the designer's solutions.

1.4 Thesis Outline

The following topics will be presented to illustrate the shape annealing method and its capabilities for the design of discrete structures.

- Chapter 2- A review of related work in both structural topology optimization and grammatical design will be presented.

- Chapter 3 - A description of the shape annealing method applied to structural design will be presented including the development of shape grammars for planar discrete trusses and single-layer space trusses. Additionally, the formulation of an optimization model for structural design will be discussed in terms of the design goals of efficiency, economy, utility and elegance.

- Chapter 4 - The implementation of the shape annealing method will be presented that includes the algorithm, computational representation, modified Lam-Delosme simulated annealing schedule, Hustin move set, a special technique for the selection of grammar rules, and dynamic normalization of the cost function. An additional method will be presented for the dynamic grouping of members by cross-sectional area or length that results in designs with a limited number of discrete sizes.

- Chapter 5 - This chapter is the first of two chapters that will present essays of discrete truss structures. Essays of planar trusses will be shown that incorporate the design goals of efficiency and economy in the generation of cantilever trusses, simply supported and fixed trusses, and transmission towers. In addition, an exploration of pseudo-tensegrity structures will be presented.

- Chapter 6 - The second group of structural essays present design applications of single-layer space trusses. Dome design will be used to explore the impact of the set of design goals (efficiency, economy, utility and elegance) on the resulting structures. An aesthetic model based on the golden ratio will then be developed and applied to the design of roof structures. Single-layer space trusses will be explored for practical roof applications that include a pool, gallery, and an octagonal airplane hanger.

- Chapter 7 - A comparison between roof truss designs conceived by human designers and an essay of roof designs generated by shape annealing will be made. In addition, the development of a practical computer tool for structural design using the shape annealing method will be proposed.

- Chapter 8 - The concluding remarks will present a summary of the work, the contributions made, and discuss method extensions and future applications.

1.5 Summary

A computational method for the design of discrete structures will be presented from an application of the shape annealing method to structural design. This method will be demonstrated on the design of both planar and three-dimensional discrete truss structures that balance practical design goals. This demonstration of the shape annealing method will achieve two goals: verification of the method's capability in generating innovative, purposeful designs, and the foundation for a computer tool that could serve to enhance the creativity and insight of designers through the generation of essays of discrete structures.

2. BACKGROUND

Related work can be divided into two categories: structural optimization and grammatical design. Structural optimization problems focus on finding a least weight design by defining the topology, shape and member sizes for a specified loading subject to behavioral constraints and in some cases geometric constraints. The discussion of grammatical design will review design grammars that define a language of designs by specifying the allowable design transformations and generative grammars that focus on searching a design language for feasible and optimal design alternatives.

2.1 Structural Optimization

The general structural layout problem has been investigated as early as Michell's (1904) analytical work that is based on Maxwell's theorem (1864) for optimal structural layout of single purpose truss structures. A detailed discussion of the theory of structural layout including both Maxwell's and Michell's work can be found in Cox (1965). The review of related methods will be divided based on the representation type, discrete or continuous material, and the optimization method employed, deterministic or stochastic. Deterministic methods provide one optimal solution from a specified starting point while stochastic methods can provide multiple near-optimal solutions. The differentiation between discrete and continuous material representations is that discrete representations assume a layout is composed of a particular structural element type, such as a truss or beam, while a continuous representation fills a defined space with material. This material then forms a structural type, such as a truss, during the optimization as a result of the required behavior and the desired percentage of material reduction.

Methods for structural optimization focus on minimizing the amount of material required for a specified performance where the design variables can include member sizes, geometry and topology. Three types of structural optimization problems exist: (1) the optimal sizing of a fixed geometry and topology layout (sizing optimization), (2) the optimal sizing and geometry of a fixed topology layout (shape optimization), and (3) the optimal sizing, shape and topology of a layout (topology optimization). The related work shown will focus on methods for topology optimization to identify where shape annealing, a discrete, stochastic method, adds to the current structural optimization methods. A survey of discrete sizing methods will also be presented to provide context for the dynamic grouping method used in shape annealing for discrete sizing. It must be noted that the methods reviewed here are just a sampling of the prevalent methods in the field of structural optimization since the amount of literature in this area is extensive. For this reason, review papers will be referred to as good sources of comprehensive information in specific domains within structural optimization.

Discussing topology optimization research from the point of view of meaningful research for practical engineering applications, Rozvany (1995) classifies solutions according to the type of optimization problem posed, practical problems or artificial problems, and the type of solution obtained, exact analytical solutions or approximate numerical solutions. Since exact solutions to practical problems are unknown in most cases, he states that the most useful research presents approximate solutions, that is numerical solutions, to complex problems. Practical problems are defined as optimization formulations that minimize an objective function subject to behavioral constraints whereas artificial problems optimize compliance, which he states, as an engineer, has little meaning in practical design.[1] In this context, shape annealing presents an approach to topology optimization that generates approximate solutions to practical design problems.

2.1.1 Deterministic Methods

A presentation of deterministic methods, including both distributed and discrete representations, for structural topology optimization can be found in Bendsoe (1995).

2.1.1.1 Distributed Material Methods

Distributed material optimization methods approach the layout problem representation as a continuum of material broken down into a grid of elements. Among the distributed material methods much work has been done with the homogenization method that discretizes a specified space into finite elements. A material density function is then prescribed over the defined space and a constraint is placed on the percentage of material used in this space that effects the type of solution generated. If this value is high, plate type solutions emerge, whereas if it is low, truss or beam type solutions emerge. The optimal density of each element is determined from the stress limit resulting on the principal stresses (Bendsoe and Kikuchi, 1988; Bremicker, et al., 1991; Diaz and Belding, 1993).

For practical design problems the most common drawback of the homogenization approach is that checkerboard patterns often emerge since the density of each discrete element varies. This poses the question of how to interpret the resulting distribution of material such that it can be manufactured. Addressing this problem the homogenization method was adapted for practical structural design (Chirehdast, et al., 1994) through the development of a three-phase method, called ISOS, that generates parameterized, manufacturable objects. ISOS uses homogenization to determine the optimal material distribution that is interpreted using vision

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[1] There is some discrepancy here since Yang and Chahande (1995) present a tool developed at Ford for topology optimization in automotive design, a practical problem, that uses a minimum compliance problem formulation.

algorithms and parameterized to define a finite element model that is then further shape optimized.

A continuous material method that varies from homogenization is a simulation of adaptive bone mineralization used to generate structural topologies (Baumgartner, et al., 1992) where again the relation to a manufacturable object is problematic. A skeleton-based method has also been presented by Stal and Turkiyyah (1996) that moves between topology design spaces through the introduction of additional design variables in the model.

2.1.1.2 Discrete Topology Methods

Rather than specifying the design space as a continuum of material, discrete topology optimization methods predispose the design solution to a structure that is made up of structural elements, either truss, beam, plate, shell or solid. The methods reviewed here present solutions to the truss topology optimization problem. Most discrete topology optimization methods solve a sizing optimization problem that uses a highly connected ground structure (Bendsoe, et al., 1994; Hemp, 1973). With these methods topology changes occur as the result of a member reducing in cross-section to a specified minimum area based on behavioral limits such that it can be removed from the structure. Pederson (1993) presented a similar method for a reduced three-dimensional ground structure. The primary disadvantage of these methods is the strong dependence of the resulting design to the ground structure upon which it was based (Bendsoe, 1995).

The previous grid-based methods reviewed only considered the sizes of members as design variables while keeping the joint positions in the ground structure fixed. One approach to including shape variables in addition to sizing variables is to create a hierarchical optimization problem such that the optimal topology is found and is then shape optimized. Methods that use a hierarchical approach to topology, shape and size optimization have been presented by Bendsoe et al. (1994) as well as Kirsch (1993) who presents an interactive tool for hierarchical truss topology design. While this is a more robust approach than fixing the grid node locations, the fundamental problem is that the space of design alternatives is limited since the coupling effects among topology, shape and sizing transformations are not considered.

To solve discrete topology methods that include simultaneous optimization of joint positions in addition to sizing variables, Rozvany and Zhou (1991) and Ben-Tal et al. (1993) have formulated ground structure approaches for two-dimensional structures. An interactive tool developed for ground structures in both two and three-dimensions was developed by da Silva Smith (1996). Similar to a ground structure, Dorn (1964) presented a method that defines the ground structure by a grid of allowable nodes upon which to lay out members. The most general approach presented for the discrete topology problem was by Spillers (1975) who moved away from defining a design universe by using a heuristic to introduce new members. The drawback to this method, especially considering the computational resources in 1975, was that the method depended on exhaustive topological search from every design state.

Additional deterministic methods for both discrete and continuous topology optimization can be found in (Bendsoe and Soares, 1993). Informative reviews of discrete topology optimization problems can be found in (Kirsch, 1989) and (Topping, 1993).

2.1.2 Stochastic Methods

In contrast to deterministic methods, stochastic approaches have been developed for topology layout that allow discontinuous objectives and constraints to be used in the optimization model since stochastic methods do not require gradient information. There is significantly less literature on the use of stochastic methods in structural optimization since these methods do not guarantee that an optimum will be found and at best only produce near-optimal solutions. However, for difficult design problems it is questionable as to whether the optimal solutions that are constrained to lie on a predefined grid are better than the near optimal solutions of stochastic methods. The stochastic methods reviewed here use either genetic algorithms or simulated annealing.

2.1.2.1 Continuous Methods

An approach similar to homogenization has been developed to generate optimal topologies using a genetic algorithm (Chapman, et al., 1994). This approach differs from homogenization in that rather than allowing the density of each element to vary continuously the elements can only have a density of either zero, which implies a hole, or one, which implies the existence of material. An approach that uses the same discrete density scheme but is combined with simulated annealing optimization has been presented for part design that considers part performance, manufacturing and material cost (Anagnostou, et al., 1992).

2.1.2.2 Discrete Methods

The class of stochastic, discrete methods is the class of structural optimization that shape annealing fits within. There is one other method in this category that optimizes a given topology using simulated annealing where changes in topology can occur by removing members and joints during the optimization process (Topping et al., 1993). Both discrete and continuous sizing variables are implemented. In contrast to shape annealing, with this method members and joints can only be removed from the structure and not added.

2.1.3 Discrete Sizing of Structures with Fixed Topology

A review of the literature in discrete sizing will be presented in order to provide context for the dynamic grouping method for sizing that will be presented in Chapter 4. The methods reviewed investigate the problem of sizing structures of a fixed topology using discrete member sizes picked from an allowable set. In this problem formulation, the topology varies by allowing a member to have zero area, implying that the member can be removed; no new members are introduced. The difference between work in optimal sizing and that of ground structure based topology optimization, which uses the same problem formulation, is the size of the initial topology. In methods for optimal sizing the initial topology is close to the size of the actual desired topology of the structure as opposed to the large ground structures used for topology optimization.

A review of numerical methods for discrete sizing including branch and bound, approximations using branch and bound and ad-hoc methods can be found in Vanderplaats and Thanedar (1991). A mixed integer formulation to the discrete sizing problem where the allowable set of sizes contains only integers can be found in (Ghattas and Grossman, 1994). Approaches using genetic algorithms to perform discrete sizing from an allowable set are (Grierson and Pak, 1993) and (Rajeev and Krishnamoorthy, 1992). None of the above-mentioned approaches have been applied to the problem of generating optimally directed structural topologies while dynamically assigning members to groups based on cross-sectional area. The shape annealing method will be shown to be capable of solving this problem.

2.1.4 Multiobjective Methods

Multiobjective approaches to structural sizing design have also been developed that trade off behavioral attributes including stress, buckling, displacement and frequency while searching for the best compromise design (Eschenauer, et al., 1990; El-Sayed and Jang, 1991; Grandhi, et al., 1993).

2.1.5 Commercial Tools

Two computational topology optimization tools for continuous material layout of structures are OptiStruct and TOP. OptiStruct is the first commercial software package for structural topology optimization developed by Altair Computing Inc. and uses the homogenization method (Brennan, 1994). TOP is an in-house tool developed at Ford that combines mathematical programming methods with finite element analysis using MSC/NASTRAN (Yang and Chahande, 1995). Both tools are limited to a continuous material representation and local, deterministic optimization. These tools are best suited to the design of least weight monolithic parts.

2.1.6 Placing Shape Annealing in the Proper Context

Many methods have been reviewed for structural topology optimization. The most prevalent continuous material method, homogenization, has been applied successfully to the generation of monolithic parts in design domains such as the automotive and aerospace industries where structural efficiency is the primary goal. In the design of structures that have both functional and visual interaction with humans the design goals are broader and cannot be modeled solely by physical laws making a discrete representation advantageous for adding problem specific design knowledge. Adding stochastic optimization to a discrete representation, as in shape annealing, enables the optimization of discontinuous design objective functions that result from discrete changes in topology and the modeling of practical design goals. The common ground for homogenization and shape annealing is planar truss design for maximal efficiency as a comparison was shown in Reddy and Cagan (1993).

Comparing the shape annealing method to the discrete methods reviewed, the main advantage of shape annealing is the introduction of new members and joints in the design such that the topology space is sufficiently explored without having to define a large design universe of possible connections. A benefit of a grammatical representation is the ability to control the geometric transformations of shapes through grammar rules. This will be illustrated by imposing a proportional system on the layout of a structure to meet visual design goals. The shape annealing method gains much advantage in its generality of the type of member that can be represented and the design goals and constraints that can be used in the optimization model. Of course the price for this generality is that only near-optimal solutions can be found since a stochastic optimization method is employed. However, for conceptual design of structures, “near-optimal” solutions are often sufficient.

2.2 Grammatical Design

Grammars were developed as a production system that specifies a set of designs, called a language, by the transformations required to generate that set (Stiny and Gips, 1980). For design problems, the interest in specifying a set of alternative design solutions is that the set can then be searched to determine optimal designs within that set. Applied to spatial design, a shape grammar defines the allowable transformations of shape, either with fixed dimensions or parametric, that can be used to generate a language of spatial designs (Stiny, 1980). Since their introduction, shape grammars have been used extensively in architecture to define languages of architectural form and style; examples are Palladian villas (Stiny and Mitchell, 1978) and Queen Anne houses (Flemming, 1986). The advantage of using a grammar as a design production system is that the language of designs defined by the grammar can be used to generate both known designs, from which the grammar was derived, and new designs in the same style.

The application of a shape grammar formalism to spatial and functional design requires either implicit or explicit specification of function in the grammar rules. Grammars that define architectural languages use form-function relations encoded in the shape grammar that imply a design object’s function from its shape. Grammars for the generation of solid models also use implicit function to define valid solids (Fitzhorn, 1990; Barnard, et al., 1993; Heisserman and Woodbury, 1994).

Since an object's function cannot always be implied from its shape, function can also be explicitly described in a grammar, called a functional grammar, through the use of labels and symbols. Functional grammars most often do not allow for spatial emergence and thus are a type of set grammar (Stiny, 1982). Mitchell (1991) presents functional grammars as shape grammars that are limited to the generation of both realizable and functional designs and illustrates this point with an example of a functional grammar for the design of primitive huts. A functional grammar has also been presented for the conceptual design of structures using architectural and structural critics to guide the design configuration (Fenves and Baker, 1987). Similar to functional grammars in their purpose, attribute grammars, a string grammar with a set of attributes that describe parametric and behavioral properties which are attached to every symbol, have been used for the configuration of discrete planar structures (Rinderele, 1991). A bond graph grammar has also been presented for the form and function configuration of mechanical systems (Finger and Rinderle, 1989).

The grammatical formalisms mentioned up until this point have primarily focused on the definition of a language of designs, except in the case of Heisserman and Woodbury (1994) who implemented the Queen Anne grammar. Generative grammars use a grammatical formalism to define a design language and a control mechanism to computationally search this language for optimal solutions. Applying grammatical formalisms to practical design configuration problems presents issues of the proper formulation of syntax, semantics and generation control as outlined by Brown (1997). In mechanical design, grammatical approaches to the generation of optimal mechanical systems have been developed for serial configurations based on string grammars (Schmidt and Cagan, 1997a) and coupled configurations based on graph grammars (Schmidt and Cagan, 1997b). Generative grammars have also been used in the generation of optimized process plans for the machining of designs defined by a language of machineable parts (Brown et al., 1995; Brown and Cagan, 1997). Additionally, a generative system has been created for the spatial layout of buildings (Flemming et al., 1988).

2.3 Summary

After reviewing the relevant research, the contributions of the shape annealing method in the domain of discrete truss topology optimization can be seen as: a discrete, stochastic structural optimization method for topology, shape and sizing design capable of expanding the design space through the introduction of new members and joints, handling multiple, discontinuous objectives, and performing discrete and continuous sizing. In the domain of grammatical design, the grammar used in this work presents a generative, parametric shape grammar that does not allow emergence and defines function implicitly in the formulation of the shape rules. Additionally, the application of grammatical design to discrete structural layout illustrates the use of syntax and semantics to achieve the functional and visual goals of structural design and simulated annealing as a means of generating purposeful designs.

3. SHAPE ANNEALING APPLIED TO DISCRETE STRUCTURES

This chapter will present an application of the shape annealing method to the design of discrete structures. First, a shape grammar will be presented for the generation of valid discrete structures based on the form-function relation described by Maxwell's rule for the stability of pin-jointed structures. Next, an optimization model will be developed based on a set of ideals for structural design that incorporate functional and visual design criteria.

3.1 Grammatical Design and Shape Grammars

A shape grammar is a means of defining the allowable transformations of shape, which are embodied in grammar rules, to generate a spatial language of designs (Stiny, 1980). Applied to functional design, shape grammars can be used to specify the desired form-function relations for the spatial layout of functional systems (Mitchell, 1990). Formally, a shape grammar is defined by a four-tuple: S, a finite set of shapes, L, a finite set of symbols, R, a finite set of shape transformations, and I, the initial shape. This four-tuple defines the language of spatial designs that can be generated from the grammar. For truss design, the set of allowable shapes consists only of a triangle where each line in the shape represents a truss member. The rules of the grammar are then formulated as shape transformations of triangles by dissection, addition and modification such that the design is always comprised of a set of triangles. Since a design is represented as a set of shapes and not as lines in space, the design can only be interpreted one way and thus does not allow for spatial emergence. This type of grammar is a special case of a shape grammar called a set grammar (Stiny, 1982). In this formulation, the shape grammar rules are parametric where the topology and labels of the shape are matched without concern for geometric information unless rule syntax for geometric constraints is specified. The initial shape is determined from a minimal connection of members between the applied load points and the support points of the problem specification.[1]

The application of shape grammars to functional design requires inherent form-function relations in the grammar rules in order to generate valid truss structures that can be analyzed. This can be accomplished implicitly by the spatial form of the rules or explicitly by the addition of syntax to grammar rules using labels. Labels can be attached to points, lines and shapes in the grammar such that the application of a rule is limited to a certain design case. For structural layout, labels can be used to incorporate behavioral and aesthetic constraints in the design

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[1] The topology of the initial structure has a minimal influence on the final design since with simulated annealing there is a high probability of accepting inferior designs at the beginning of the exploration process such that the design will always move away from the initial structure. However, through the use of reverse grammar rules the initial structure may be revisited.

generation. For example, a label attached to a point can be used to represent the boundary condition of the analogous joint for analysis purposes or a constraint on the allowable spatial transformation of the point to suit aesthetic or economic design constraints. Labels can also used to indicate the behavior of a member based on feedback from the design analysis. In the designs presented, rule syntax has been used to formulate geometric constraints that ensure the generation of designs with specified geometric properties such as a minimum angle between members, minimum member length, and desired aesthetic proportions.

Applying a shape grammar to the configuration of functional systems requires functional knowledge that must be placed either in the grammar rules, which generates the form of a design, or in the optimization model, which interprets this form. The control mechanism used with a shape grammar determines where knowledge is best placed; a rule-based configuration method places most of the knowledge in the rules while a generate and test method generally places knowledge in the test mechanism[2]. Since simulated annealing is a generate and test control mechanism, the purpose of the grammar is to define valid topologies that can be interpreted in a meaningful way. This places only base level knowledge in the grammar itself whereas higher level knowledge is incorporated in the optimization model. Placing the knowledge in the optimization model is advantageous for two reasons: (1) unbiased exploration of topologies that could lead to the generation of innovative forms, and (2) a single grammar can be applied to different problem domains, within the class of structures defined by the grammar, and used with different optimization models. While at first it was thought that grammars with problem specific syntax would improve the capability of the shape annealing method in generating optimally directed, purposeful designs, it was found that not only are specialty grammars not helpful but in some cases restrict the design generation. This point will be discussed further in Chapter 8.

Two grammars, a planar truss grammar and a space truss grammar, have been developed for the design of discrete truss structures. Space trusses, often called space truss structures or just space structures, are three-dimensional truss structures. The rules of the shape grammar are divided into geometry and topology modification rules where the lines in the grammar represent truss members and thus only truss topologies can be generated (Figure 3.1). The allowable geometric modifications include the shape modification rule that changes the location of a single joint in a design and the size modification rule that changes the cross-sectional area of a single member. Together these rules with shape annealing perform shape and sizing optimization of a structure with a fixed topology.

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[2] A continuum exists between rule knowledge and the knowledge level of the control mechanism: a generate and test method with a high level of knowledge in the generate stage can become more like a rule-based configuration method.

Since the purpose of the grammar is to define the set of valid truss structures, topology rules are formulated using the form-function relation found in Maxwell's rule for the stability of pin-jointed structures (Maxwell, 1864):

$$n = r + b - (d*j), \qquad \text{Eq. 3.1}$$

where: n = degree of determinacy ($n < 0$ => mechanism; $n = 0$ => determinant; $n > 0$ => indeterminant),
r = number of reaction forces,
b = number of bars,
d = degrees of freedom (for 2D: $d= 2$; for 3D: $d= 3$),
j = number of joints.

While this is not a sufficient condition of stability it is a necessary one. It is possible for a structure to obey Maxwell's rule and be unstable due to geometric instabilities. The implication of this rule on truss layout is that for planar structures in order to not alter the determinacy of the existing structure two members must be added for each node added. Similarly, for a three-dimensional structure, three members must be added for each node added. However, if the structure is indeterminant, additional members can be added to the structure to decrease the number of redundant members in the structure while still obeying Maxwell's rule.

Following Maxwell's rule for planar truss layout, topological modification of a structure can be made by either dividing an existing shape (rule 1), or adding a shape to an exterior member in the design, denoted by the label 'f', (rule 3); see Figure 3.1. Note that topology rules are created in pairs so that any modification can be reversed, except for rule 5, which is its own reverse. The application of a topology reversal rule is monotonic and in theory any previous design state can be reached through the sequential application of reverse rules[3]. The grammar rules are fully parametric and can incorporate additional geometric syntax based on considerations of good structural design such as: (1) no two members may intersect without a joint, (2) members cannot overlap, and (3) a specified minimum angle between members must be maintained.

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[3] Spatial emergence does not exist in the representation presented so that any rule is always monotonic.

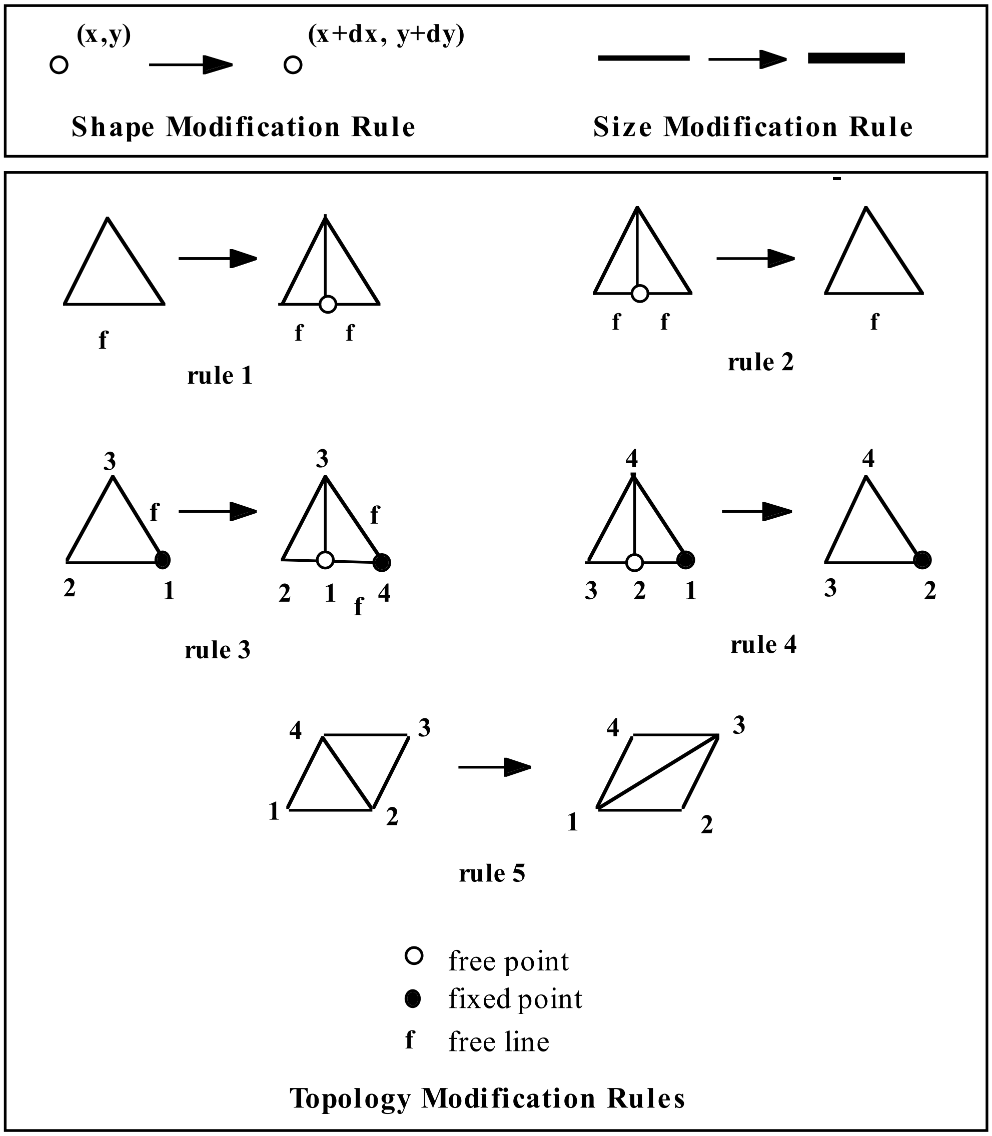


Figure 3.1: Planar truss grammar

An additional rule to switch an existing connection was added (rule 5) to the grammar that does not change the determinacy of the structure but alters the topology. The effects of this rule can be seen in Figure 3.2 where the same topology is formed through the combination of the dividing rule (rule 1) and the switching rule (rule 5) as resulted from the adding rule (rule 3). The addition of rules, such as the switching rule (rule 5), provides an additional means of generating the same type of topology but does not lead to any new topologies that could not have been generated using only the dividing and adding rules. It is beneficial to keep the grammar as simple as possible to allow the optimization to guide the design generation in the direction of desirable topologies. But, additional rules, such as Rule 5, are not detrimental to the grammar and are left in the grammar since they provide a simpler means of obtaining a new topology rather than applying the reverse rule and then applying a different forward rule.

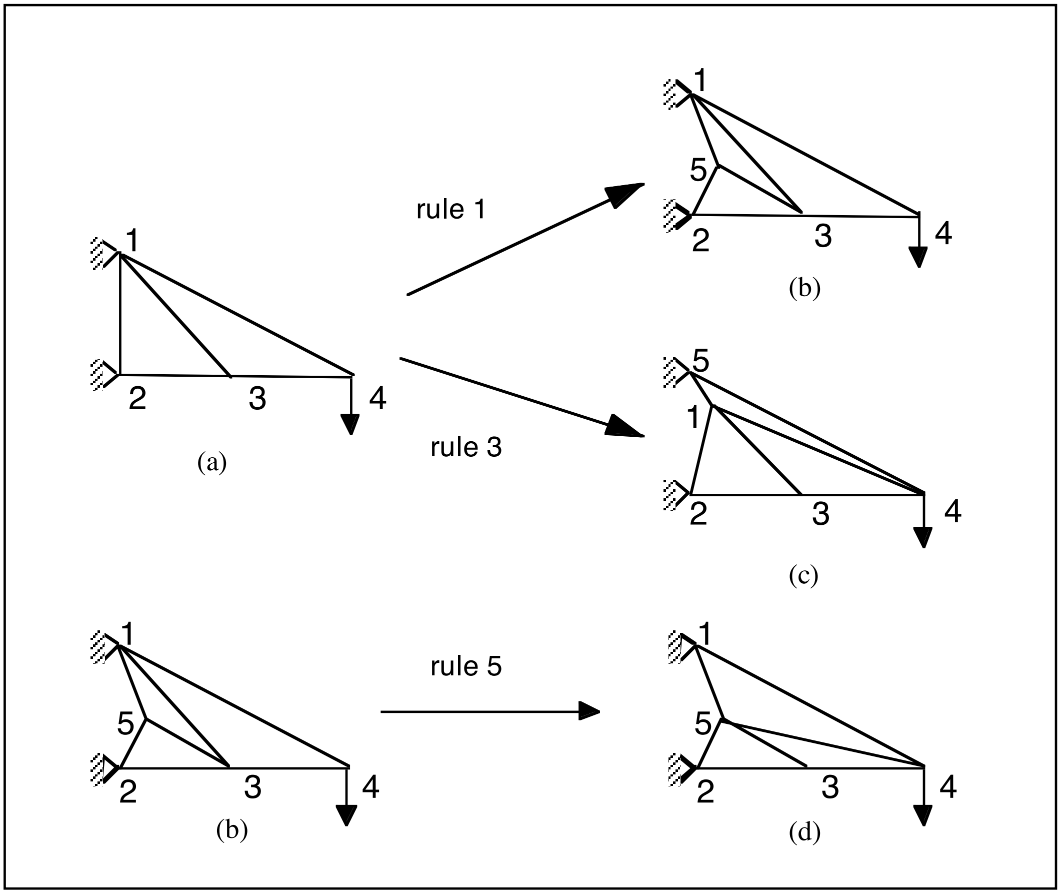


Figure 3.2: Example of rule applications

Expanding the planar grammar to three dimensions following Maxwell’s rule for space truss layout results in two new topology rule pairs, rules 6 through 9; see Figure 3.3. The rule to divide a shape, rule 1, can be also used in three-dimensional structures to either reduce the number of redundant members in the structure or can be supported in the third-dimension, indicating a support such as the ground or a wall, so that the net effect on the determinacy of the structure is zero. Another difference between the three-dimensional grammar and the planar grammar is that the shape modification rule now uses a projection function, z(x, y), to generate the third dimension of the structure.

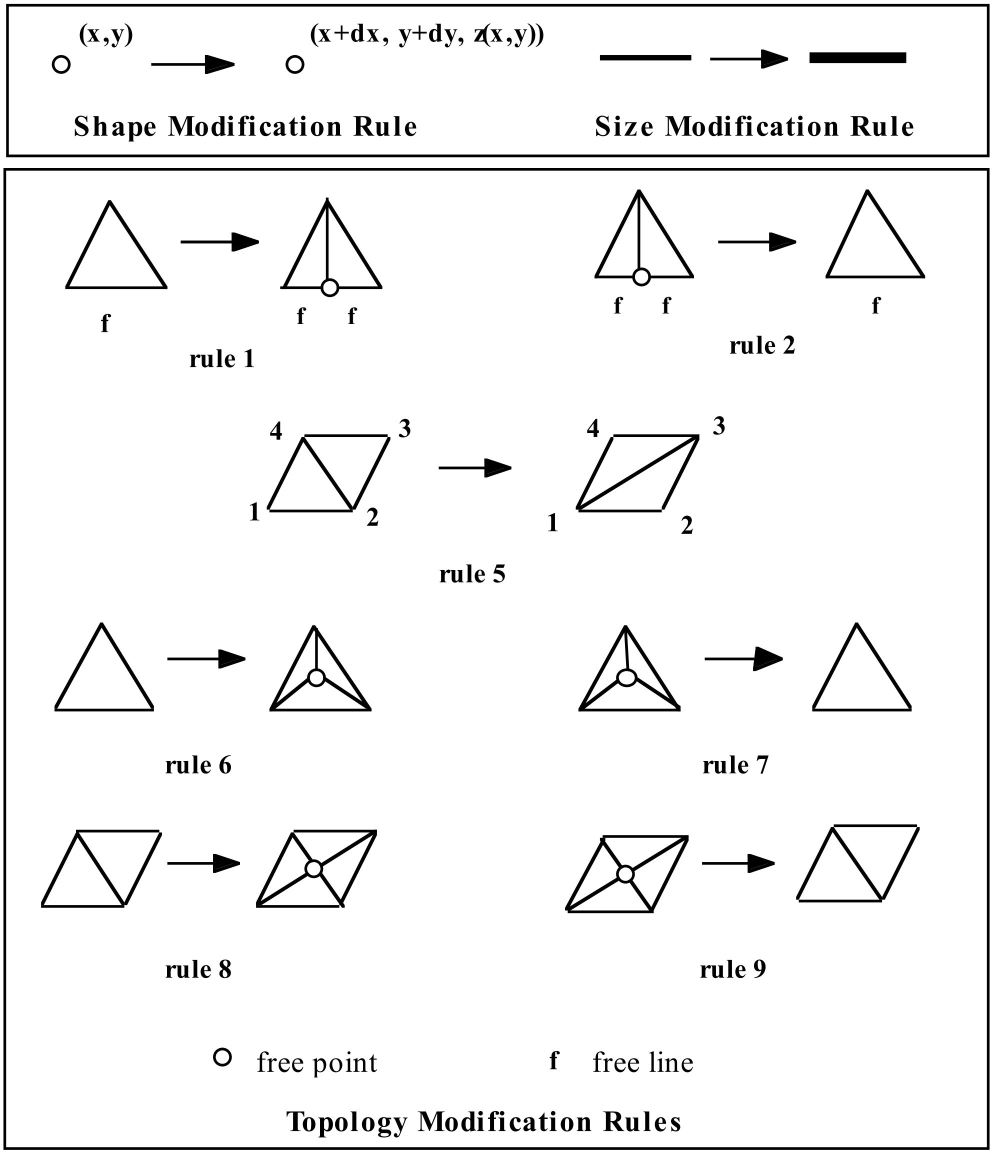


Figure 3.3: Space truss grammar

It was mentioned before that the shape grammar was implemented such that emergence was not supported in either the computational representation or in the formulation of the grammar itself. A brief example, shown in Figure 3.4, illustrates the rationale behind this approach. Assume that emergence is allowed when applying grammar rules. If the divide rule (rule 1) is applied to the exterior shape 1-2-4 such that it divides the connection between joints 1 and 4, then the lines 1-3 and 2-5 overlap without a joint and a four-sided shape between this intersection and joints 3, 4 and 5 is created. Since the current grammar is limited to triangles, the four-sided shape can not be recognized by the grammar. Continuing, if one of the new triangles formed by the previous operation is then recognized and divided, as shown in the next design, the rule attempts to make a connection where no joint exists. Improvements could be made to the grammar such as adding rules for four-sided shapes and formulating constraints such that a shape cannot be modified unless it consists of three joints but these modifications do not solve all of the problems.

The main difficulty with allowing emergence is that computationally emergence is a difficult and time consuming process. If emergence were supported, rather than knowing up front the exact shape interpretation of the design, the design would have to be parsed for multiple interpretations. Controlling the design modifications also becomes difficult. Stiny, in his comments on set grammars (Stiny, 1982), noted that set grammars are particularly useful for applications that need the multidimensional properties of shape grammars but with the well-behaved nature of standard production systems. By excluding emergent shapes from the possible shapes for topology rule application, the design generation is more controlled leading to less constraints on the application of rules and a computationally faster algorithm.

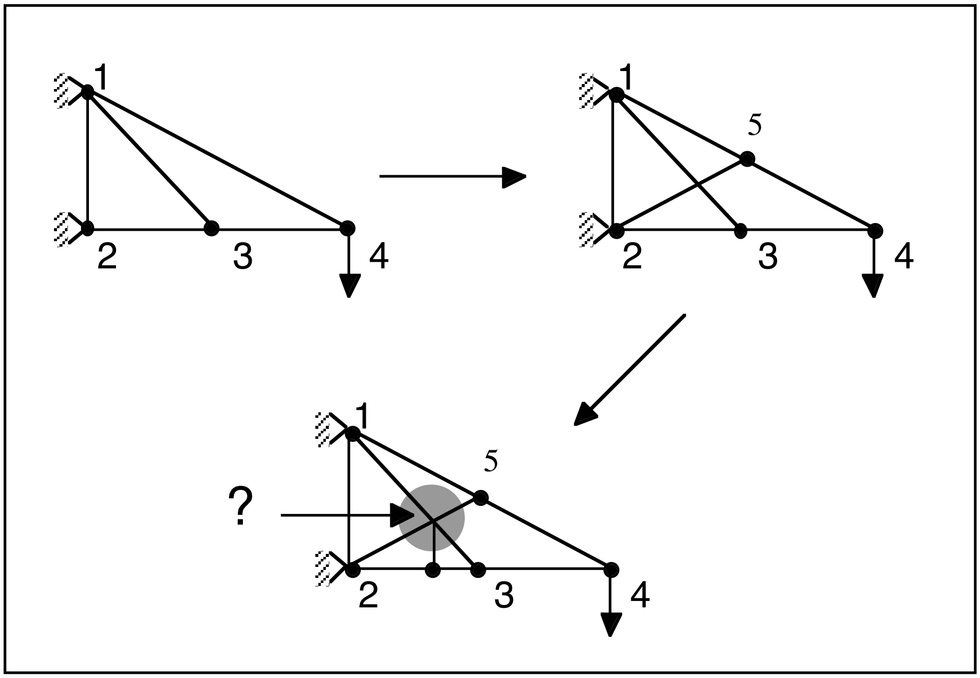


Figure 3.4: Example of problems entailed in allowing emergence

3.2 An Optimization Model for Structural Design

We have seen that a shape grammar can define the relationships between form and function in the design of discrete structures to generate valid structural topologies. Since an infinite number of spatial configurations can be generated from the grammar, designs must be interpreted and evaluated according to design criteria in order to compare design quality among all possible designs. The formulation of an optimization model provides a mechanism to measure design quality as the satisfaction of a set of design objectives and constraints. The spatial form of a design is configured by the grammar and the function is then interpreted based on the computational models for the design criteria. An optimization model for conceptual structural design will now be presented that reflects the design criteria of civil engineers and architects alike such that discrete truss structures with different purposes can be interpreted and evaluated.

Billington in his interpretation of structures provides the following definition: structural design is a balance between three ideals: efficiency (minimum materials), economy (minimum cost), and elegance (maximum aesthetic expression) (Billington, 1983). While Billington, a civil engineer, uses this definition to interpret towers and bridges, structures that are primarily designed by civil engineers, this definition also incorporates the goals of architects since it includes the interpretation of a structure as a visual form in addition to a physical object. The one element missing from this definition, perhaps because it is affiliated more with architectural design, is the utility of the structure, or how the structure will be used. The intent of modeling these higher level design goals in the optimization model is to provide a measure of design quality that reflects important design goals in the conceptual design stage. Design criteria that model detailed design considerations are not included in this work since the focus is on conceptual design but any important criteria that can be articulated can be included in the optimization model.

Structural design is a complex problem that incorporates many competing design goals. The optimization model formulated uses individual computational models for efficiency, economy, elegance, and utility as they relate to the design of conceptual discrete structures. While most research in structural optimization methods focus on structural efficiency as the sole design goal, the multiobjective model presented here allows for a broader range of design criteria that impacts the resulting layout of structures. Now, rather than basing the design quality purely on mass, a cost function that is a summation of weighted objectives determines the design quality, or how well a design satisfies the specified set of objectives. The "balance" among design objectives that Billington describes is achieved when there is a balance among tradeoffs modeled in the cost function.

Optimization models can vary among problems in the domain of discrete structures. In general, structures achieve two goals: control of forces and control of space (Billington, 1983). For example, in the design of a bridge, a structure whose primary purpose is to control forces, the main design goals are efficiency and economy where elegance is usually a secondary factor. Although the aesthetics of a bridge are important since bridges are symbols of society, most likely these aesthetic values are derived from the efficiency and economy of the structure rather than on pure merit of the visual form. When commenting on the elegance of a structure, engineers often refer to the superior use of materials or clever details in the construction. On the other hand, in the design of buildings the primary design goal is to control the space subject to restrictions on efficiency and economy (someone must pay for the building). So, by modeling a wide range of objectives the designer can choose the appropriate formulation or formulations of the optimization model to create tradeoffs in the design generation that must be balanced so as to achieve the goals of the design task at hand. We will now investigate design issues appropriate to conceptual structural design and their quantification for the definition of an optimization model.

3.2.1 Efficiency

The first design goal, structural efficiency, is the most widely used structural design goal and the simplest to model computationally with the use of computational analysis methods (see Chapter 2). The goal of structural efficiency is to generate the lightest design that meets the required set of behavioral and geometric constraints. A light structure may be desired for the cheapest design based on the cost of raw material alone or, in the case of structures for aerospace, the least amount of mass to propel. Most applications of least weight structures tend to be found in aerospace where the cost of propelling the structure outweighs the high costs associated with manufacturing the intricate geometries that often result from structural optimization.

3.2.1.1 Model of Efficiency

The formulation for structural efficiency used to generate all structures in this work is the basic structural optimization problem:

min: mass,

s.t.: stress constraint, $\sigma < \sigma_{allow}$,
buckling load constraint, $P_i < P_{critical}$, and
displacement constraint, $\delta < \alpha_{allow}$,

that is formulated in the optimization model as:

$$\text{efficiency objective cost} = \text{efficiency weight} * \text{mass} \qquad \text{Eq. 3.2}$$

$$\begin{aligned}\text{efficiency constraint cost} = {} & (\text{stress weight} * \sum_{i=1}^{\text{num members}} (1 - \frac{\sigma_i}{\sigma_{allow}})) + \\ & (\text{buckling weight} * \sum_{i=1}^{\text{num members}} (1 - \frac{P_i}{P_{critical}})) \\ & (\text{displacement weight} * \sum_{i=1}^{\text{num members}} (1 - \frac{\delta_{i,1}}{\delta_{allow}}) + (1 - \frac{\delta_{i,2}}{\delta_{allow}}))\end{aligned}$$

Eq. 3.3

This model of efficiency reflects the tradeoff between the mass of the structure and the behavior of the structure. A factor of safety against failure from stress violations or buckling can be incorporated into the allowable stress or the critical buckling load calculation. The displacement constraint is calculated from the vertical displacement at each end of a member relative to the member length since an absolute limit on displacement is difficult to define. The allowable displacement is calculated as the vertical displacement at one end of the member that corresponds to a 2° rotation about the member about the opposite end.

3.2.2 Economy

The second design goal, structural economy, considers the costs associated with realizing a design, which often depends on more than just minimizing the mass of the structure. The overall cost of a structure depends on the cost of purchasing and fabricating members as well as constructing the structure where a tradeoff exists between them. For example, in a country where labor is cheap but material cost is high it would be more important to reduce the material cost. But, on the other hand, in a country where labor is expensive but material is cheap it will be more advantageous to reduce construction costs. The tradeoff of economy with other design goals also must consider how many structures will be built. For instance, the influence of economy in the design of a transmission tower that will be duplicated thousands of times is much larger than for the design of an intricate roof that will only be built once. Thus, the influence of costs for building the structure can range from very influential to minor importance depending on the quantity to be built and the geographic location. These factors can be articulated in the optimization model.

Construction costs consider the costs associated with building the structure. These costs can be associated to design attributes such as the number of joints and complexity of joints. Through the weighting of the objective function the relative importance of the joint cost could be modified depending on the design application. In the layout of a bridge, the cost of joints may be high compared with the structural mass whereas for a smaller structure, such as a roof truss, the cost of joints may be low when compared to structural mass.

Material costs are dependent on the purchase or fabrication of the materials required to construct the structure and are difficult to model since the relation between the amount of material to be purchased and the sizes of the required members, both lengths and cross-sections, versus the cost of material is hard to quantify. Cross-sections can be either standard sizes or continuous sizes that will be fabricated. In most design applications, using standard sizes eliminates the need to move the resulting continuous sizes to the closest standard size after the optimization has been completed, which moves the design away from the optimum. Estimating the cost from member length is a more difficult problem since it consists of considering the uniform lengths of members for ease of cutting from stock lengths and the amount of waste material generated when cutting from stock lengths.

In this work considerations for material costs have been incorporated through a method for the dynamic grouping of members into groups based on cross-sectional area or length. The optimal grouping problem involves finding a structural design with an optimal number of groups and the optimal sizes for each group. Area groups are a group of members with the same cross-sectional area while length groups are a group of members with lengths that fall within a small range based on a specified tolerance. Given a problem specification, the optimal number of groups and the cross-sectional area of each group are determined during the design process. Combining the design objectives of efficiency and a limited number of distinct member sizes, a tradeoff is created between achieving the minimum mass of the structure and having fewer distinct member cross-sectional areas with a corresponding economy of scale.

Current layout methods that generate topologies with a limited number of distinct cross-sections place members a priori into predefined groups where the cross-sectional area variables of all members in a group are linked to the same design variable. The optimal grouping problem is different since for each topology generated the members are grouped dynamically, where individual members within a group are assigned a common cross-sectional area, either continuous or discrete, but still maintain an independent cross-section variable. Generating topologies with a limited number of continuous cross-sections will result in cost savings if members are to be fabricated, while, if materials are to be purchased, additional cost benefits will be gained if the group cross-sections are assigned based on a specified set of standard sizes.

3.2.2.1 Model of Economy

The incorporation of optimal grouping in the shape annealing method determines the optimal number of groups and the optimal cross-sectional area for the resulting area groups. The influence of the number of groups in a design has been modeled as a design objective, a soft constraint and as a hard constraint. Formulated as an objective the group objective function is:

$$\text{group objective} = \text{group weight} \;*\; e^{\left(\text{iteration}/\text{max iterations}\right)} \;*\; \text{number of groups}^{2}, \qquad \text{Eq. 3.4}$$

where the group weight specifies the relative importance between the number of groups and other competing objectives. Since this objective is discrete, the value of group weight determines the relative cost of adding a new group to a structure. Among other design objectives, the greatest competition is most likely between the number of groups and the mass of the structure. Comparing the objectives of minimum mass and a minimum number of groups, by increasing the group weight, the cost of adding a new group increases and thus the tradeoff with mass, that is, the amount of mass that the structure must be reduced by in order for the design to have the same objective value, also increases. The group penalty function was chosen to exhibit the following behavior: a strong effect with the number of groups and an increase in the relative importance of the number of groups in the objective function as the design progresses.

Imposing a restriction on the number of allowable classes rather than treating it as an objective models a slightly different economic design goal. In this case there is a specified minimum number of classes that when achieved the number of classes has no effect on the design quality. But, if the number of classes exceeds this limit, the design is penalized. Formulated as a soft constraint the group constraint function is:

$$\text{group constraint} = \text{group constraint weight} * \max\{0, \text{number of classes in the design} - \text{number of allowed classes}\}. \qquad \text{Eq. 3.5}$$

The group constraint weight for violating the allowed class constraint is determined based on a penalty function, which will be described in Chapter 4, that adjusts so that the cost of violating the class constraint is lower at the beginning of the process than at the end. The dynamic weight and the decreasing tolerance for grouping classes work together to group members into a limited number of tightly grouped classes.

The last model that influences the number of distinct member sizes in a structural layout imposes a hard constraint on the number of groups. In this formulation the number of classes is fixed so that all members in a design are divided among the specified number of groups, although, the cross-sectional area of each group is still allowed to vary. This model of economy transforms the design problem from determining how many groups are optimal to determining the optimal cross-sections for a fixed number of groups. This formulation eliminates the tradeoff between the number of groups and the mass and is advantageous if the optimal number of classes for a problem is known a priori. A demonstration of this model will be presented in Section 5.2.3 through an example that investigates designing with tensegrity principles.

To examine the tradeoff between minimum mass and the number of groups, consider a fixed determinant structural topology under a single loading condition. If each member in the fixed layout is allowed to have a distinct cross-sectional area the optimal mass will result by requiring maximum efficiency of all members through setting each member at its stress or buckling limit. Imposing a restriction on cross-sectional area decreases the efficiency of the structure by preventing some members from being at this limit. By varying the structural topology and shape, the structure can compensate, to an extent, for this restriction on cross-sectional area through a search for optimal groupings of members. Thus with group restrictions the design will not be optimal with respect to mass alone but rather with respect to the relative importance of mass and number of groups reflected in the weighted objective function. The tradeoff between mass efficiency and member grouping is less clear for indeterminant structures or structures with multiple loading conditions and will be explored in the examples.

3.2.3 Utility

The third design goal modeled, utility, incorporates the intended purpose, or usage, of the structure into the design process. For example, in the design of a discrete dome if the structure is to be used purely to carry loads to the ground then the form of the resulting structure is less important than the structural behavior. But, if the structure is to be used as an enclosure, either as a house or an exposition center, then it is important that the structure enclose the space to provide protection from the environment. The design goal of utility provides information about the design usage to the optimization enabling the generation of appropriate designs.

A distinction is made in this optimization model between function and behavior where utility models the function of a structure. Behavior is defined as what the structure does, i.e. carry a specified load to the supports, while function is defined as the intended purpose of the structure, i.e. a roof or a bridge (Finger and Dixon, 1989). For example, different functions are required from a bridge than a roof since the purpose of a roof is to enclose a space while the purpose of a bridge is to provide a crossing. But, the required behavior of both structures to not fail under environmental loads is the same.

The design goal of utility can be formulated as either part of the problem specification in terms of the geometric configuration that represents the physical design space or as a design goal in the optimization model. The difference in the two formulations depends on whether utility can be traded off in the design. Consider the design of roofs. Since the purpose of the roof is to cover the structure below there is no tradeoff between a design that meets this goal and one that does not. Thus the designs of interest consist only of structural layouts that meet this constraint. By providing additional spatial syntax to the shape grammar forms that comply with this constraint can be generated.

Alternatively, some geometric constraints can be considered as tradeoffs. Returning to the design of a roof truss there may be a desired pitch of the roof to meet. If the design solution does not meet this pitch precisely the roof will still function but will have associated behavioral effects on other portions of the design, like the design would need to be able to carry a larger load due to snow accumulation. Utility incorporates geometric considerations that affect the usage of the structure in order to generate purposeful structural layouts.

3.2.4 Elegance

The fourth, and last, design goal considered in the optimization model for discrete structural design is elegance. The generation of elegant structures is perhaps the most difficult of the structural design goals to model. Without a model of elegance, shape annealing generates forms following the philosophy that: "beauty results from functional efficiency," (Holgate, 1992, p.222), or that elegance follows efficiency. This statement implies that a model of elegance is not necessary since it will merely appear if the structure generated is functionally efficient. But, through the additional considerations of economy and utility, the design form is altered from being the most efficient form and thus it is questionable whether or not the generated design will be elegant. Even if a functionally efficient design is desired, the argument can be made that a designer may not consider functionalism beautiful. We have seen that form-function relations can be modeled in the shape grammar and now we will investigate the tradeoff created between the visual form of a structure and its function by the incorporation of a computational model for visual elegance in the optimization model.

The design goal of elegance can either compete or cooperate with other design goals, mainly competing with the efficiency of the structure. On one end of the spectrum are utilitarian structures like transmission towers and standard bridge designs where the elegance of the design often arises from the functional requirements of the design problem. In this case there is a strong tie between the function of the structure and what is considered aesthetically pleasing. On the other hand are structures where there may be more freedom in the form of the structure to make it visually pleasing to the eye such as roof design. For example, as an architect commented during the study of roof truss design in Chapter 8, “there is nothing more boring than a flat roof”. For this case, while a design based on structural function alone may be flat, through the addition of a visual design goal, a design that balances efficiency and visual elegance could be generated. This balance of functional and visual objectives is very important in structural design (and one in which architects and civil engineers often disagree) since even a structure whose primary goal is visual impact should not portray an image of excessive waste. This leads back to Billington’s ideal for structural design in achieving a balance between an economical structure and a visually pleasing structure.

A computational model of elegance interprets a structure as a visual form and creates a new goal in the layout of a structure that it be pleasing to the eye regardless of functional implications. In structural design there is a high coupling between a visually pleasing structure and a functionally efficient structure. By adding the design goal of elegance the position is taken that the problem of making a structure stand up is entirely different from that of making it visually pleasing (Schofield, 1958, p.4) and Schofield goes on to remark that defects in a structure’s function will only hinder one’s enjoyment of it.

For this work, a model of elegance will be formulated using proportional systems that have been used in architectural design to design visually pleasing structures. Using the previous design goal models, the proportions of members in a structure were based on functional considerations alone through models for structural efficiency and economy and syntax in the grammar rules that specified geometric constraints based on design usage. While there has always been a strong tie between proportion and function in proportional system theory, for example the theory of Vitruvius who studied engineering and architectural proportions (Schofield, 1958), the proportional system applied here is solely a visual interpretation of the design and not functional. However, the functional implications and tradeoffs involved with adhering to an aesthetic model will be investigated.

In the interpretation of designs, the word “elegance” can be used to refer to formal, material and associative properties (Mitchell, 1990). Here, a computational model of elegance is based on formal elegance, or elegance derived from formal qualities such as symmetries, proportion, rhythm, uniformity and variety that can be determined purely from geometric properties. This interpretation of elegance only uses a portion of what was meant by Billington since he used the word elegance to mean the expressive power of a structural form which is a combination of all parts of aesthetic value. Since material and associative aesthetic values are dependent on personal interpretation, and often relate back to functional properties that are already included in the optimization model, this portion of aesthetic value is left to be evaluated by the designer when evaluating designs.

A computational model based on formal elegance, termed an aesthetic model, defines a set of aesthetic values for interpreting designs. This aesthetic model gives an aesthetic measure such that designs can be compared for satisfying the desired aesthetic objectives. If design A has an aesthetic measure x and design B has an aesthetic measure y, the relation can be formed that design A is more aesthetically pleasing then design B or vice versa. Similar formulations for computational aesthetics are discussed in Stiny and Gips (1978) who formulate computational models for various aesthetic theories and measures such as a quantitative interpretation of the popular phrase “unity in variety” and Birkhoff’s aesthetic measure.

The aesthetic models that have been created in this work are more simplistic than the models presented by Stiny and Gips and are used to determine a measure of the visual proportion of members in a design. Many proportional systems for structural design have been used in the past, Vitruvius’ account of Greek proportion, the harmonic proportions of the Renaissance, and the golden section, which was also first used by the Greeks (see Schofield, 1958, for a historical discussion of proportional theory). Two aesthetic models are created here: one model uses a design goal of uniformity and attempts to standardize the spatial breakdown of the geometric design space by proportioning the size of all members in a design to be near in length. A second aesthetic model provides a relative proportional system that uses the golden ratio as a basis to compute an aesthetic measure of individual shapes based from the relative lengths of its members. The development of this model will be shown in Section 6.2. The application of both aesthetic models will be explored in the design of roofs in Section 6.3.

3.3 Summary

Applying the shape annealing method to the design of discrete structures requires the development of a grammar to generate valid, stable structures that can then be interpreted by the optimization model. Shape grammars were presented for the design of two and three-dimensional discrete structures that model the form-function relation found in Maxwell's rule for the stability of pin-jointed structures. While the interpretation of structures could be based solely on functional efficiency, such as in most structural optimization, additional design goals of economy, utility and elegance are added to the optimization model to allow for the generation of structures that balance practical design goals.

4. IMPLEMENTATION

Shape annealing has been presented as a design technique for configuration optimization of discrete structures. This chapter will present the general algorithm used in the implementation, the representation, a description of the simulated annealing techniques used, and a scheme for dynamic normalization and weighting of the cost function.

When it was first presented, the shape annealing method was shown to be capable of generating optimally directed structural topologies that avoid geometric obstacles while satisfying stress and Euler buckling criteria (Reddy and Cagan, 1995). The conclusion of the work by Reddy and Cagan identified two main areas of improvement to the implementation:

1. the improvement of convergence, especially in the case of multiple constraints such as stress, Euler buckling and geometric constraints, and
2. a more sophisticated annealing schedule.

Addressing these issues and improving on the implementation by Reddy and Cagan, five main additions were made:

1. a new data structure that includes shape adjacency relations,
2. implementation of the modified Lam-Delosme annealing schedule,
3. application of the Hustin move set,
4. dynamic normalization and weighting of constraint violations, and
5. interfaces to external finite element analysis tools.

After a brief presentation of the general shape annealing algorithm each of these topics will be addressed. For the reader who is not interested in the details of the implementation, the discussion of these topics can be skipped.

4.1 Algorithm

The shape annealing method, as applied to structural design, builds structures using a shape grammar (Stiny, 1980), optimizes the structures with the stochastic optimization method of simulated annealing (Kirkpatrick, et al., 1983; Swartz and Sechen, 1990; Ochotta, 1994) and analyzes the structures using the finite element method. The algorithm used follows the diagram in Figure 4.1: first, an initial structure is generated from a minimal connection of structural members between the applied loads and support points of the problem specification. If economy through member groups is included as a design goal, the members are grouped accordingly; the method for dynamic grouping will be presented later. Next, the structure is loaded and analyzed using the finite element method. The cost function is then evaluated from the specified optimization model, which can include models for structural behavior, geometric constraints, economic considerations or aesthetics. The initial design is automatically accepted. Next, a rule from the shape grammar is applied to the structure to create a new design that is then analyzed and its cost is calculated. The costs of the new design and the previous design are used in the Metropolis algorithm to determine whether to accept the new design or revert to the previous design. A better design is always accepted while a worse design may be accepted based on a probability function (Metropolis, et al., 1953). A rule from the shape grammar is applied again to the structure to create a new design and the process continues iteratively until the annealing schedule terminates or the design has converged, or frozen. The resulting design is then presented to the designer in the form of a description of topology and geometry, including both the location of joints and the sizes of members. At the designer's discretion, some members in the design may have the minimum allowable cross-sectional area and can be removed as long as the structure remains stable. However, a designer may choose to leave the members in the design since they add negligible mass but may provide additional visual benefits.

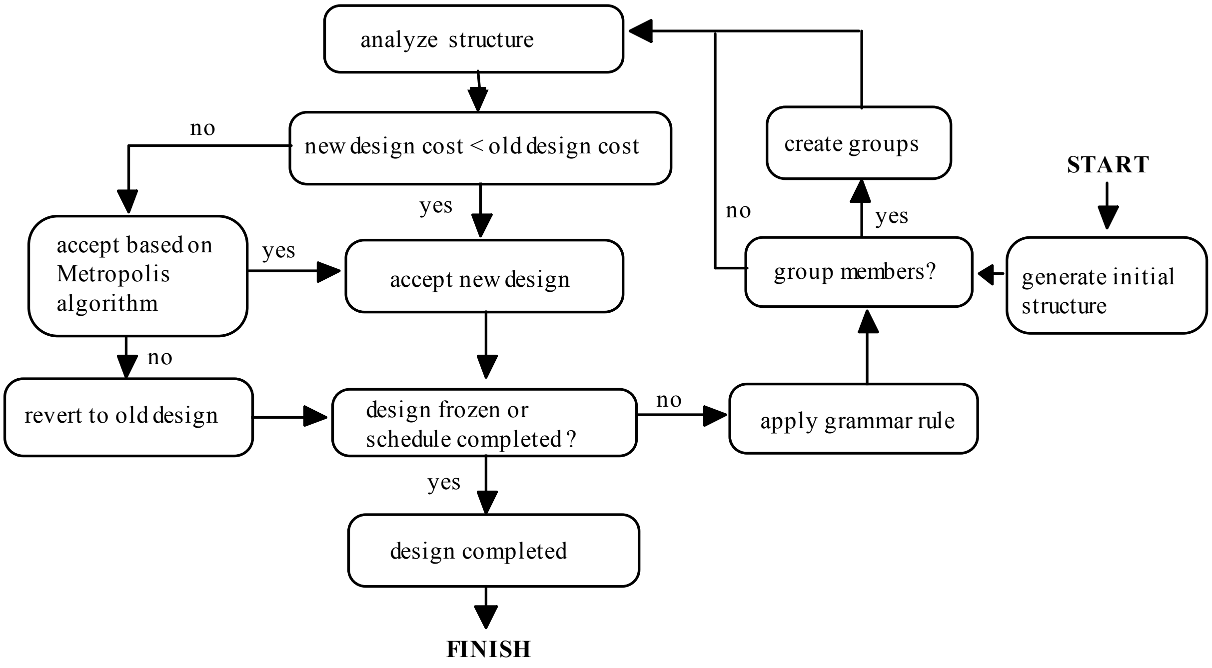


Figure 4.1: Shape annealing algorithm for structures

4.2 Data Structure

The current implementation of the shape grammar, a labeled, directed graph, is based loosely on a winged-edge solid boundary representation (Baumgart, 1975; Heisserman and Woodbury, 1994). A winged-edge data structure is a directed graph with adjacency relations between components in the graph. Since this representation is applied to planar objects, rather than solids, the manipulation of the representation does not ensure manifold designs or follow Euler's law. An analogy is made between the components of the labeled graph and the components of the shape grammar. The representation separates the description of geometry and topology information with a link between the two and includes the following components:

1. Geometry
    - point, the geometric location of a node in the boundary graph,
    - line, the geometric description of one or more line-halves,
2. Topology
    - root node, the starting node in a shape that refers to one geometric point,
    - line-half, a directed connection in the graph from node a to node b with the direction a → b, (Each line-half has references to one geometric point that is the root node, a, of the line-half and a geometric line. If a line-half is adjacent to another shape in the design it will have a mating line-half. If the line-half is an exterior line in the design there is no mating line-half.)
    - shape, a closed, directed subgraph of three or more line-halves that represents an n-sided polygon,
3. Labels
    - a set of labels that can be associated with any of the above components in the boundary graph.

A graphical description of a labeled boundary graph for a planar truss is shown in Figure 4.2. This example consists of two shapes with three line-halves each, four geometric lines, and four labeled points. The labels are used to indicate the type of each point. Shapes consist of a directed subgraph of line-halves, in this case three for a triangle. A shared line, as in the line between point two and four, is indicated by the line-half containing a reference to the mating line-half and vice versa. Currently, since the labeled graph represents planar shapes rather than solids, it is not necessary that each line-half have a mate. The exterior boundary of a design is then represented by the series of line-halves without mates. There is only one geometric description of each line that is referenced by one or more line-halves. The advantages of using a labeled boundary graph to represent discrete structural forms are: (1) the capability of representing different polygons within the same design without representing each line separately, (2) the separation of geometric and topological information for ease of changing either independently, (3) adjacency information between shapes for parametric control in the grammar and evaluation of relative proportions, and (4) a straight forward extension to representing three-dimensional shapes such as for double-layer space trusses and frames.

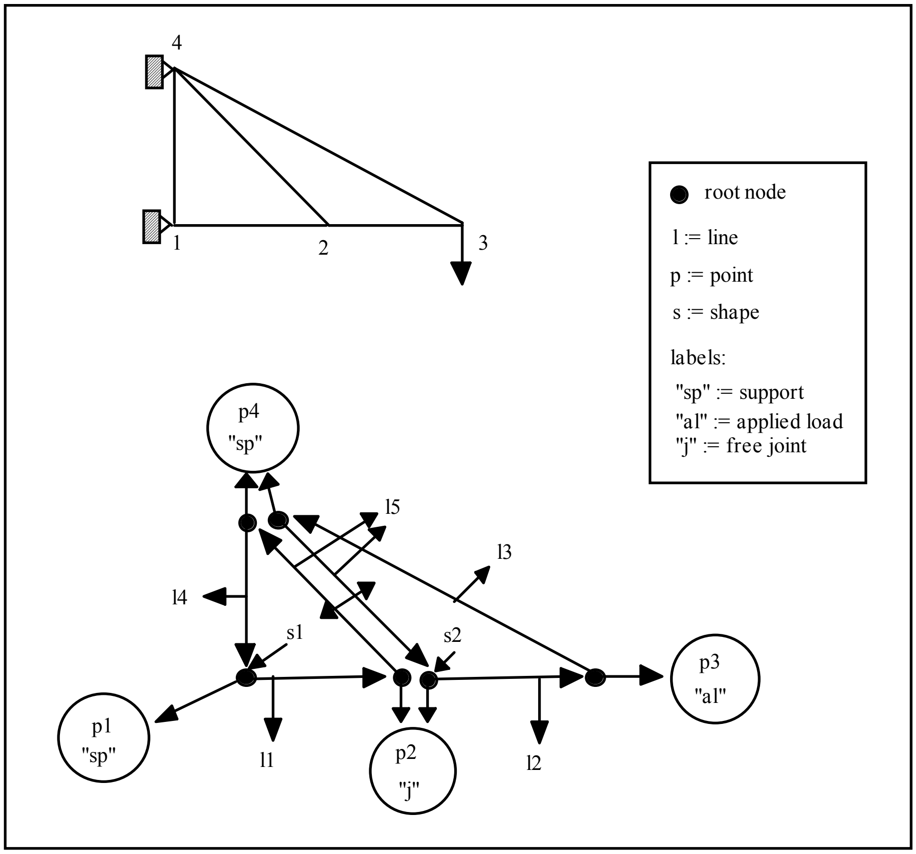


Figure 4.2: Graphical description of the data structure

4.3 Optimization

The problem with using a generative grammar for design generation is that the number of designs in the design language is immense and the design space is complex. Combining optimization with a generative grammar allows for exploration of the design space and the generation of optimally directed designs. Simulated annealing, a stochastic technique, is the optimization technique used in this work to provide the capabilities for global optimization, exploration of a complex space and handling discontinuous objectives. Simulated annealing is an adaptive search technique that is based on an analogy to the annealing of metals where a temperature schedule is used to define the probability of selecting inferior designs. Following the analogy to annealing metals, the design process is heated to a high temperature that results in very random design selection, comparative to the molecules of a metal being in a highly random energy state. The design process then follows a cooling schedule such that the final design will be a minimum, comparative to the molecules of a metal reaching a stable minimum energy.

Simulated annealing can be broken down into the following components: the annealing schedule, the move selection technique, and the cost function formulation including objective costs and constraint violation costs. The annealing schedule, or cooling schedule, defines the temperature of the process, the move set defines the probability of selecting rules and the cost function is an evaluation of each design, or energy state. The next section discusses the important aspects of applying simulated annealing to the configuration of discrete structures.

4.3.1 Modified Lam-Delosme Annealing Schedule

A modified Lam Delosme annealing schedule (Swartz and Sechen, 1990) was implemented to provide a robust, fast simulated annealing technique that required fewer statistical calculations when compared to other schedules. This schedule is based on the full Lam Delosme schedule which was optimized for the VLSI placement problem. An assumption is made that the same schedule can be extended to the topological layout problem but will be shown successful in Chapters 5 and 6. The advantages of the modified Lam Delosme schedule are a known number of iterations, fewer statistical calculations than the full Lam Delosme schedule, and faster convergence than the vanilla schedule (Swartz and Sechen, 1990). These qualities have allowed shape annealing to solve more complex problems involving up to four simultaneous constraints, stress, buckling, geometric constraints and member groups.

The modified Lam Delosme schedule is based on following a specified accept rate function, where the accept rate is defined as the number of accepted moves out of total moves over a fixed statistical interval. The target accept rate function versus the actual accept rate for one annealing run is shown in Figure 4.3. At each iteration the temperature is adjusted based on the comparison between the actual accept rate and the target accept rate. If the actual accept rate is higher than the target this implies that too many designs are being accepted and the temperature is decreased, thus "cooling" the process. In contrast, if the actual accept rate is lower than the target not enough designs are being accepted and the temperature is increased, thus "heating" the process. This annealing schedule is characterized by three regions of temperature: an initial quench, a simmer, and a freeze as shown in Figure 4.4. During the initial quench large moves have a high probability of being accepted so that the design space is explored and the design moves sufficiently away from the initial structure so as not to bias the final topology. The simmer region is where most of the profitable design takes place since it is more controlled than the initial quench but not as restrictive as the freeze region. Freezing criteria have been added to detect if a design has reached its minimum, allowing for the same annealing schedule to be used for problems of varying difficulty. Since simulated annealing is a stochastic method based on an intricate schedule that defines the design process, significant improvements can be made by applying dynamic methods to determine schedule variables.

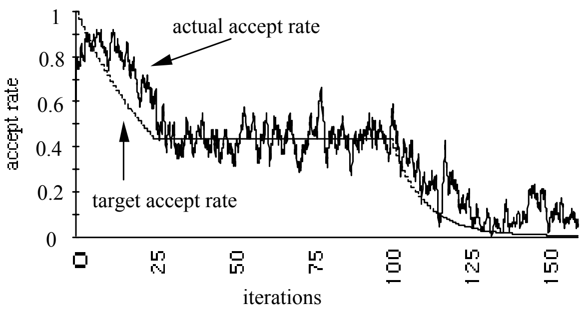


Figure 4.3: Accept rate over annealing schedule

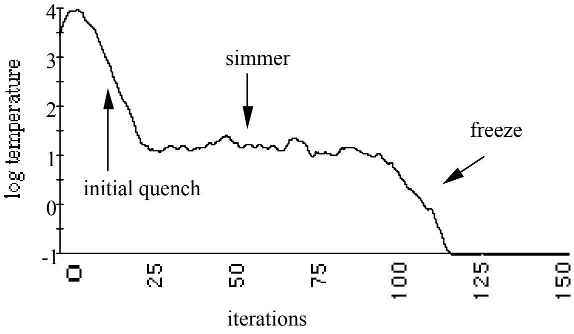


Figure 4.4: Temperature over annealing schedule

4.3.2 Rule Selection

The generation of a new design in the shape annealing method requires the application of one of the three types of grammar rules: shape, sizing, and topology. The rules of the shape grammar are randomly chosen based on individual rule type probabilities that change over the annealing schedule. The determination of probabilities for rule selection is a tricky process since it can bias the generation of designs to a particular notion of how the design should evolve. Two methods for setting the rule selection probabilities will be presented in this section. The first is a static scheme that defines probability trajectories and the second is a dynamic scheme based on the merit of the rules. While the static method predisposes the design generation to a certain probability of transformations, the dynamic method will be shown to adapt to the design problem and generate similar quality results without having to change the defined trajectory values until satisfactory designs are generated.

4.3.2.1 Rule and Move Size Trajectories

The static method for rule selection sets rule trajectories at the beginning of the annealing process that change at a fixed rate over the annealing schedule, see Figure 4.5. These trajectories were chosen such that the probability of selecting a topology modification rule is greater in the beginning of the annealing process, while the probability of selecting a shape modification rule is greater at the end of the schedule. The rationale behind these probabilities are that they lead to larger moves through topology modification at the beginning of the annealing process and small perturbations on the design through shape and sizing modification at the end of the design process. The probabilities for rule selection do not change during the simmer section of the annealing schedule.

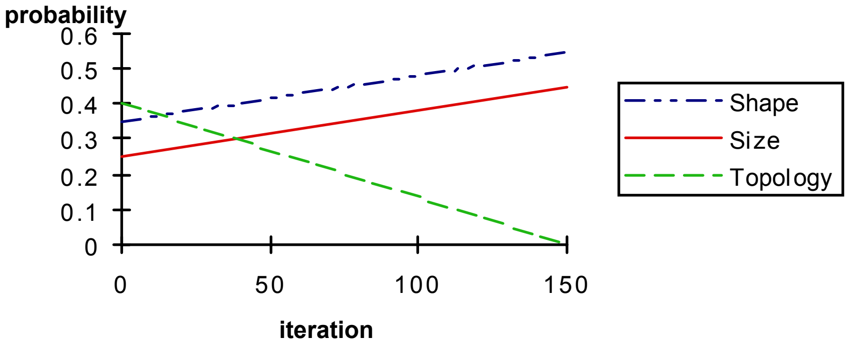


Figure 4.5: Trajectory rule selection probabilities

Along with setting the probabilities for selecting a rule type the move size must also be set to define the geometric range of the rule. For example, if the rule to move a joint location is chosen, the move size specifies a square (or cube in 3D) neighborhood, centered around the point, in which the point can be moved. This move size decreases linearly throughout the annealing schedule which in turn decreases the window in which a move can take place. This fits within the general simulated annealing philosophy of making large moves at the beginning of the design process and small perturbations at the end of the process.

4.3.2.2 Hustin Move Set

The Hustin move set is a dynamic method for rule selection that determines the probability for selecting a rule based on rule qualities (Hustin, 1988). The main advantages of the Hustin move set are: (1) the method can adapt to the problem through the selection of the best rules, (2) the possibility of escaping a local optimum increases since there is a probability that topology moves can be applied at anytime during the annealing schedule, and (3) an indication of the quality or appropriateness of each topology rule pair.

With the Hustin approach to move selection the definition of rule and move size trajectories described in the previous section is no longer required. Although the previous method worked effectively, it is based on subjective intuition about how a design should evolve.

Also, the trajectories are defined by perturbing the values until the designs generated seem satisfactory. The purpose of implementing the Hustin move set is to base rule selection on an objective measure of the effectiveness of a rule. Since a rule is selected based on quality, there is now no preconceived notion of how the selection of topology, shape or sizing rules should or should not be made. The defined trajectories shown previously were chosen such that the topology modifications occurred in the beginning of the design process and tapered off towards the end with the an increase in shape and sizing rules. This may not be the best way for all design problems to be solved.

To define probabilities of rule selection based on merit, each rule is assigned a quality measure that determines the probability of selection. A rule quality, Q_r, is calculated for each rule, r, according to Equation 4.1 as the summation of the absolute change in cost for each accepted rule divided by the number of times the rule was attempted:

$$Q_r = \frac{\sum_{\text{accepted rules}} \|\Delta \text{cost}(r)\|}{\text{number of rules attempted}} . \qquad \text{Eq. 4.1}$$

The rationale behind this quality metric is that rules that cause large changes in cost, either positive or negative, will result in a high quality if these rules are accepted. This will normally occur in the beginning of the annealing schedule when the probability of accepting large inferior moves is high. Towards the end of the annealing process, if these same rules are no longer accepted as frequently the quality of the rule will decrease. On the other hand, rules that cause small changes in the cost function but are accepted frequently will also result in high qualities. The calculation of rule qualities is performed at each iteration based on the statistics collected for the previous iteration while rule qualities remain static during a single iteration. In order that each rule is attempted enough times to calculate statistics, a minimum probability for each rule is defined.

A Hustin move set has been implemented to calculate the probabilities for the selection of rule type (topology, shape, sizing) and in the case of parametric rules (shape, sizing) the move range as well. Topology rules are grouped in pairs, the forward and reverse rule, and the pair is assigned an overall quality. Within a pair of topology rules the probability of selecting the forward or reverse rule is specified and indicates the probability of backtracking. The reason topology rules are paired is that in a particular iteration a reverse rule may be applied and accepted often such that the quality is high but results in a sparse structure at the end of the iteration. Thus the rule quality would be high for the subsequent iteration but there would be few, if any, shapes where the rule would apply. While this is not detrimental to the design process, it is however a waste of computation time.

The probability hierarchy for the Hustin move set is shown in Figure 4.6 with the root node representing the summation of all rule probabilities, or one. A parametric rule is defined by the rule type, either shape or sizing, and the move size, a number that defines the range of the parametric rule, (-move size, +move size). A fixed number of ranges are set in the beginning of the annealing process as increments over the total range of moves desired. A good number of ranges has been found to be six. Whereas with the static trajectories only one range existed at any one time, now, different move ranges can be active in the rule set. The examples will show that the selection of the ranges of parametric rules works much like the previous static trajectories in that larger ranges are more active in the previous stages of the design with smaller ranges more active towards the end of the design. The difference between the two methods is that with the dynamic method there is always a chance that larger rules can be selected and thus possibly jump out of a local optima.

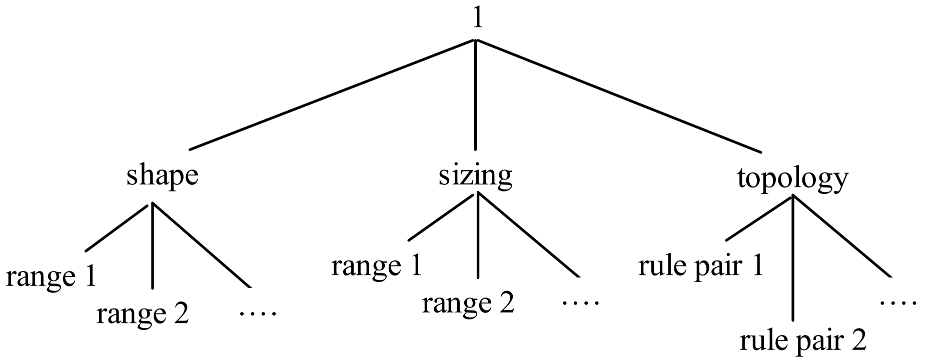
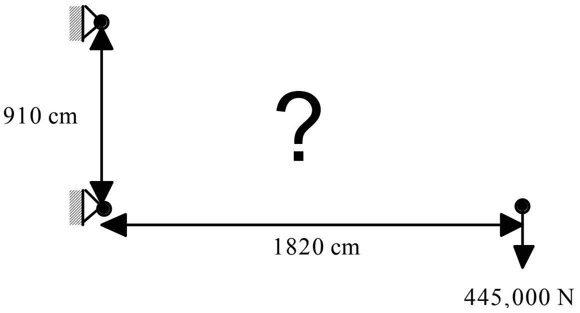


Figure 4.6: Hustin rule probability tree

To illustrate the effectiveness of the Hustin move set a study was performed for three possible implementations within the shape annealing method. The example problem used is a minimum mass layout for a cantilever truss problem subject to stress and buckling constraints; see Figure 4.7. In this section we will only consider the numerical results while the designs will be shown in Section 5.1. Although this seems to be a simple problem it proves to be a good test of the optimization method since a local optimum exists that is a two bar solution with an objective value of 3947.41; see Figure 4.8. Four different cases of the Hustin method were considered where each case was run ten times and the statistics are presented in Table 4.1. The first case, (a), is the base case that uses the static trajectories described in the previous section. The second case, (b), calculates rule qualities for the top level of the rule hierarchy, the rule type (shape, sizing, topology), while the move range for the shape and sizing rules is determined using the same trajectories as were used for the base case. This case does not show promise as it results in an increase in the mean objective. The next case, (c), calculates rule qualities for the parametric rules based on rule type and range while keeping the rule quality of the topology rules grouped together. This formulation seems to have about the same merit as not using a Hustin move set except that the best solution found has improved. The last case, (d), calculates qualities for the each rule individually, or rule pairs for topology rules. Again the mean and standard deviation of the costs do not improve but the best solution found is improved. For all cases including the base case, (a), the two bar solution was generated two in ten times.



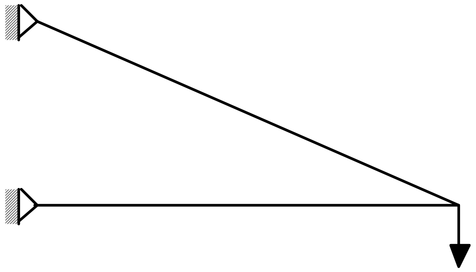


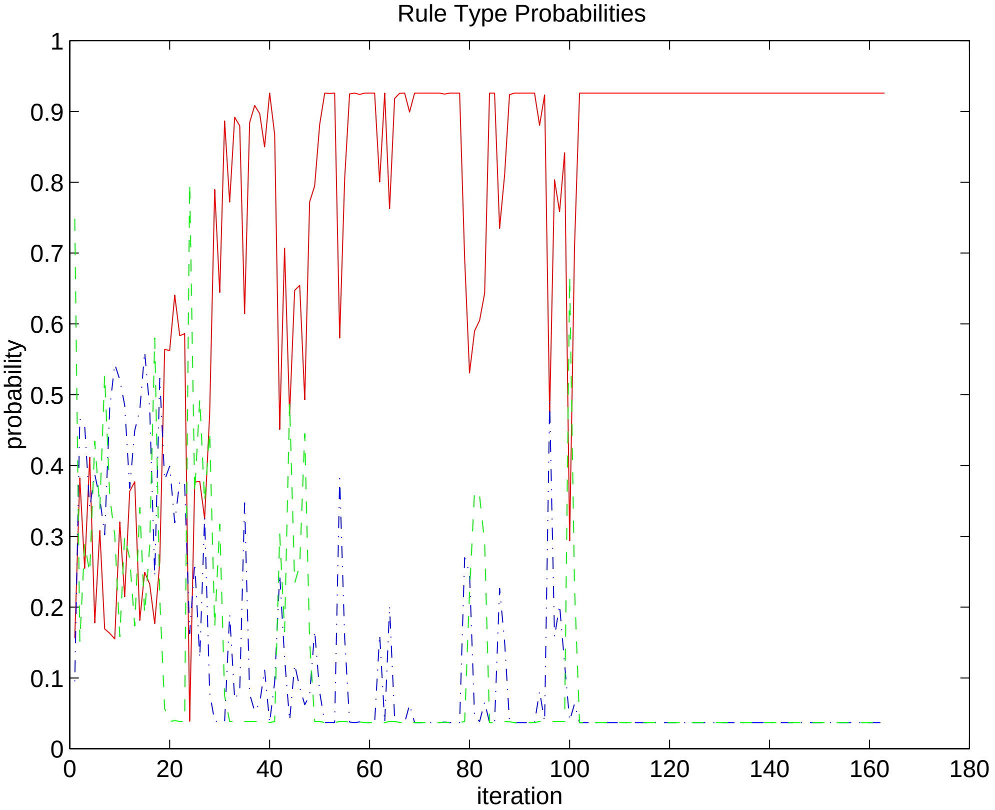
Figure 4.7: Cantilever truss problem description

Figure 4.8: Two bar solution

Table 4.1: Hustin move selection tests

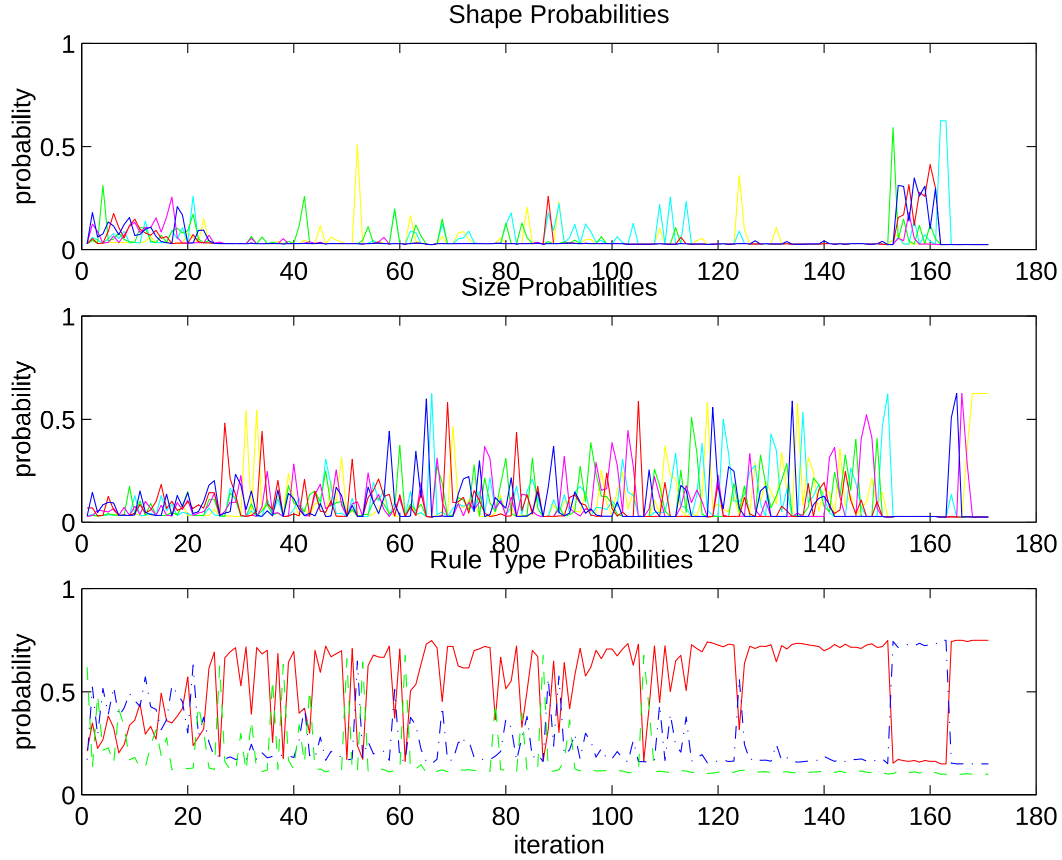
Case	Hustin Selection	Mean Objective	Standard Deviation	Minimum	Maximum
(a)	none	2177.95	1000.77	1222.79	3947.41
(b)	rule type	2570.40	836.22	1302.71	3947.41
(c)	rule type and move range	2167.34	995.08	1186.6	3947.42
(d)	rule type, move range and topology pair	2171.52	1009.2	1057.38	3947.42

The rule type selection of the Hustin move set can now be compared to the static trajectories of Figure 4.5. The graphs in Figures 4.9 through 4.11 show the probabilities for rule selections over the design process for the best design in each case. These graphs show that the previous logic for setting the static trajectories was somewhat correct since the large parametric rules (darker lines) have higher probabilities in the beginning of the design process while the smaller moves (lighter lines) have higher probabilities towards the end. However, the probabilities for topology rules is quite different than the static method with surges in topology rule probabilities throughout the design. Rather than generating most of the topology at the beginning of the design process and then manipulating the shape and sizing, these graphs indicate that multiple cycles of this process can occur in any one design generation. Schedule data is shown in Figures 4.12 for cases (a) and in 4.13 for case (c) to illustrate that the annealing schedule works better, i.e. follows the desired accept rate more closely, with the Hustin move set.



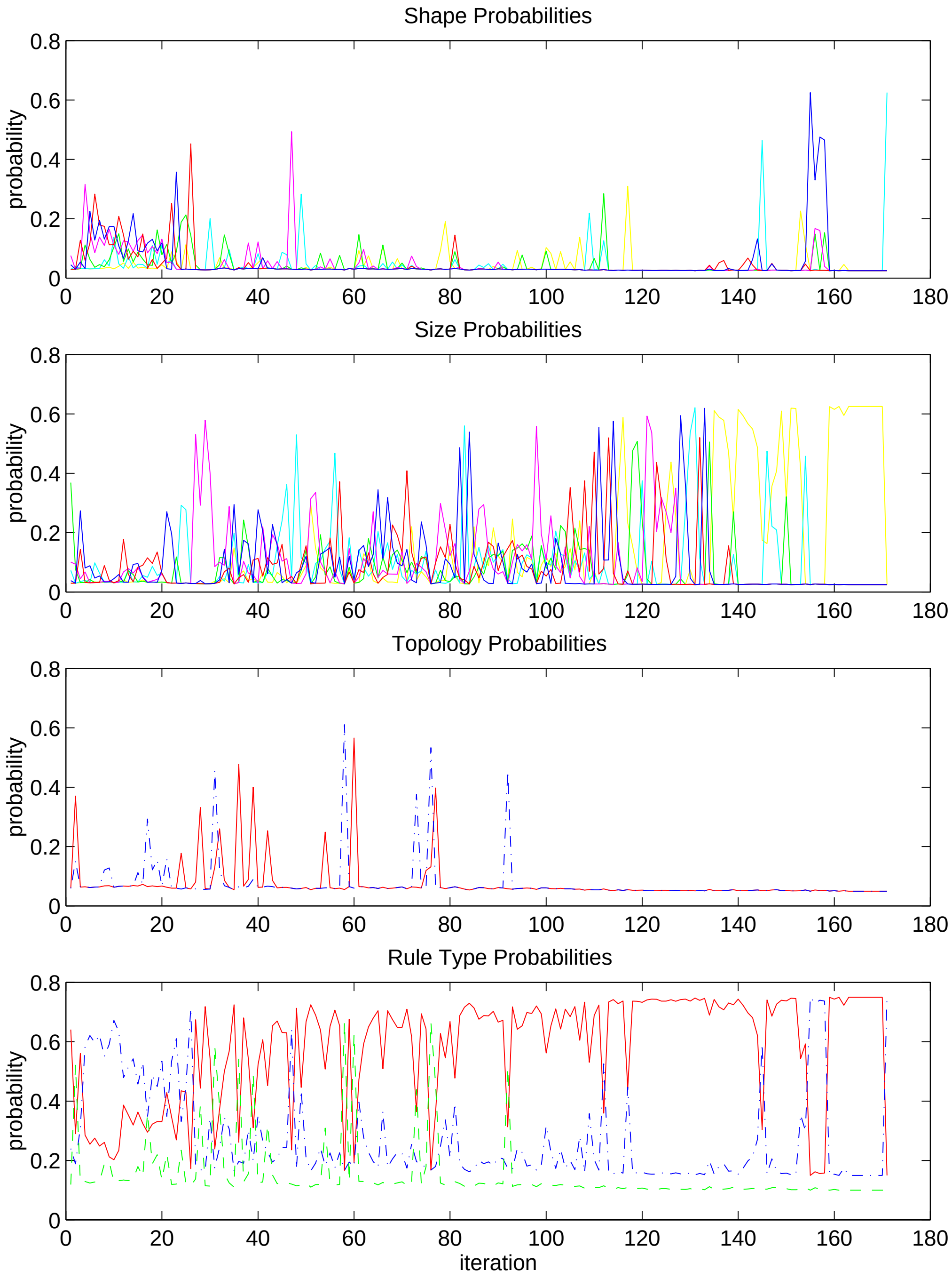
Legend for Rule Type Graph: - sizing rules, - - - shape rules, -.- topology rules

Figure 4.9: Rule type probabilities for case (b)



Legend for Rule Type Graph: - sizing rules, - - - shape rules, -.- topology rules

Figure 4.10: Rule probabilities for case (c)



Legend for Topology Rules Graph: - divide rule, -.-shape rule

Legend for Rule Type Graph: - sizing rules, - - - shape rules, -.- topology rules

Figure 4.11: Case (d) rule probabilities

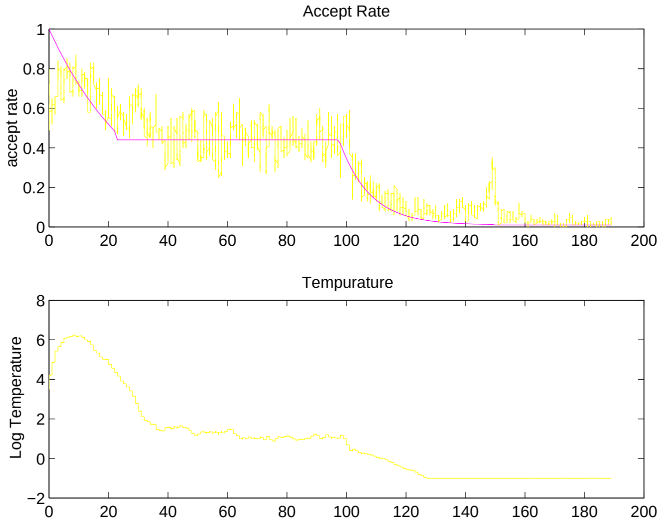


Figure 4.12: Schedule data for case (a) with the static trajectories

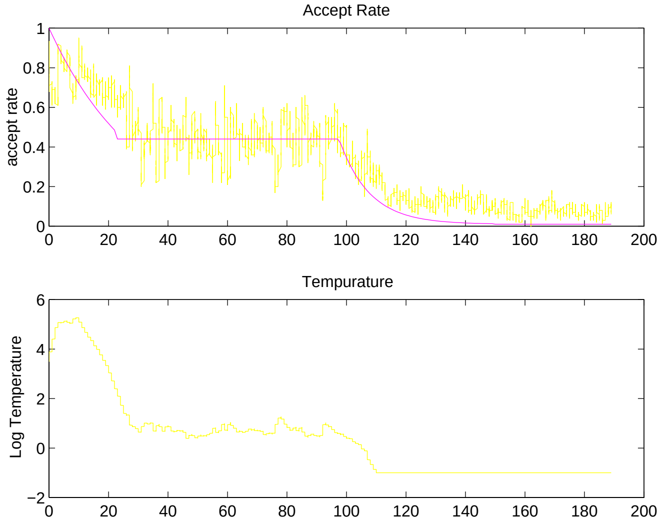


Figure 4.13: Schedule data for case (c) with the Hustin move set

Using rule quality to determine topology rule selection has several interesting impacts on the shape grammar. Shape grammar rule qualities can be interpreted as dynamic state labels, a formalism used in grammars to imply when a design rule is applicable in the design process, based not on the modeled design process but rather on the merit of the rule in a particular design situation. Comparing rule qualities among grammar rules can provide for an assessment of the relative quality of rules to indicate non-beneficial rules. Also, if a grammar rule is implemented that is not appropriate for a particular design problem and generally results in detrimental designs, the rule will only be selected based on the minimum probability. An additional impact of the Hustin move set is on the shape annealing method itself. Previously the grammar rules were selected without regard to their functional impacts on the design. This resulted in a form-driven strategy where the function of a design follows the generated form but is selected in the optimization based on the functional evaluation alone. Basing the rule selection on the functional merit of a rule increases the coupling between form and function in the design process creating more of a function-driven strategy.

Although it is not clear that the Hustin move set provides any great advantage over the static trajectories for this problem, the advantage of this method is that no schedule parameters were changed in order to achieve similar results. Previously the static trajectories were tweaked until good solutions were found. The disappointment of the method was that it was not able to steer clear from the local optimum for this problem (two bar solution) any more so than the static method.

4.3.3 Cost Function

The cost function is used in the optimization method to determine the quality of a design. This function is formulated as the combination of a multicriterion objective function (efficiency, economy, utility and elegance) and dynamically weighted constraint violations (stress, Euler buckling, displacement and overlap of geometric obstacles):

$$\text{cost function} = \text{objective cost} + \text{constraint cost}, \qquad \text{Eq. 4.2}$$

$$\text{objective cost} = \sum_{i=1}^{l} (\text{objective weight}_i * \text{objective value}_i), \qquad \text{Eq. 4.3}$$

$$\text{constraint cost} = \sum_{j=1}^{m} (\text{constraint weight}_j * (\sum_{k=1}^{n} \text{constraint violation}_{j,k})), \qquad \text{Eq. 4.4}$$

where:

l = number of objectives,

m = number of constraints, and

n = number of loads.

This cost function also supports multiple independent loading conditions. For each independent loading condition the structure is analyzed for violations of the behavioral limits (stress, buckling and displacement) and these violations are summed resulting in the total violation for all loading conditions.

4.3.3.1 Objectives

A summation of the design objectives and their weights comprise a total quality measure for a design. The objective weights are set *a priori* by the designer and represent the tradeoff among objectives. The objective weights incorporate a scaling factor in order to keep all objectives in the same approximate numerical range. The objective weights have been formulated in two different ways: (1) such that they correspond to the relative tradeoff with the structural mass that is given a weight of one, and (2) such that all weights sum to one indicating a ranking of importance among design goals. The first formulation will be used in the examples of planar trusses in Chapter 5 and geodesic-like domes in Section 6.1 while the second formulation will be used in the design of roofs in Section 6.2. The primary difference between the two approaches is the modeled tradeoff. While the first formulation imposes a priority of structural mass and all other design goals are treated relative to this primary design goal, the second formulation treats all design goals equally and attempts to specify the relative tradeoff among all design goals. Setting weights in multicriteria optimization is a difficult and somewhat arbitrary task and the second formulation has shown to make modeling the desired tradeoffs an easier task.[1]

4.3.3.2 Constraints

Constraints in the optimization model are soft constraints, implying that they can be violated during the design process, that direct the design to a functionally feasible state. The constraints that can be included in structural design are allowable stress and buckling limits, allowable joint displacements either absolute or relative to the attached members, and overlap of geometric obstacles.

4.3.3.2.1 Constraint Weights

Each constraint has an associated dynamic weight, or constraint weight, that acts as a penalty function when combined with the optimization to push the constraint violation to zero as the design progresses. The constraint weights are dynamically modified based on feedback from a predefined decreasing target violation (Ochotta, 1994); see Figure 4.13. If the violation of a constraint is higher than the target violation the constraint weight is increased in order to increase the influence of this constraint violation in the cost function; if the constraint violation is equal to the target violation the constraint weight is not changed since it is working effectively; if the constraint violation is lower than the target violation the constraint weight is decreased thus decreasing the influence of the constraint violation in the cost function and allowing more designs with this violation to be accepted. Each constraint weight changes individually allowing the search to be driven by the largest term in the cost function: either the constraint with the greatest violation or the dominating design objective value. Since simulated annealing attempts to

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[1] Utility functions are another option for a multiobjective optimization formulation but are not considered here since the expense of creating purposeful utility functions is quite large.

minimize each term in the cost function, constraint violations are driven down by the end of the schedule. This implies that the only feasible design generated may be the final design. However, since in the cost function everything is a tradeoff it is not necessarily true that a feasible design will be generated.

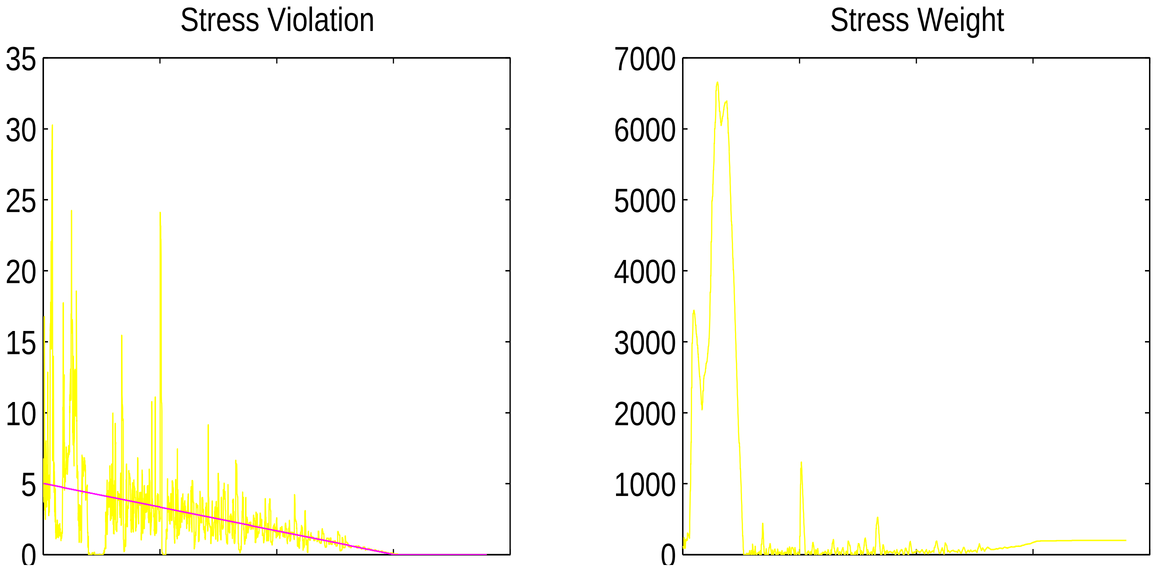

Figure 4.13: Dynamic weighting

4.3.3.3 Dynamic Normalization

Scaling is an important concern in optimization formulations that use a cost function consisting of a summation of individual cost terms (objective costs and constraint costs). To address the issue of scaling, a study of dynamic normalization for both the constraints and objectives was performed. Dynamic normalization is beneficial in making the cost function more robust for different magnitude objective costs and constraint costs. The purpose of this study was to find a technique that improved convergence and provided better avoidance of local optima by maintaining important balances in the cost function: (1) the balance among constraint violation costs, (2) the balance among objective costs, and (3) the balance between the objective cost and the constraint cost.

Independent normalization factors were added to each term in the cost function and updated dynamically to allow the normalization to adapt to the problem. The addition of these normalization factors results in new objective and cost function formulations shown in Equations 4.5 and 4.6 where the independent normalization factors are determined from either the minimum or maximum value of that term (objective value or constraint violation) from the previous iteration:

$$\text{objective cost} = \sum_{i=1}^{l} \left(\text{objective weight}_i * \frac{\text{objective value}_i}{\text{objective normalization factor}_i}\right), \qquad \text{Eq. 4.5}$$

$$\text{constraint cost} = \sum_{j=1}^{m} \left(\text{constraint weight}_j * \left(\frac{\sum_{k=1}^{n} \text{constraint violation}_{j,k}}{\text{constraint normalization factor}_j}\right)\right), \qquad \text{Eq. 4.6}$$

where:

l = number of objectives,

m = number of constraints, and

n = number of loads.

The normalization factors provide a relative measure compared to the best or worst value seen from the previous iteration. Since a constraint violation is in the range $(0,\infty)$ the constraints are normalized only by the maximum value of the previous iteration to avoid the problem of normalizing by a zero. Since the constraint weights change dynamically they were not used in the determining the normalization factor in order to avoid coupling issues.

An additional method for dynamic normalization was considered that uses the vector norm of all constraint violations that is then used to normalize the total constraint cost rather than normalizing individual terms:

$$\text{constraint cost} = \frac{1}{\|\text{constraint violations}\|} * \sum_{j=1}^{m} \left(\text{constraint weight}_j * \left(\sum_{k=1}^{n} \text{constraint violation}_{j,k}\right)\right), \qquad \text{Eq. 4.7}$$

where:

$\text{constraint violations} = \{\text{constraint violation}_1, \text{constraint violation}_2, \ldots, \text{constraint violation}_m\}$

m = number of constraints,

n = number of loads.

Since it is difficult to anticipate which normalization scheme will perform better with simulated annealing a study of four different normalization schemes was performed for the same example problem used in the Hustin tests; the layout of a truss subject to stress and buckling constraints. Two forms of the objective function were considered: a single objective of mass, and two weighted objectives: mass and the standard deviation of the length of members. Since the purpose of normalization is to make the cost function more robust, this study will show that appropriate normalization techniques used with simulated annealing can lead to better quality solutions as well as improved convergence.

For each of the six normalization cases ten runs were performed with the results shown in Table 4.2. This is not a statistical survey but rather a quest for significant improvements made by adding a dynamic normalization scheme. Case (a) is the base case with no normalization. The local optimum is indicated by the maximum value found, 3947.42, and was generated two out of ten times. For case (b), which normalizes the constraint cost by the norm of the constraint violations, the cost function showed significant improvement over the base case in both quality and the range of the solutions found. Also, the two bar solution was avoided. Cases (b) and (c) consider scaling among constraints and show that normalizing by the maximum value produces better quality results based on the mean objective but the standard deviation remains unchanged. Since normalizing by the maximum value has proven advantageous for the constraints, case (c) considers scaling between the objective cost and the constraint cost by normalizing both the objective values and constraint values by the maximum. This leads to scaling problems between the objective cost and the constraint cost since the constraint weights are dynamic and in the range $(1, \infty)$ while the objective cost is now predominantly in the range $\{0,1\}$ (note: it can occur that a new minimum objective value is found in which case the objective cost will be greater than one).

The next normalization cases consider multiple objectives, or scaling between objective values. The intention of weighting schemes for multiple objectives is to specify the tradeoff among objectives. The purpose of including a normalization scheme would allow for a clearer and less ad-hoc means of setting these weights since the range of objective function values would be known up front. For each case the normalization of constraints was performed using the maximum value from the previous iteration since this case showed the most promise in the previous tests. Case (f) considers normalizing the objective values by the minimum value of the objective from the previous iteration while case (e) uses the maximum value. The objective values listed are the non-normalized values for the objective function. These tests of normalizing schemes for the objective function indicate that the objectives are best left without a normalization factor.

The results of this study show that among the cases considered it is advantageous to normalize the constraints by the maximum value and to use constant normalization, or scaling factors, for the objectives. This formulation of the cost function improved both the convergence of the method and the quality of solutions generated versus the previous implementation by Reddy and Cagan; see Table 4.3. However, the value of the cost function is highly coupled with other parts of the annealing process such as the Hustin probabilities and the temperature schedule. Since these parts of the algorithms were left unchanged the conclusions made are in this context. Figure 4.14 shows the schedule data for the best design in case (f).

Table 4.2: Normalization case data

Case	*Objective*	*Normalization of Objectives*	*Normalization of Constraints*	*Mean Objective*	*Standard Deviation*	*Min*	*Max*
(a)	mass	none	none	2171.52	1009.20	1057.20	3947.42
(b)	mass	none	norm	1511.58	359.27	998.42	2076.80
(c)	mass	none	maximum	1279.88	343.84	852.92	2073.84
(d)	mass	maximum	maximum	2694.25	860.46	1432.50	4226.73
(e)	.2*mass + .8*σ_1	constant	maximum	393.62	58.64	274.86	471.42
(f)	.2*mass + .8*σ_1	minimum	maximum	586.57	108.90	365.36	699.06
(g)	.2*mass + .8*σ_1	maximum	maximum	525.58	70.83	431.89	651.54

Table 4.3: Convergence comparison between old and new implementations

	mean mass	*standard deviation*
current implementation	1247 kg	349 kg
Reddy and Cagan (1995)	2923 kg	477 kg

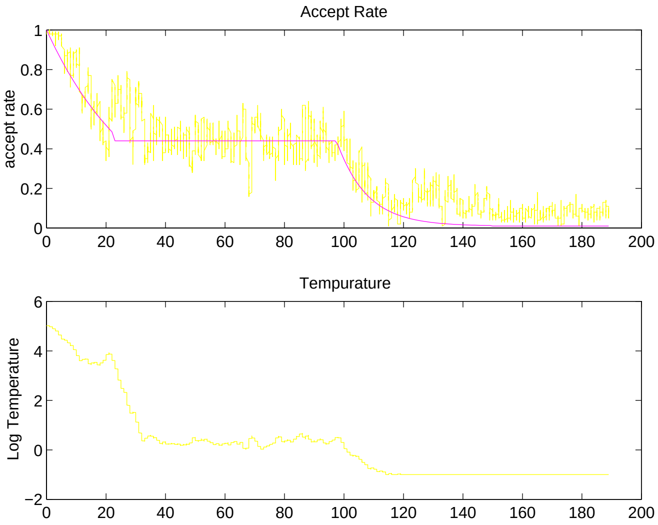

Figure 4.14: Schedule data for the best design in case (f).

4.4 Dynamic Grouping

To determine the groups of members within the shape annealing method, for each new structure the members are regrouped based on cross-sectional area. This process takes place at each iteration after a new design has been generated but before the analysis is performed so that the analysis reflects the cross-sectional areas imposed by the grouping. To group members, they are first sorted into increasing order by cross-sectional area. The first group is set to the smallest member cross-sectional area. The range of cross-sectional areas for this group has as its lower limit the first cross-sectional area and as the upper limit the first cross-sectional area plus a defined tolerance. Members are added to this group as long as they fall in this range. Once a member is found that is larger than the upper limit of the current group range a new group is created with the new cross-sectional area as the lower limit and the new cross-sectional area plus the tolerance as the upper limit. This process continues until all members are assigned to a group. The cross-sectional area variable for each member in a group is then reset to the calculated group size which is the average cross-sectional area of all members in the group. If discrete sizes are desired the group size can be assigned to the closest discrete size to the calculated average cross-sectional area from a defined set. Members within a group still maintain independent cross-sectional area variables after the members are grouped. Members at or near the minimum allowable cross-sectional area are not considered in the grouping since they may be removed from the structure as long as the structure remains stable. If a member of minimum area is not removed from the structure for stability or aesthetic reasons the cross-sectional area is reset to the smallest group cross-sectional area. Another alternative would be to allow an extra group of the minimum cross-sectional area.

The tolerance used in grouping the members defines the range of cross-sectional areas within a group that will be forced to a common group size or cross-sectional area. The tolerance is determined from the user defined variables, the start tolerance and the end tolerance, where the start tolerance is larger than the end tolerance. The tolerance is then calculated over the annealing schedule and decreases linearly from the start tolerance to the end tolerance allowing looser groupings of members at the beginning of the annealing and tighter groupings towards the end. The tolerance is always smaller than the maximum change in cross-sectional area so that upon application of the rule to change the cross-sectional area of a member the member may move between groups. The same method was applied to the grouping of members by length with the distinction that member lengths were not changed due to coupling effects.

4.5 Analysis

In order to evaluate the designs for structural behavior, structural analysis of each design must be performed. For ease of both truss and beam design, analysis of designs is performed using external finite element tools through interfaces to both FElt (Gobat and Atkinson, 1994) and MSC/NASTRAN. For all designs generated in this work FElt was used since although MSC/NASTRAN provides broader modeling capabilities the overhead involved is too great. Analysis is the most time consuming part of the design process and generally accounts for 80% of the computation time. Computation time improvements could be made through the integration of analysis within the shape annealing method and will be discussed in Chapter 8.

4.6 Summary

The implementation of the shape annealing method was presented which consists of a graph-based representation, a modified Lam-Delosme simulated annealing schedule, and structural analysis using the finite element tools. Additional techniques to the annealing process, a Hustin move set and dynamic normalization and weighting of constraint violations, were added to improve the solution quality and convergence.

5. ESSAYS OF DISCRETE STRUCTURES, PART 1: PLANAR TRUSSES

Planar truss systems are the foundation for many structures such as roofs, bridges and towers. This chapter presents an exploration of the language of planar truss design to create essays of cantilevered trusses, bridge trusses, and transmission towers. All structures presented were generated using rules from the planar truss grammar illustrated in Chapter 3. Since planar truss structures are often utilitarian structures the primary design goals considered are efficiency and economy whereas elegance is only incorporated as a constraint on symmetry, the simplest model of formal aesthetics. Utility of the structure is incorporated in the design problem as a problem constraint, where necessary, and is modeled by geometric obstacles and support locations.

5.1 Design for Efficiency

The design for efficiency alone provides a base investigation of the capability of shape annealing in finding optimally directed least weight structures. This example presents an essay of design alternatives for a cantilever truss problem where the standard solution is a ten-bar truss. Solutions from shape annealing to the least weight design of a truss subject to stress and Euler buckling limitations are shown in Figure 5.2[1]. The material properties for these designs are listed in Table 5.1 with the parameters used in the generation listed in Table 5.2. These designs can be compared with the a shape optimized ten-bar truss layout that has a mass of 2129 kg. An additional design goal was added to standardize member lengths, such as in the ten-bar truss, by minimizing the standard deviation of all lengths, denoted σ_l[2]. Designs generated for this multiobjective problem are shown in Figures 5.3 and 5.4.

A few interesting characteristics of the relation between form and function can now be noted from this essay of cantilevered truss designs. While the topologies of the two least weight solutions in Figure 5.2 are similar, by adding a design goal of uniformity the solutions generated become quite different (Figures 5.3 and 5.4). The structures in Figure 5.3 present two interesting forms that solve the same problem. While the design in Figure 5.3(a) attempts to make the entire design uniformly divided, the design in Figure 5.3(b) seems to be a merger between two design styles joined by a vertical compression member. The division of structural topologies into styles within a design will be a recurrent trend in the designs presented. Figure 5.4 presents two designs to the multiobjective

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[1] For each essay of structures a note concerning the scaling of line widths in the illustrations will be made. While an attempt was made to show a structure as it would look if built, this was not always possibly due to the limitations of line widths (a line with a width less than one pixel could not be drawn). For this set of designs the dimensions of members can be compared among designs, but there is a scaling factor of approximately 2 when comparing a members width to the overall geometry of the design.

[2] Here the model of uniformity has functional implications and is not considered as an aesthetic design goal although the optimization model is the same as the model for visual uniformity.

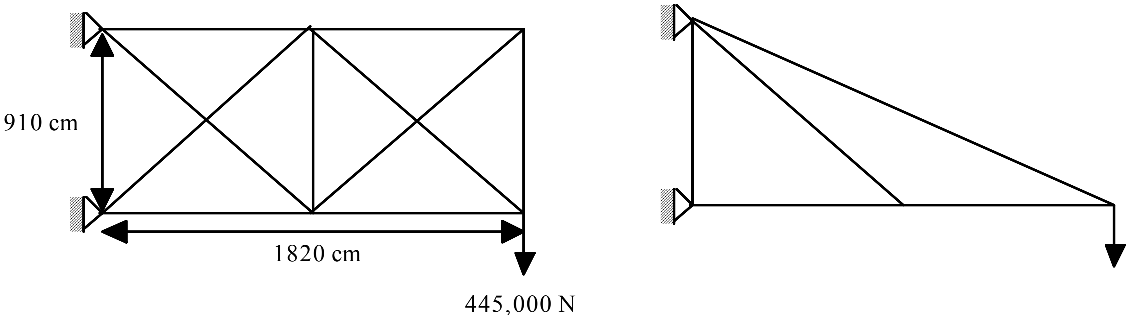
problem that have drastically different topologies but the same quality, that is the same objective value, which provides the designer with options for visual style without loosing design functionality.

Table 5.1: Material properties for cantilever truss problem (Figures 5.2-5.4)

Material Property	
modulus of elasticity, E	*$6.88 E^6$ N/cm^2*
allowable tensile stress	*17,200 N/cm^2*
allowable compressive stress	*17,200 N/cm^2*
mass density, ρ	*.0027 kg/cm^3*

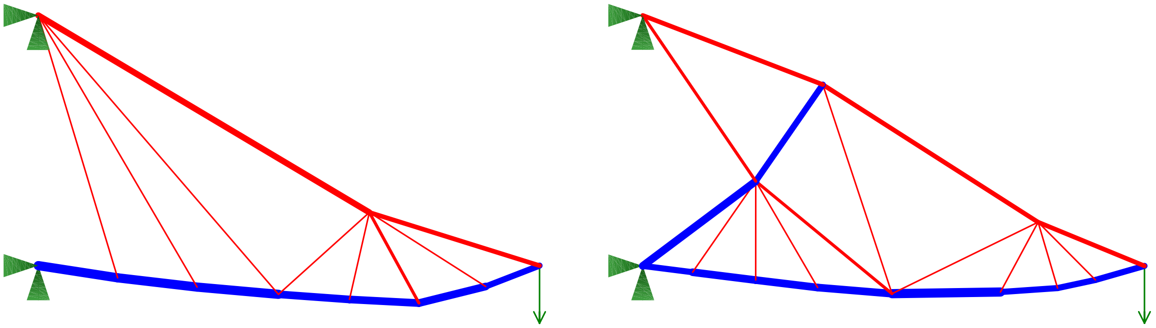
Table 5.2 Method parameters for cantilever truss problem (Figures 5.2-5.4)

Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum number of members	25
maximum area	none	number of iterations	170
member areas	continuous	number of designs per iteration	200
minimum member length	15 cm	planar truss topology rules	1-5
minimum angle between members	1°	rule selection	Hustin
intersections between members	not allowed	constraint violation normalization	yes
member shape	solid rod		



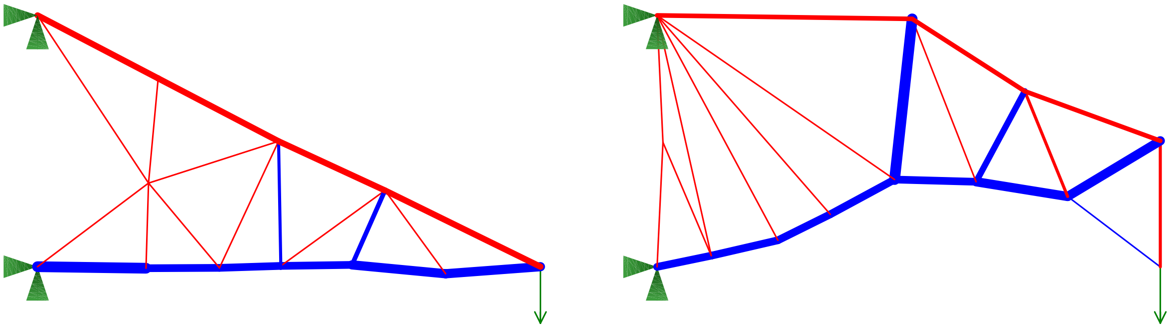
(a) ten bar truss; mass = 2129 kg (b) initial shape

Figure 5.1: Cantilever truss problem specification



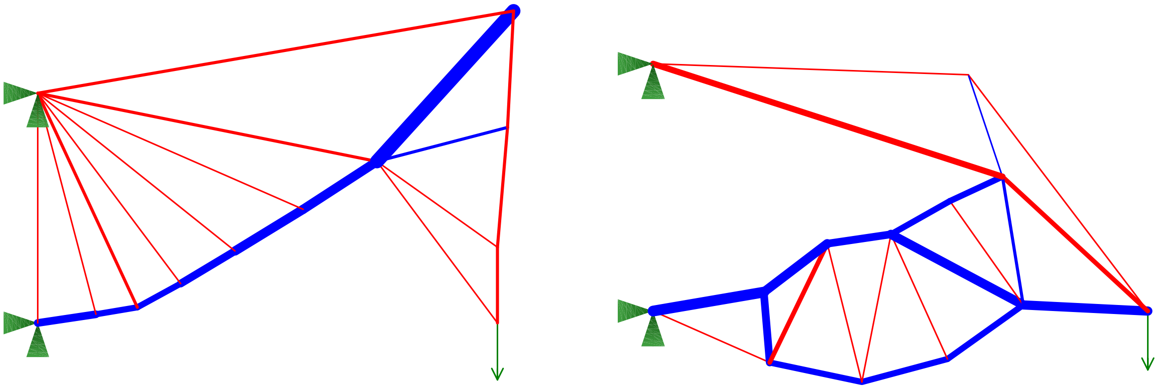
(a) mass = 853 kg (b) mass = 991 kg

Figure 5.2: Cantilever truss designs for minimum mass



(a) mass = 1038 kg, σ_l = 84 cm (b) mass = 1505 kg, σ_l = 158 cm

Figure 5.3: Cantilever truss designs for minimum mass and uniform member length



(a) mass = 1426 kg, σ_l = 299 cm (b) mass = 1428 kg, σ_l = 300 cm

Figure 5.4: Cantilever truss designs with similar objective values and different forms

5.2 Design for Efficiency and Economy

Since the design of discrete structures for pure efficiency has few practical applications, the optimization model is expanded to include a model of economy resulting in the generation of an essay of planar trusses that consider the tradeoff between efficiency and economy. The design goal of economy considers the cost of a building structure and is a function of the cost of purchasing and fabricating materials as well as the labor involved in constructing the structure. Adding the design goal of economy to the optimization model enables the generation of designs that reflect a practical design goal that is often considered in the design of utilitarian structures such as bridges, towers and arches. Economy is modeled in the design generation by using a method for the dynamic grouping of members by cross-sectional area, which was discussed in Chapter 4, and by adding a term to the optimization model that is a function of the number of groups. The number of groups in a structure can be introduced into the optimization model as a constraint on the number of allowable groups, as an objective function to minimize, or as a hard constraint using a fixed number of groups. Results from the first two formulations will be presented using examples of bridge trusses, arches and transmission towers. The principles of tensegrity structures will then be used as an example of generating designs with cables and compression bars, or structures with only two allowable groups.

5.2.1 Constraining the Number of Discrete Cross-Sections

Generating topologies for optimal grouping, where the number of allowable groups is three, will be demonstrated on three problem specifications: an arch, a truss and a symmetric truss. By adding a soft constraint on the number of desired cross-sectional areas a tradeoff is created between the mass of the structure and the number of cross-sectional area groups. Two types of boundary conditions are considered in these examples. The material properties for theses examples are listed in Table 5.3 with the method parameters listed in Table 5.4. The three examples presented are as follows: the first example has both supports fixed resulting in an arch-like structure, the second is simply supported resulting in a truss-like structure, and the third example requires the design to be symmetric. Although the arch and truss problems only differ in one boundary constraint the resulting designs portray different characteristics demonstrating the method's ability in finding suitable forms to meet the required function. For all examples, an obstacle is placed in the space below the supports to force the design to remain above the line of supports. Obstacles are geometric constraints that can be used by the designer to push the design into a prescribed area or keep the design out of restricted areas. Among the designs presented observations can be made concerning the tradeoff between mass and the number of groups in the design and the effects of this tradeoff on the design form.

Table 5.3: Material properties for Figures 5.5-5.7

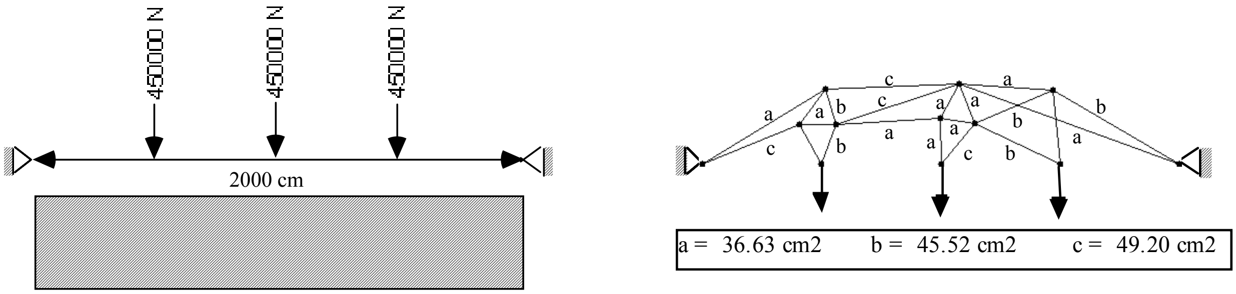
Material Property	
modulus of elasticity, E	6.88 E^6 N/cm^2
allowable tensile stress	17,200 N/cm^2
allowable compressive stress	17,200 N/cm^2
mass density, ρ	.0027 kg/cm^3

Table 5.4 Method parameters for Figures 5.5-5.7

Method Parameter		*Method Parameter*	
minimum area	.06 cm^2	allowable number of groups	3
maximum area	none	maximum group tolerance	3 cm^2
member areas	continuos	minimum group tolerance	.6 cm^2
minimum member length	.3 cm	number of iterations	170
minimum angle between members	1°	number of designs per iteration	200
intersections between members	allowed	planar truss topology rules	1-4
maximum number of members	50	rule selection	static trajectories
member shape	solid rod	constraint violation normalization	none

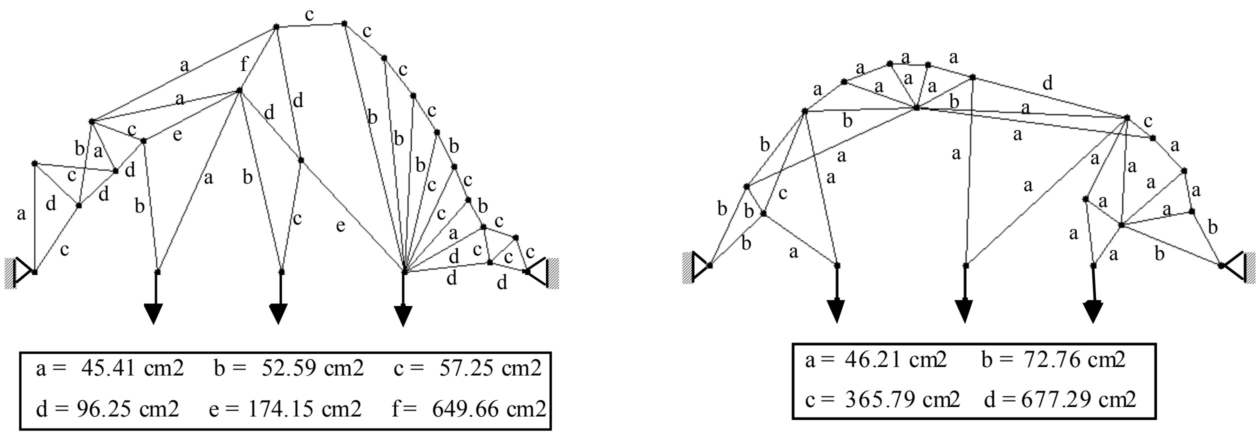
5.2.1.1 Arch Design

The first example is the design of an arch truss for the problem specification shown in Figure 5.5(a). The arch structure shown in Figure 5.5(b) complies to constraints on stress, the number of member groups and avoidance of an obstacle underneath the supports while the designs in Figures 5.5(c) and 5.5(d) additionally comply to a constraint on Euler buckling. The design for stress only, witch does not consider Euler buckling, in Figure 5.5(b) converged to the three allowed groups and resembles an arched truss bridge found in bridge design. The designs in Figures 5.5(c) and 5.5(d) did not converge to the three allowable groups, but resulted in solutions with six and four groups respectively. The design in Figure 5.5(c) having four groups is actually 1.4 % lighter than the design in Figure 5.5(b) with six groups illustrating that for these designs there is essentially no tradeoff between mass and the number of groups within the range of four to six groups. All designs for this problem are asymmetrical since there is no constraint on symmetry and thus exact symmetry is not expected.



(a) arch problem specification

(b) mass = 853 kg (stress only)



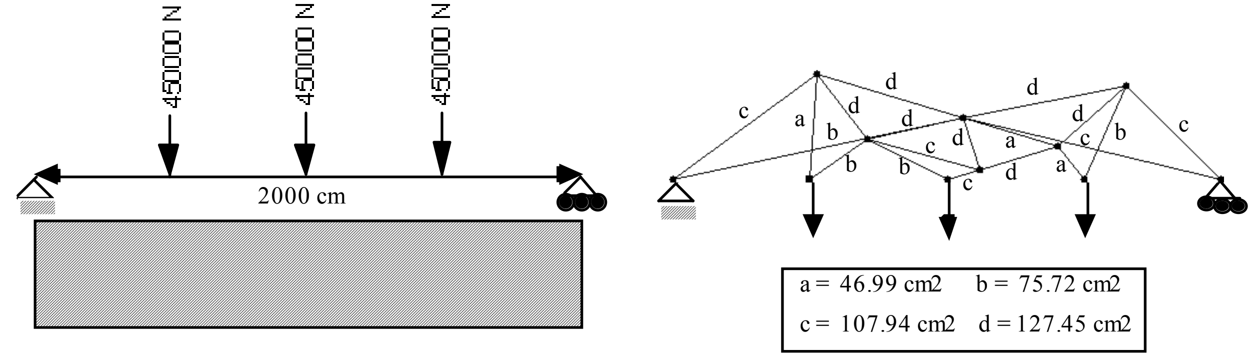
(c) mass = 3, 321 kg

(d) mass = 3, 274 kg

Figure 5.5: Arch design problem

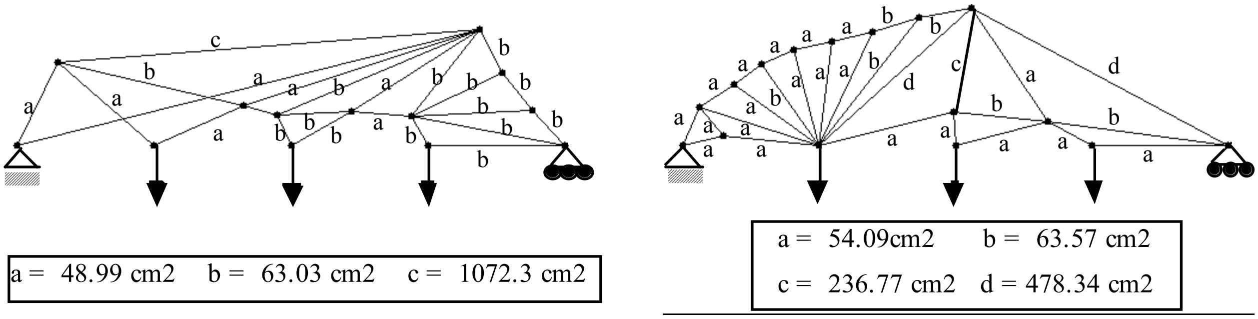
5.2.1.2 Simply Supported Truss Design

The next example is the design of a simply supported truss structure, shown in Figure 5.6(a), considering constraints on stress, the number of groups and avoidance of a geometric obstacle underneath the supports. The structures in Figure 5.6(c) and Figure 5.6(d) have an additional constraint on Euler buckling. The first design shown in Figure 5.6(b) has a mass of 2,225 kg and results in four groups rather than the desired three groups since the tradeoff with mass in this particular design is too large. The designs in Figures 5.6(c) and 5.6(d), which include buckling, converge to different solutions having masses of 5,989 kg and 3,845 kg respectively. The structure in Figure 5.6(c) converged to the three allowed groups but has a much higher mass than the structure in Figure 5.6(d) that has four groups. These designs show that there is an increase in mass in order to meet the allowed number of groups. Examining the forms of the generated structures, the design in Figure 5.6(c) is a unified solution whereas the design in Figure 5.6(d) is a combination of two types of design solutions.



(a) simply supported truss problem specification

(b) mass = 2, 225 kg (Stress Only)



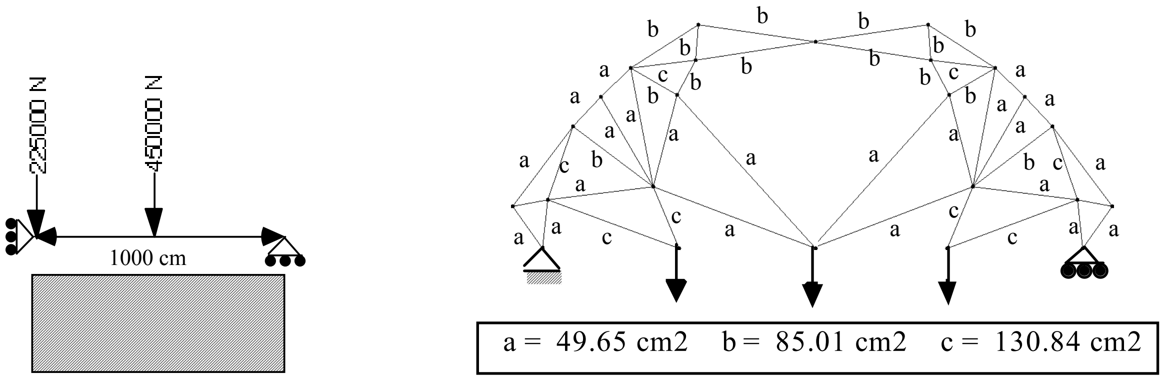
(c) mass = 5,989 kg

(d) mass = 3,845 kg

Figure 5.6: Truss design problem

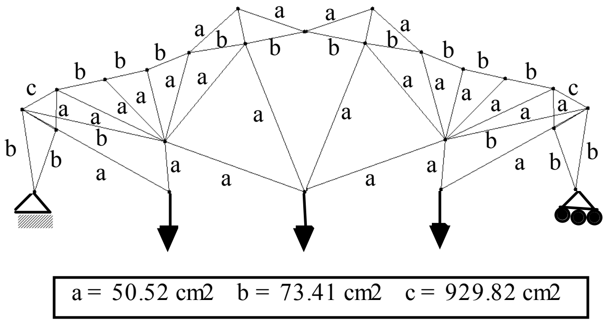
5.2.1.3 Imposed Symmetry

As discussed with the arch problem, the generation of a perfectly symmetric design is not expected using this method. For this reason, this example requires the structure to be symmetric throughout the design generation. The same specifications are used as the example in the previous section but with only one half of the problem modeled where a center line is added as a line of symmetry; see Figure 5.7(a). Any point that lies on the line of symmetry during the design process is given a boundary condition such that there is no displacement perpendicular to the line of symmetry. When the design is reflected across the line of symmetry to create a complete design, any member that lies on the line of symmetry is not reflected so that members do not double up.



(a) symmetric simply supported truss problem specification

(b) mass = 2,946 kg



(c) mass = 2,798 kg

Figure 5.7: Symmetric, simply supported truss designs with three allowed cross-sectional areas

The designs shown in Figures 5.7(b) and 5.7(c) both converged to the three allowable groups specified and have similar masses of 2,946 kg and 2,797 kg respectively for different topologies and group sizes. Comparing the symmetrical design in 5.7(c) to the asymmetrical design in 5.6(c), both with three groups, the symmetrical solution is 53% lighter than the asymmetrical solution. The asymmetrical design in 5.6(d) with an additional group has a closer mass to 5.6(c) but the symmetrical design is still 37% lighter. One reason the symmetric designs show better convergence to the group constraint is that the maximum number of members allowed in the symmetric case is reduced, thus reducing the design space and making it easier to arrange members into groups.

5.2.1.4 Discussion of Results

The results show that imposing a constraint on the number of allowable groups in the structural layout optimization problem affects the mass of the structures generated. An investigation into the increase in cost associated with decreasing the number of allowable groups is now presented. This analysis was performed for the arch example specification shown in Figure 5.5. The constraints used in this analysis are stress, an obstacle below the supports and the number of allowable groups. For each increment in the number of allowable groups ten annealing runs were performed yielding ten best designs and ten final designs for each increment. The mean final cost is the mean of all ten final designs, while the mean best cost is the mean cost of all ten best designs. The cost function used is the tracking cost function based on a constant weight of 1000 for all constraint violations. Baselines were generated using the same problem specifications but with no constraint on the number of allowable groups.

The results from this investigation are shown in Figure 5.8. Comparing the mean final cost curve against the mean final cost baseline indicates that for this problem specification there is always an increased cost associated with a constraint on the number of allowable groups below ten. The cost significantly increases for allowable groups below four implying that there is an increase in tradeoff between mass and number of allowed groups. The fairly flat region on the cost curve between four and nine groups justifies the initial assumption that the changing topology and shape can compensate, to some extent, for a decrease in the number of allowed groups. Note that as the number of allowed groups increases from ten to infinity the group imposed cost will approach the baseline cost.

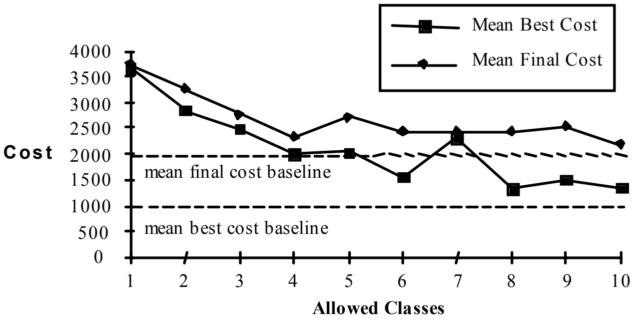


Figure 5.8: Cost vs. allowed groups

Imposing a soft inequality constraint on the number of allowed groups is one approach to including the effects of member groups in truss design. With the current constraint method there is no advantage to finding a design with fewer groups than the allowable number unless there is a decrease in mass, because at this point the constraint is not active. The effect of including the number of groups in the design objective function creates a continuous tradeoff between the number of groups and the mass of the structure and will be explored in the following section.

5.2.1.5 Minimizing the Number of Discrete Cross-Sections

The layout of trusses for a minimum number of groups relative to the mass of the structure is now considered. This formulation for generating topologies with optimal groups will be demonstrated on two problem specifications, a symmetric transmission tower with asymmetric loading and a symmetric truss. Both examples have constraints on stress and Euler buckling and use the method parameters listed in Table 5.5. Since the generation of a perfectly symmetric design is not expected from this method both examples impose symmetry by representing only one-half of the design throughout the design generation where the final design is reflected to yield a full symmetric design. For analysis purposes symmetric boundary conditions are imposed if the loading is symmetric or if the loading is asymmetric the design is reflected during the design generation to create a full analysis model.

Table 5.5: Method parameters for Figures 5.9 and 5.10

Method Parameter		*Method Parameter*	
minimum area	.06 cm^2	maximum group tolerance	3 cm^2
maximum area	none	minimum group tolerance	.6 cm^2
member areas	continuous	number of iterations	170
minimum member length	15 cm	number of designs per iteration	200
minimum angle between members	10º	planar truss topology rules	1-4
intersections between members	not allowed	rule selection	static trajectories
maximum number of members	50	constraint violation normalization	none

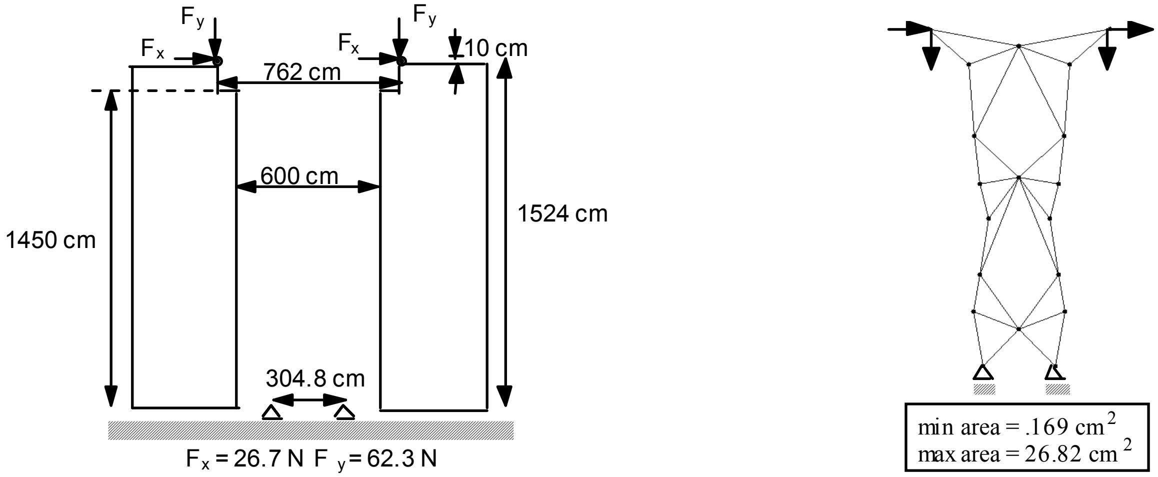
5.2.1.6 Transmission Tower

The transmission tower problem specifications shown in Figure 5.9(a) are based on the planar tower problem in Vanderplaats and Moses (1972). Three geometric obstacles were used in the problem formulation as a model of the physical design space: two obstacles are to the sides of the supports and one is below the supports representing ground level. The side obstacles were chosen to allow access to the power lines that are represented by the applied load F_y. The second applied load, F_x, represents the wind load that creates an asymmetric loading. Since the structure is required to be symmetric the wind load the resulting structure withstand wind coming from either direction. The material properties of the members are listed in Table 5.6.

Table 5.6: Material properties for tower designs (Figure 5.9)

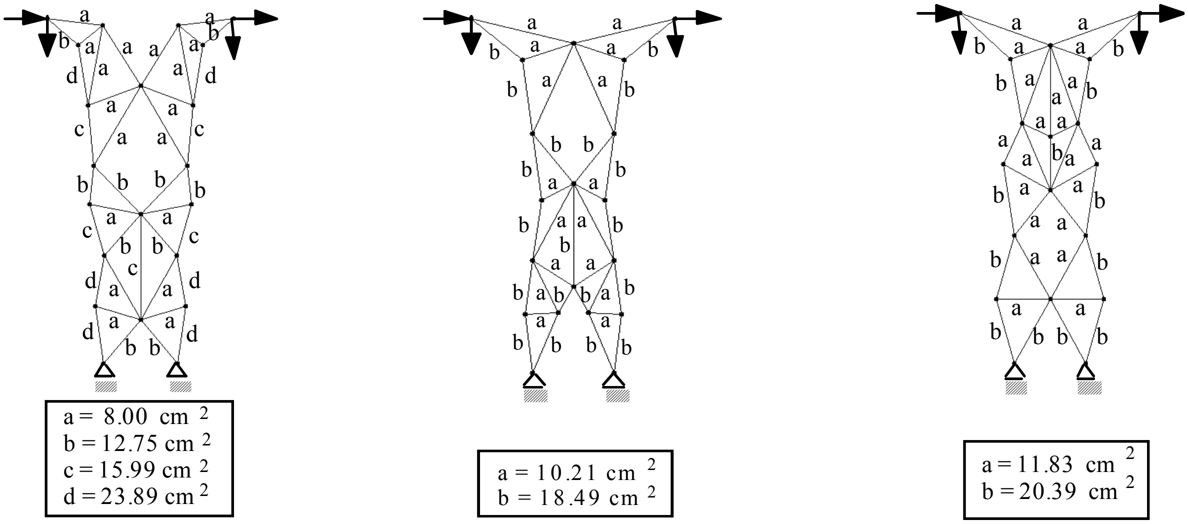
Material Property	
modulus of elasticity, E	2.067 E^7 N/cm^2
allowable tensile stress	13,790 N/cm^2
allowable compressive stress	10,340 N/cm^2
mass density, ρ	.0083 kg/cm^3
member shape	tube, d/t = 10

The structure in Figure 5.9(b) shows the best design generated for this problem without a restriction on cross-sectional area. This design, with a mass of 850 kg, is comparable to the best designs found by Vanderplaats and Moses (1972) who used shape and sizing optimization of a fixed topology to produce optimized structures with masses ranging from 844 kg to 866 kg. Figures 5.9(c) through 5.9(e) show the best design, which is determined by the cost function, from 30 designs generated for each group weight. The group weight determines the relative importance of minimizing the group objective in the objective function. All of the designs with the group restriction result in an increase of mass over the continuous sizing of design 5.9(b) as expected. The group sizes determined by the algorithm for designs 5.9(d) and 5.9(e) with only two groups are in the middle of the range of the cross-sectional areas of design 5.9(b) where there is no group restriction. When the number of groups increases to four in design 5.9(c), the group sizes move towards the extremes of the range of cross-sectional areas in design 5.9(b). When the group weight is increased from two to five, Figures 5.9(c) and 5.9(d) respectively, solutions are found with an increase in mass but a decrease in the number of groups from four to two. This increase in mass is due to the increase in importance of minimizing the number of groups. Comparing designs 5.9(d) and 5.9(e), increasing the group weight from five to ten results in a design with the same number of groups and an increase in mass. This increase in mass could be due to the algorithm focusing more on minimizing the number of groups rather than the mass causing certain moves that lead to design 5.9(d) to be rejected when generating design 5.9(e). Again, the shape annealing method does not guarantee that a global optimum will be found but rather that optimally directed designs are produced.



(a) tower problem specification

(b) mass = 850 kg



(c) group weight = 2

mass = 1045 kg

(d) group weight = 5

mass = 1121 kg

(e) group weight = 10

mass = 1196 kg

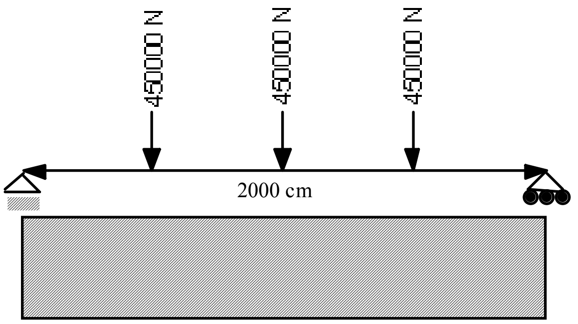
Figure 5.9: Transmission tower designs for optimal groupings

5.2.1.7 Truss Problem

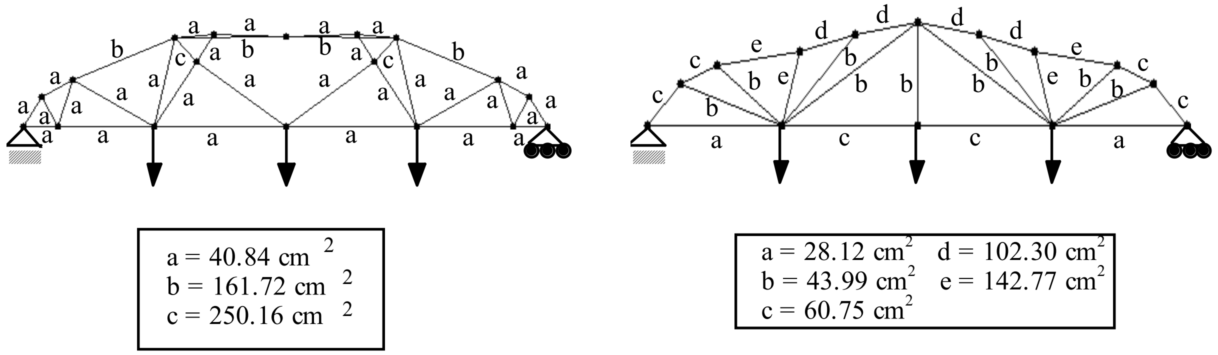
This example presents the design of a standard truss structure, like that for a bridge, using steel with material properties listed in Table 5.7. There is one geometric obstacle placed below the supports to represent ground level; see Figure 5.10(a). The group weight is set to 20 for the designs shown in Figure 5.10. To illustrate the tradeoff between mass and the number of groups for a single objective function formulation two designs with similar cost values but different topologies are shown. The design in Figure 5.10(b) has a higher mass than that in 5.10(c) but two fewer groups, trading off economy of scale with mass.

Table 5.7: Material properties for symmetric truss design (Figure 5.10)

Material Property	
modulus of elasticity, E	2.067 E^7 N/cm^2
allowable tensile stress	25,000 N/cm^2
allowable compressive stress	25,000 N/cm^2
mass density, ρ	.00785 kg/cm^3
member shape	solid bar



(a) truss problem specification



a = 40.84 cm^2
b = 161.72 cm^2
c = 250.16 cm^2

a = 28.12 cm^2 d = 102.30 cm^2
b = 43.99 cm^2 e = 142.77 cm^2
c = 60.75 cm^2

(b) mass = 4923 kg
cost = 5485

(c) mass = 4131 kg
cost = 5524

Figure 5.10: Truss designs for optimal groupings

5.2.1.8 Discussion of Results

An investigation of the tradeoff between structural mass and the distinct cross-sections in the structure is now shown. Specifically, the increase in mass associated with an increase in the importance of groups is investigated by increasing the group weight in the objective function and observing the resulting structural masses. This analysis was performed for the transmission tower example shown in Figure 5.9. Group weights of two, five and ten were considered with 30 designs generated for each group weight. A comparison of mass and number of groups for all designs generated in this study is shown in Figure 5.11. When the group weight is set to two, the majority of designs found have three and four groups, while when the group weight is increased to five, the majority of designs have two and three groups. When the group weight is increased to ten, only designs having two and three groups are found.

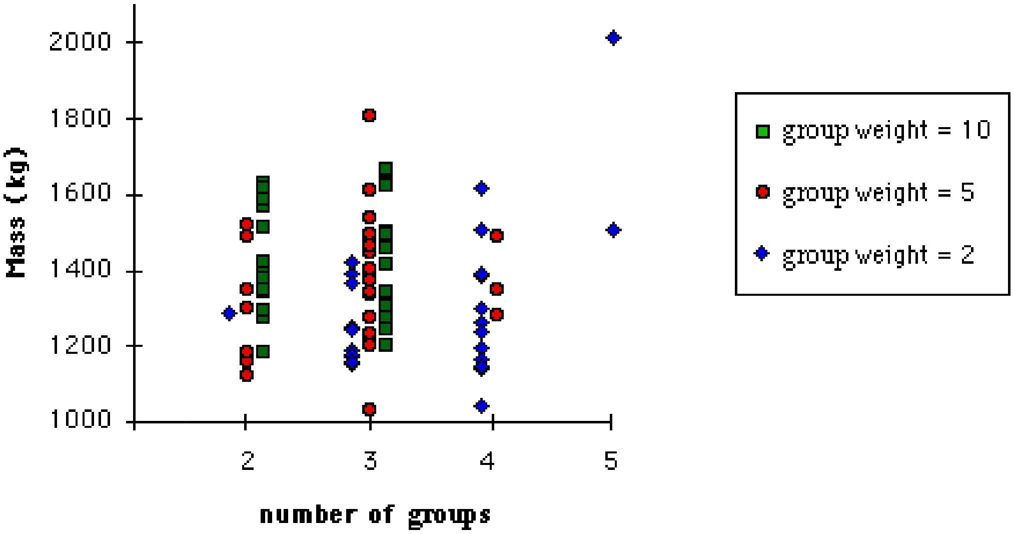


Figure 5.11: Comparison of group weights

As the group weight increases the mean number of groups decreases and the mean mass increases. This is shown in Table 5.8, which includes statistics for all thirty designs for each group weight. Additionally, since the group sizes are not determined a priori, the range of group sizes increases as the number of groups increases, as shown in Figure 5.11. Because simulated annealing can get trapped in local optima it is standard practice to take the best design out of every three designs generated. Using this heuristic, a second set of statistics is shown in Table 5.9 calculated from a total of 10 designs for each group weight. For the tower problem, the algorithm tends to get trapped in a local optimum on average one in ten times; thus, the convergence statistics are significantly improved by using the one in three heuristic.

Table 5.8: Convergence Statistics for All Tower Results

	Group Weight		
	2	*5*	*10*
mean mass, kg (σ)	1359 (210)	1418 (226)	1489 (203)
mean cost (σ)	1473 (251)	1720 (908)	3058 (5804)
mean number of groups (σ)	3.73 (.70)	2.90 (.61)	2.53 (.51)

Table 5.9: Convergence Statistics for the Best 1:3 of All Tower Results

	Group Weight		
	2	*5*	*10*
mean mass, kg (σ)	1221 (111)	1262 (139)	1333 (118)
mean cost (σ)	1314 (105)	1394 (154)	1541 (112)
mean number of groups (σ)	3.40 (.70)	2.70 (.67)	2.40 (.52)

5.2.2 Application: Generating Pseudo-Tensegrity Designs

We will now investigate placing a hard constraint on the number of groups throughout the generation of a structure. Tensegrity systems are structural systems designed using only cables and compression members and as such a fitting application for the design of structures with a fixed number of groups. The word "tensegrity" is a contraction of "tensile-integrity" coined by R. Buckminster Fuller in his patent (1962), although the first models were built by Snelson in 1948 (Motro, 1992), a sculptor and student of Fuller's. The functional rationale Fuller gives for tensegrity systems stems from the design of suspension bridges. Fuller's goal with tensegrity systems was to reduce the size of large compression members, such as the large compression towers used in suspension bridges, and distribute compression throughout the structural system to gain greater economy. This would result in what Fuller described as "islands of compression in a sea of tension," (Baldwin, 1996).

5.2.2.1 Principles of Tensegrity

Multiple interpretations have risen since the introduction of tensegrities concerning what is or is not considered a tensegrity system since these systems can be investigated from different viewpoints: system principles, geometry or mechanics. These viewpoints of tensegrity will be described along with the interpretation used in this work to formulate an optimization model for the design of tensegrity systems using shape annealing. The interesting point of tensegrity systems is that their application in structural design, except as sculptures, has not yet been established and as Fuller remarks the best applications of the tensegrity principle may not even be in the field of structures (Pugh, 1976). The purpose of this investigation is to generate structures with only two cross-sectional area groups, which have different material properties, subject to external loading so that the application of pseudo-tensegrities to a standard structural design problem may be studied.

In a review of tensegrity system research, Motro formulates the following definition of tensegrity combining multiple viewpoints: "Tensegrity systems are systems whose rigidity is the result of a state of self-stressed equilibrium between cables under tension and compression elements and independent of all fields of action, " (Motro, 1992, p.77). Breaking down this definition one important component to model is a self-stressed system. Self-stressed systems can arise from three cases of structural layouts:

1. an indeterminate system, that is more members than are required for stability according to Maxwell's rule,
2. an ill-conditioned system, systems that follow Maxwell's rule but are in general unstable except as a result of certain geometric conditions
3. and, systems that are mechanisms according to Maxwell's rule but whose stability is a result of certain geometric conditions.

The last two cases are special cases of Maxwell's rule that Maxwell himself anticipated (Maxwell, 1864). However, the stability in the latter cases is limited such that any external force applied to the system causes the system to collapse while the force is applied. Thus, these systems will not be investigated for applications with external loading.

In Motro's definition of tensegrity systems he notes that his definition does not incorporate aesthetics or efficiency of the systems. Looking at the aesthetic attributes of tensegrity systems requires the viewpoint of the architect or artist whose interest in these systems is mainly geometric. The striking visual effect that Snelson's sculptures achieve is created by tying large compression members together using thin prestressed wires such that the compression members appear to float in mid-air since the wire is not visible at a distance. Adding this visual effect of tensegrity systems to Motro's definition also reflects Fuller's tensegrity model of systems with discontinuous tension and continuous compression.

The efficiency of tensegrity systems should also be noted. In an investigation by Calladine (1978) of Fuller's tensegrities in the context of Maxwell's rules he remarked that the proclaimed efficiency of such systems was due to the exploitation of prestressed wire in the systems such that four out of the five members were wires as opposed to compression struts. Calladine goes on to say that it is questionable whether there is much to gain from making the structure so sparse through the use of prestressed wire such that any external action on the system renders it useless. Efficiency will be looked at not through the use of prestressed wire but in the same manner as was used in previous examples by minimizing the mass of the structure subject to stress and Euler buckling constraints. The goal in using wire as a structural member is that its high strength to weight ratio, since steel wire has four times the allowable stress as steel bars, will encourage their wide use for carrying tension.

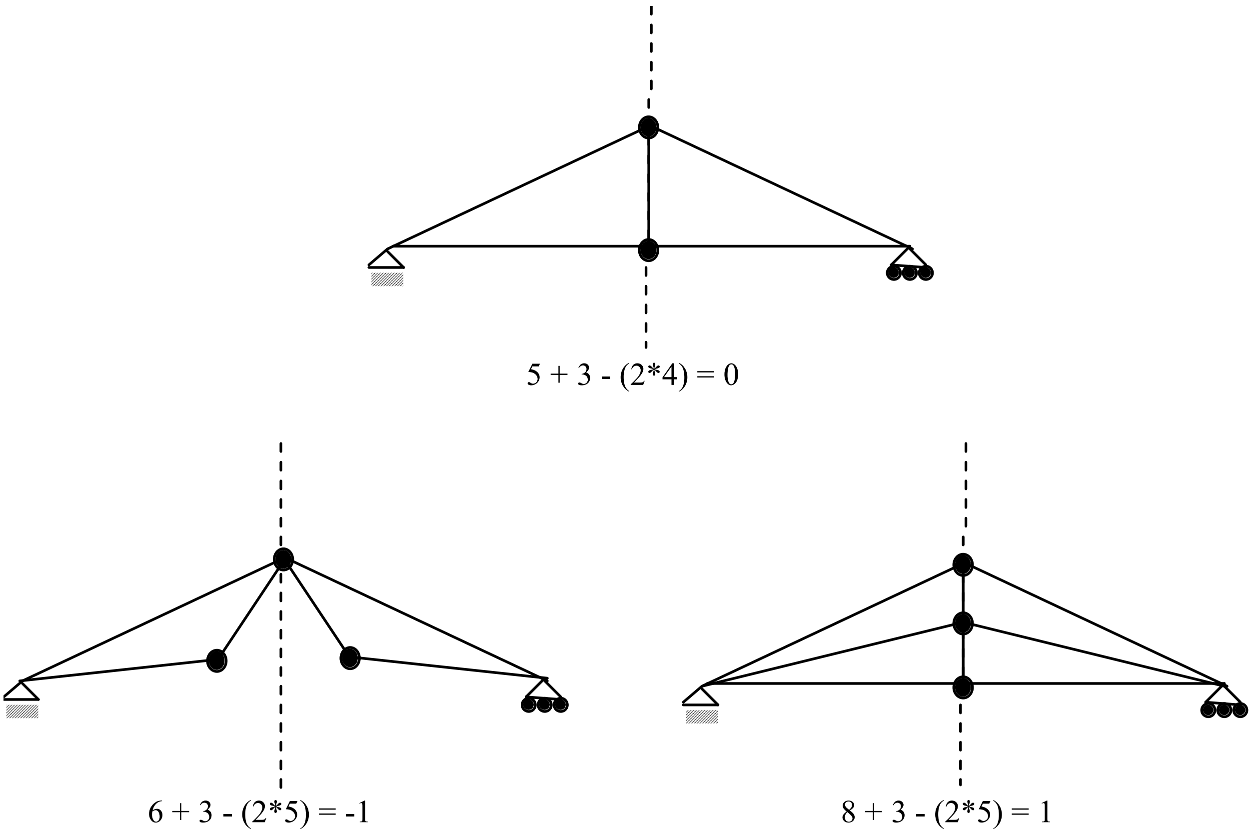
The other aspect of Motro's definition worth noting is that tensegrity systems are "independent of all fields of action". The interpretation used for this model, which is in accordance with Calladine's work, is that the mechanics model of tensegrity systems should only be simply supported such that the stability is not gained through external bearings such as with arch action. This also rules out suspension bridges from being tensegrity systems since although they are redundant structures they are stable due to restoring forces from external bearings.

5.2.2.2 Pseudo-Tensegrity Optimization Model and Results

Based on the previous discussion, the problem formulation for designing structures using tensegrity principles are simply supported, redundant structures whose members consist of steel cable and steel tubes arranged such that no joint connects more than one steel tube. Each structure is parsed to calculate the visual tensegrity objective as:

$$\text{tensegrity objective} = \sum_{1}^{\text{num joints}} (\text{number of tubes connected to joint} - 1). \qquad \text{Eq. 5.1}$$

Since the structural model requires the wire to be in tension with the tubes either in tension or compression, the tensegrity objective is visual rather than functional. However, the goal is that designs using wire for tension members, where possible, will have greater efficiency than those that use tubes to carry tension. In order to generate a redundant structure the problem was modeled as a symmetric design for the application of grammar rules but was reflected for the analysis. Through this combination redundant structures and mechanisms can be generated either by creating a joint on the line of symmetry or from joints originally along the line of symmetry that move away from the line of symmetry. An illustrated example of generating both mechanisms and redundant structures is shown in Figure 5.12 with the corresponding evaluation of Maxwell's rule.



5 + 3 - (2*4) = 0

6 + 3 - (2*5) = -1

8 + 3 - (2*5) = 1

Figure 5.12: Effects of reflective symmetry on truss determinacy

Three different problem formulations will be presented based on the problem specification shown in Figure 5.13 where symmetry is constrained across the dashed line. All problems use the material specifications for steel wire and steel members from Table 5.10 and the method parameters from Table 5.11. The designs in Figure 5.14 present solutions using standard size steel tubes for the compression members while the designs in Figure 5.15 use continuous sizing of solid steel bars. The design in Figure 5.16 is a problem variation that allows the support to move vertically and uses standard size steel tubes for the compression members.[3]

Table 5.10: Material properties for pseudo-tensegrity designs (Figures 5.14-5.16)

Material Property	*Steel Wire*	*Steel Tube or Bar*
modulus of elasticity, E	1.655 E^7 N/cm^2	2.067 E^7 N/cm^2
allowable tensile stress	48,403 N/cm^2	11,160 N/cm^2
allowable compressive stress	not allowed	11,160 N/cm^2
mass density, ρ	.00785 kg/cm^3	.00785 kg/cm^3
maximum area	10 cm^2	96 cm^3

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[3] The line widths in the designs in Figures 5.14 through 5.16 use a scaling factor of approximately 2 when comparing a members width to the overall geometry of the design.

Table 5.11: Method parameters for pseudo-tensegrity designs (Figures 5.14-5.16)

Method Parameters		*Method Parameters*	
minimum area	.01 cm^2	maximum number of members	50
member areas	discrete tubes/ continuous bars	number of iterations	170
minimum member length	15 cm	number of moves per iteration	200
minimum angle between members	1°	planar truss topology rules	1-6
intersections between members	allowed	rule selection	static trajectories

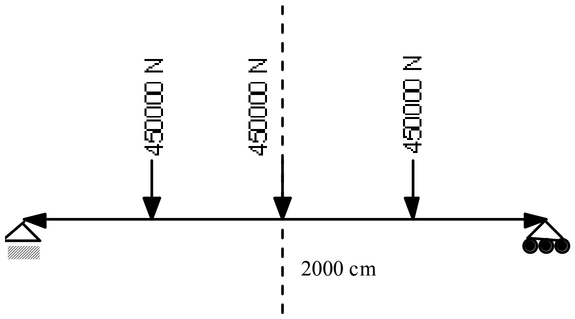
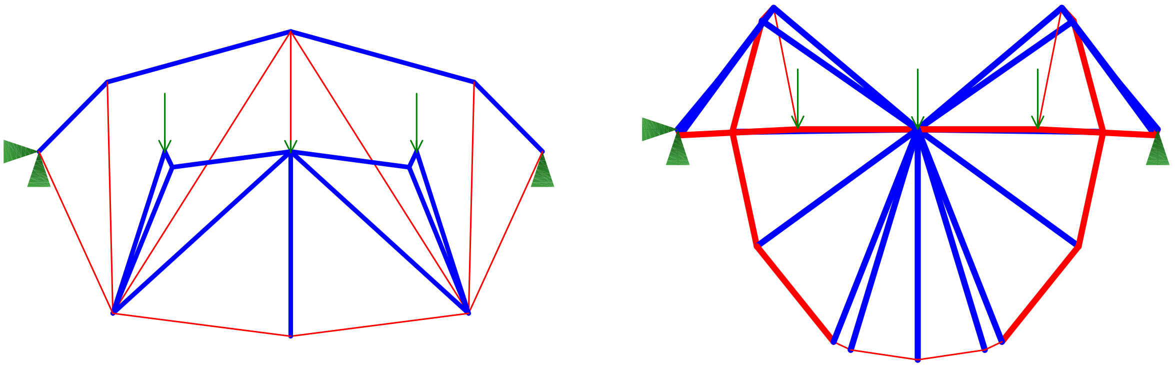


Figure 5.13: Pseudo-tensegrity problem specification



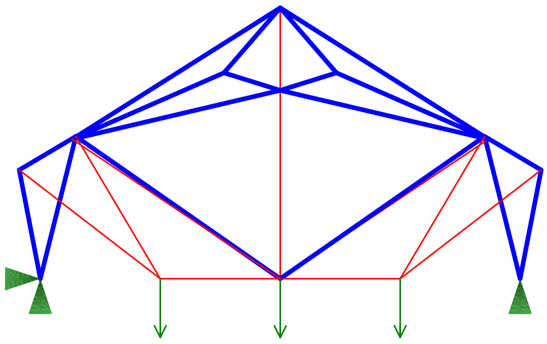
(a)

mass	2544 kg
wire size	10 cm^2
tube size	27.74 cm^2
rendundant members	2

(b)

mass	5626 kg
wire size	10 cm^2
tube size	36.00 cm^2
rendundant members	0

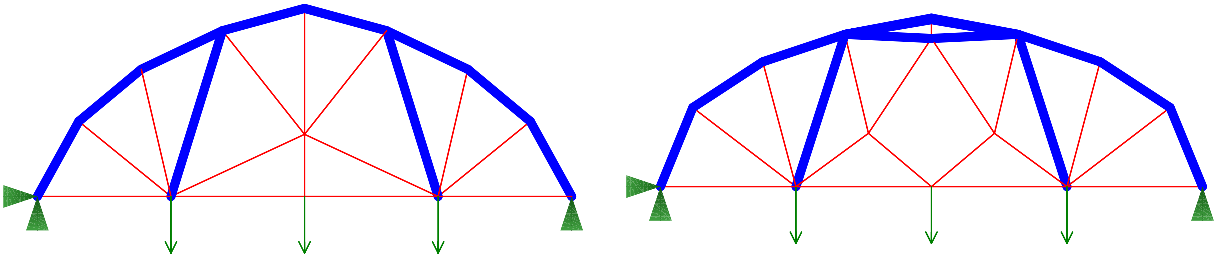
Figure 5.14: Pseudo-tensegrity designs with standard size steel tubes



(c.)

mass	2971 kg
wire size	9.58 cm^2
tube size	27.74 cm^2
rendundant members	1

Figure 5.14 (cont.): Pseudo-tensegrity designs with standard size steel tubes



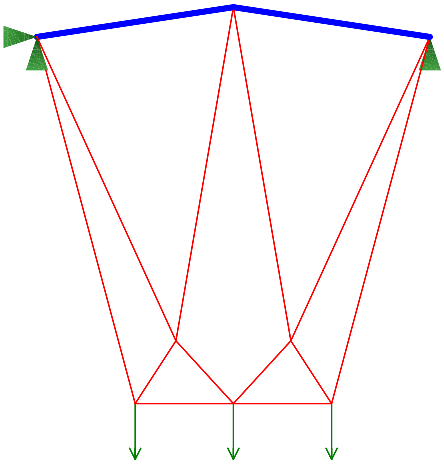
(a)

mass	2626 kg
wire size	9.89 cm^2
tube size	66.85 cm^2
redundant members	1

(b)

mass	3157 kg
wire size	9.23 cm^2
tube size	70.74 cm^2
redundant members	3

Figure 5.15: Pseudo-tensegrity designs with solid steel bars



mass	1314 kg
wire size	7.09 cm^2
tube size	35.99 cm^2
redundant members	1

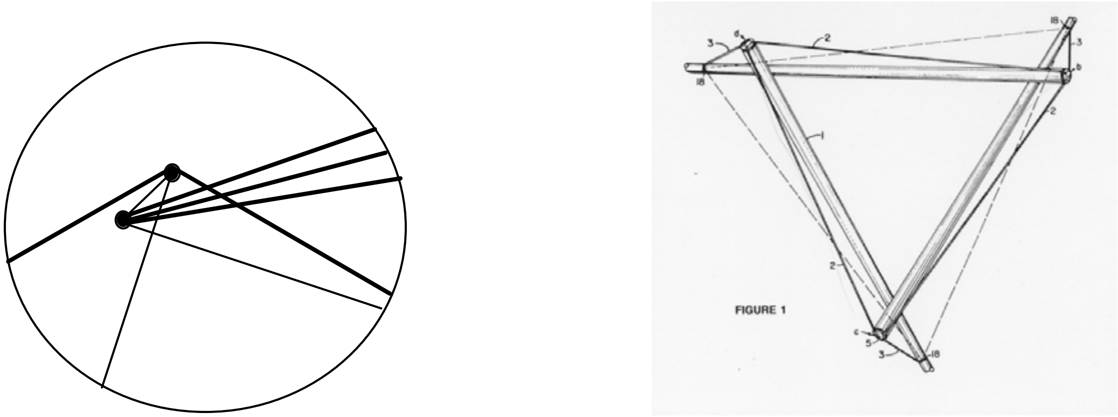
Figure 5.16: Pseudo-tensegrity designs with moving vertical supports

5.2.2.3 Discussion of Results

The results show that no design achieved the aesthetic principle of tensegrity in having continuous tension with discontinuous compression. However, components of some designs contain characteristics of tensegrity systems. Comparing the connections of the most complex joint in the design in Figure 5.14(c), a close-up of the system is shown in Figure 5.17 (a), we can see that the construction of this joint is similar to that in Fuller's tensegrity patent shown in Figure 5.17(b). For this joint, rather than connect all compression members at the same joint a new joint is added to offset the compression members with the use of tension wires.

Other observations can be made about the designs generated. The design in Figure 5.14(b) also contains an interesting relation in that, rather than an outer compression ring, the members that comprise the circular portion of the design are in tension supported by compression spokes. This is counter to the normal occurrence of structures such as a bicycle wheel with spokes in tension attached to a compression rim. However this design is much heavier than design in Figure 5.14(a) putting the efficiency of discontinuous compression and continuous tension in question except when a high level of prestress can be maintained in the tensional members.

Even though the visual goals of tensegrity systems were not satisfied, the generated structures do adhere to the mechanics definition of tensegrity systems as self-stressed systems comprised of tension wires and compression members. One extension of this model could be to use prestressed wire that may aid in satisfying the visual objective since low levels of compression could then be maintained in the wire. Nonetheless, the forms generated for the designs with steel tubes, Figures 5.14 and 5.16, conform to the model specifications of tensegrity systems and produce alternative forms than would be expected for the problem specification.



(a) Close-up of joint from Figure 5.14 (c)

(b) Fuller's tensegrity patent (Fuller, 1962)

Figure 5.17: Comparison of joints between the design in Figure 5.14(c) and Fuller's tensegrity patent

5.3 Summary

The first part in an exploration of a language of discrete structures was presented. This exploration resulted in essays of planar trusses for applications of a cantilever truss, arches, bridge trusses, transmission towers and pseudo-tensegrity structures. The styles of the generated structures resulted from the optimization model that included the design goals of efficiency, economy and elements of tensegrity systems. Adding a model of efficiency to the optimization model creates a tradeoff between the number of distinct cross-sections in a design, either absolute or relative to an allowed number, and the mass of the structure. The resulting designs present an essay of trusses with optimally directed topologies as well as a limited number of groups and the cross-sectional areas of each group according to the economies of scale. The extreme case of limiting the number of discrete cross-sections was presented in the design of pseudo-tensegrities.

6. ESSAYS OF DISCRETE STRUCTURES, PART II: SPACE TRUSSES

The second class of discrete structures that will be explored are space trusses, often referred to as space structures or space frames. Space trusses are most commonly used for enclosing space, such as roofs and exhibition halls, since they have the advantages of spanning large distances without the need for intermediate supports. Additionally, they can be more economical than solid shell structures since they are lightweight and easier to fabricate.

Space trusses can be classified as single-layer or double-layer discrete structures in three-dimensional space that can take the shape of flat surfaces or curved surfaces of multiple dimensions (Orton, 1988). The structures that will be explored in this chapter are single-layer space trusses projected onto curved surfaces of one and two dimensions and onto a pyramid. The topology layout for a space structure is designed on a plane and then projected onto the desired surface, which is defined as a dependent function of the planar x and y dimensions.

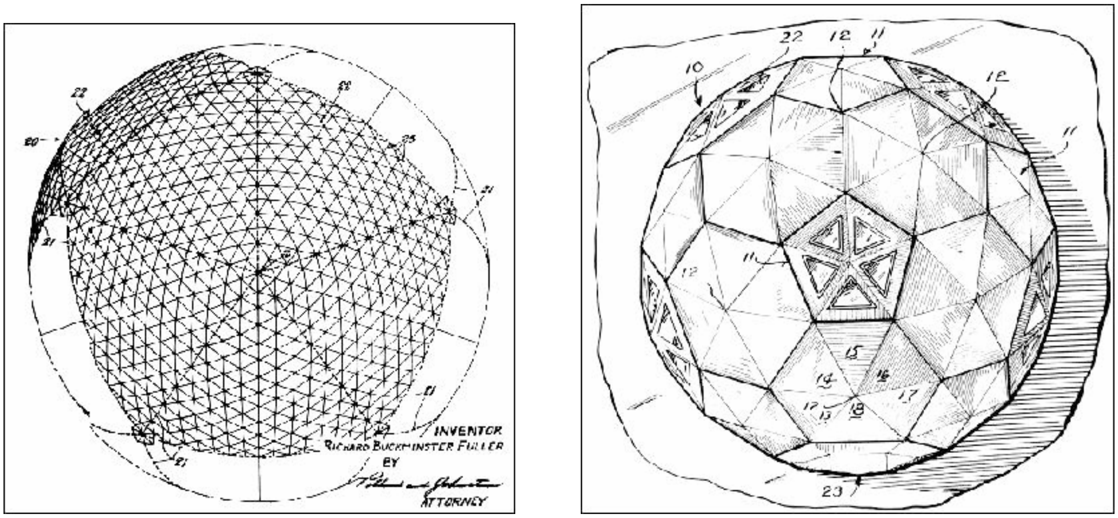
The first essay of space trusses will present the design of geodesic-like domes for a series of design goals including efficiency, economy, utility and elegance as well as for a comparison to shape optimization. The aesthetic model of elegance will then be expanded using the golden ratio proportional system to measure the relative proportions of shapes within a design. A series of essays will then be presented that use the new aesthetic model and explore the use of space trusses for complex-shaped roof design based on the design specifications of existing structures. This extension to existing structural design problems will address the issue of scaling since often, discrete space trusses involve the use of hundreds of members. The layout of discrete structures as the structural support for a glass pool roof, a glass barrel vault roof for a gallery and a conoidal shaped roof for an octagonal airplane hanger will be presented.

6.1 Designing with Geodesic Patterns

Geodesic domes, originally invented by R. Buckminster Fuller (1954), have been used for a wide range of purposes including temporary exposition structures, scientific test centers and housing (Baldwin, 1996). Domes incorporate many interesting design goals since they are considered the strongest, lightest and most efficient building system (Prennis, 1973). As domical structures, geodesic domes enclose the maximum space with minimum surface area in which the layout of the structure is influenced by both the desired span and height as well as the method of layout. Economic design goals include minimizing the number of different strut lengths and the number of distinct cross-sectional areas as well as minimizing surface area to present the least possible surface are to the environment and thus minimize heat loss (Baldwin, 1996). Additionally, economy is added to the design optimization model through dynamic grouping of members by cross-sectional area or length. Since Fuller sought to construct a structure using members of a single length, the uniform break down of a sphere is also considered as a design goal, which reflects design for efficiency and elegance. A measure of spatial uniformity is calculated from the standard deviation of the length of all members in a design. A design with a more uniform breakdown and thus a lower standard deviation of length is considered to be of greater aesthetic and economic value than a design with a more random breakdown. The objective function formulated for dome design minimizes the weight of the structure, the surface area, and the number of distinct cross-sections or length while maximizing the enclosed volume and the uniformity of the breakdown.

6.1.1 Generating Geodesic-Like Structures

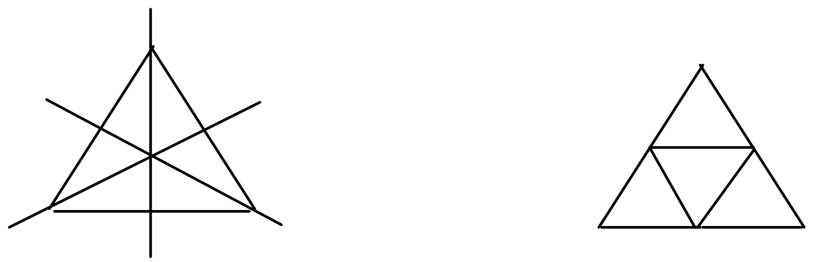
The layout of Fuller's geodesic domes consists of regulated patterns of self bracing triangles to produce a maximally efficient spherical structure (Figure 6.1). Geodesic domes are fractional parts of spherical tensegrities that consist of a planar truss system. Geodesic layouts are created by subdividing the faces of one of the five platonic solids and projecting these subdivisions onto a spherical plane (Prenis, 1973). A study of the standard rules used to generate geodesic patterns will be shown in order to quantify them in the form of a shape grammar for use in a non-conventional sequence to generate innovative geodesic-like forms.



(a) "Building Construction", 1954 (b) "Hexa-Pent", 1970

Figure 6.1: **Two of Buckminster Fuller's geodesic dome patents**

The space truss grammar that was presented in Chapter 3 as a general grammar for three-dimensional trusses will now be presented in the context of dome design from which rules 5 through 7 were originally developed. The grammar rules are based on two standard methods for subdividing the faces of platonic solids, called the triacon, class I, and alternate, class II, methods (Figure 6.2). The triacon method derives its name from the subdivision of the faces of a triacontahedron from which it was first developed. Breakdowns are referred to by the frequency with which the original sides of the triangle are subdivided, represented as "nv" for n divisions on one side. For example in Figure 6.2, "2v" states that each original side was divided in two. With the triacon breakdown the original sides of the triangle can be removed, although it is not necessary, and the new lines extended to the adjacent triangles; this is indicated in Figure 6.2 by the lines that extend past the original sides. In the alternatebreakdown the original sides of the triangle remain intact. Further discussion of standard breakdowns and general dome design techniques can be found in Prenis (1973) or Sheppard, et al., (1974) as well as numerous sites on the world wide web[1]. Calculating the angles and strut lengths for a geodesic design is a tedious task for which a computer program, DOME (Bono, 1996), has been developed that generates standard dome breakdowns for class I and II domes based on geodesic math. The dome designs that will be presented here are quite different from standard layouts since although they are based on geodesic breakdowns the designs themselves are not required to adhere to geodesic patterns. However, it will be shown that designs generated from the grammar can approach geodesic patterns.



2v triacon breakdown
Class I

2v alternate breakdown
Class II

Figure 6.2: Classes of geodesic dome breakdowns

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[1] Web sites on geodesic domes include: www.bfi.org, www.teleport.com/~pdx4d/dome.html, www.netaxs.com/~cjf/fuller-faq-4.html, www.wnet.org/bucky/dome.html, www.crisis.com/~rjbono.

Subdividing the members of a dome serves to achieve behavioral, economic and utilitarian design goals. For example, dividing a member in half decreases the length of each individual member that in turn decreases the required cross-sectional area of compression members. However, the total length of the two new members is greater than the single old member since the bisection point is projected onto the specified ellipsoid thus forming two legs of a triangle above the old member. Therefore, in order to reduce the weight of the structure and increase the efficiency, subdividing creates a tradeoff between decreasing cross-sectional area and increasing member length. Adding the utilitarian goals of maximizing enclosure space and minimizing surface area create additional tradeoffs involved in subdividing members. A higher frequency of subdivisions allows the structure to more closely approximate a sphere that will lead to a maximum enclosure space for minimum surface area.

The shape grammar shown in Figure 6.3 generates a two-dimensional truss layout that is projected onto a curved plane defined by an ellipsoid of a given height and span. The base of the dome is circular, defined by a uniform span in both the x and y directions. These parameters specified by the designer act as functional constraints on the shape grammar since they define the desired enclosure space. An ellipsoid is used rather than a sphere since often with spherical domes much vertical enclosure space is wasted and, yet, an ellipsoid can represent a sphere if so desired. An additional parametric constraint is placed on the shape modification rule in order to keep all points of the design within the circular base of the defined ellipsoid. Figure 6.4 shows a top down view of two dome layouts generated by hand from the grammar, illustrating that the grammar is capable of generating standard geodesic forms through the sequential application of shape grammar rules. For the triacon pattern layout the specified rule is applied under reflected quarter symmetry while for the alternate pattern layout the specified rule is applied under rotational quarter symmetry.

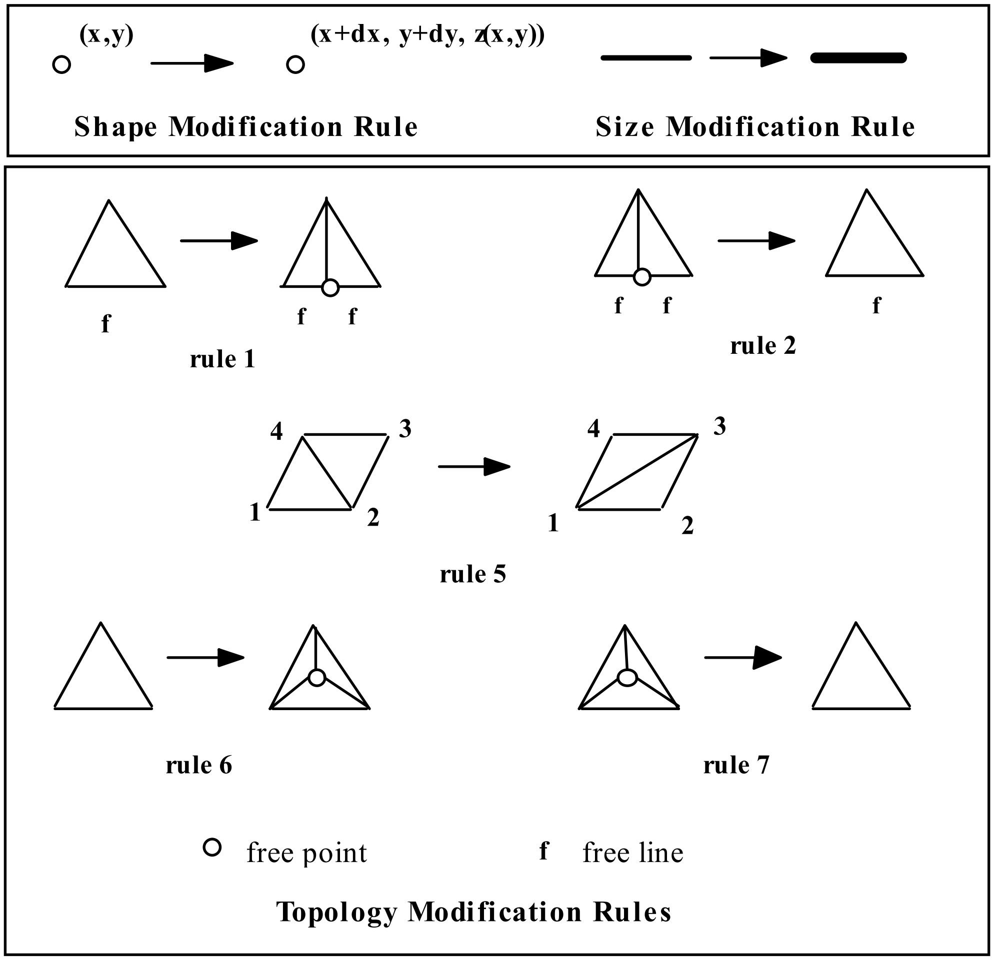

Figure 6.3: Space truss grammar for dome design

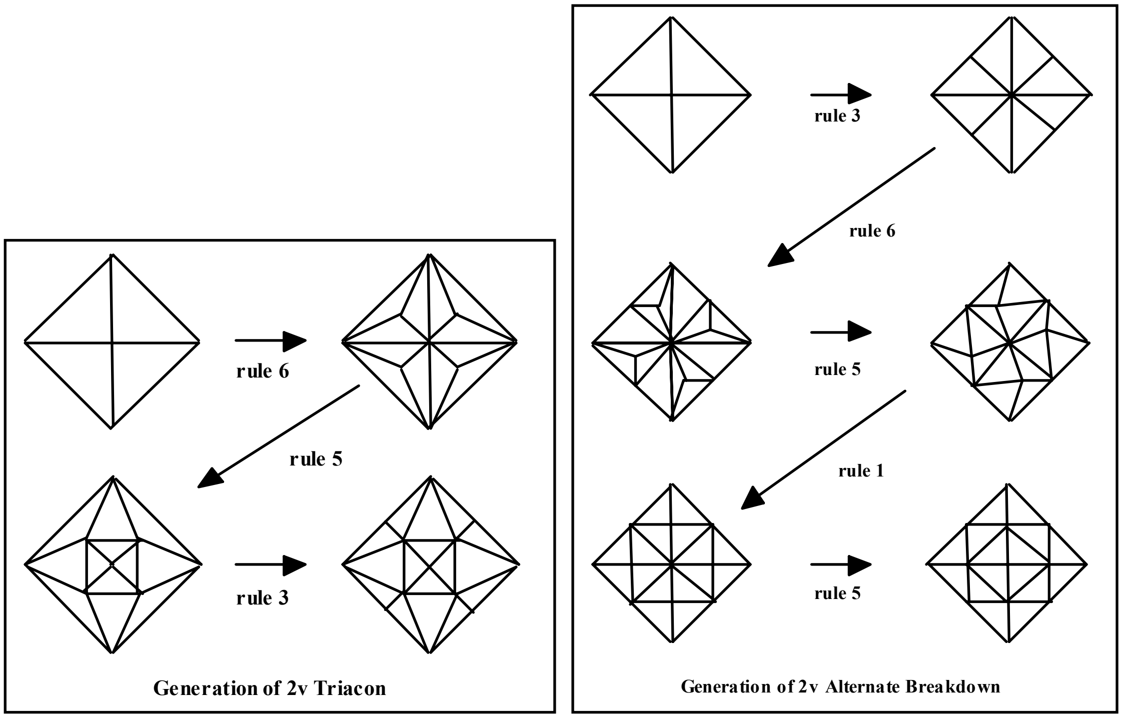

Figure 6.4: Generation of standard geodesic forms

6.1.2 Results

The dome grammar presented will now be used to generate solutions for two dome design problems. The first problem compares the shape annealing method to traditional shape optimization while the second problem explores innovative dome design at both a spatial and functional level based on geodesic patterns. Spatial dome layout is investigated through an optimally directed search using the geometric design goals of geodesic structures, that is, to closely approximate the defined ellipsoid by maximizing enclosure space and minimizing surface area. This problem will then be extended to the functional layout of domes and explore the effect of changing the semantics included in the objective function, or design goals, on the styles and quality of solutions generated. For all designs generated points in the planar layout are projected onto the surface defined by an ellipsoid to generate three-dimensional structures. For all designs presented a top down view as well as a three-dimensional view are shown.[2]

6.1.2.1 Comparison to Structural Optimization

The first problem is based on the shape optimization of a space truss presented by Pederson (1973). Figure 6.5 shows the fixed layout for Pederson's shape optimization

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[2] The width of the lines represents the diameter of the members uniformly scaled across all figures.

problem and the corresponding initial layout for shape annealing. The objective of the design is to maximize efficiency of the structure subject to four independent loading conditions applied in the z-direction:

1. -3×10^5 N at joint 1,
2. $-.3 \times 10^5$ N divided equally among all free joints,
3. -1.5×10^5 N at joint 1 and -1×10^5 N at joints 4 and 5, and
4. -1.5×10^5 N at joint 1 and $-.7 \times 10^5$ N at joints 2, 3 and 4.

Parametric constraints on the problem are as follows: joint one is restricted to move only in z which changes the overall height of the dome and thus the ellipsoid upon which the planar layout is projected. Joints two through five are restricted to move along the orthogonal axis on which they lie in the initial layout. Additionally, the design is required to maintain one-eighth symmetry. The material properties used are listed in Table 6.1 and the method parameters are listed in Table 6.2.

Table 6.1: Material properties for shape optimization comparison (Figures 6.6 and 6.7)

Material Property	
modulus of elasticity, E	2.10 E^7 N/cm^2
allowable tensile stress	13,000 N/cm^2
allowable compressive stress	-10,400 N/cm^2
proportional stress limit, σ^L	-13,000 N/cm^2
radius of gyration, α	1.0
factor of safety against buckling	2.5
specific weight, υ	.077 N/cm^3

Table 6.2: Method parameters for shape optimization comparison (Figures 6.6 and 6.7)

Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	number of iterations	170
maximum area	none	number of designs per iteration	200
member areas	continuos	space truss topology rules	1-7
minimum member length	.3 cm	rule selection	static trajectories
minimum angle between members	10°	constraint violation normalization	none
maximum number of members	20/50	intersections between members	not allowed
member shape	tube, d/t=25		

The allowable force before buckling for each member is calculated using the following formulae:

	Tensile	Compressive Elastic	Compressive Plastic
Allowable Force	$\sigma * a$	$-\left(\frac{a}{c}\right)^2$	$p^L + \sigma^c\left(a - c\sqrt{-p^L}\right)$

where:

$a \equiv \text{cross} - \text{sectional area}$

$l \equiv \text{member length}$

$\sigma \equiv \text{member stress}$

$$c \equiv \frac{l\sqrt{n}}{\pi\alpha\sqrt{E}}$$

$$p^L \equiv \frac{-l^2 s^{L^2}}{\pi^2\alpha^2 En}$$

Shape annealing designs have been generated for two cases: (1) requiring one-eighth symmetry as in Pederson's shape optimization problem, and (2) allowing asymmetric designs. The designs requiring one-eighth symmetry were allowed a maximum of twenty members in the one-eighth segment of the design whereas the asymmetric design were allowed a maximum of fifty members in the entire design. Three designs generated for the symmetric problem are shown in Figures 6.6 (a) through (c) with weights of 66,757 N, 67,679 N and 68,503 N respectively. Shape optimization of the topology shown in Figure 6.5 results in weights of 55,162 N to 65,482 N for different parametric constraints[3] (Pederson 1973). The designs generated from shape annealing are not the exact optima but are close to the numeric range of the optimal solutions without using prior topological knowledge of optimal topologies.

For a given dome topology an optimal solution exists in which all members are at their limit for at least one loading condition thus giving an optimal height. By allowing the topology to change, a distinct topology may be found that decreases the optimal height of the structure since the objective is to minimize weight subject to Euler buckling. The heights of the designs generated by shape annealing (6.33 m to 7.62 m) are considerably less than the shape optimization solutions for which the height ranges from 9.25 m to 11.24 m. This could be advantageous depending on the purpose of the design since vertical space is often wasted and increases the surface area that must be covered which in turn increases the energy costs of maintaining the building. This demonstrates the benefit of design exploration over deterministic

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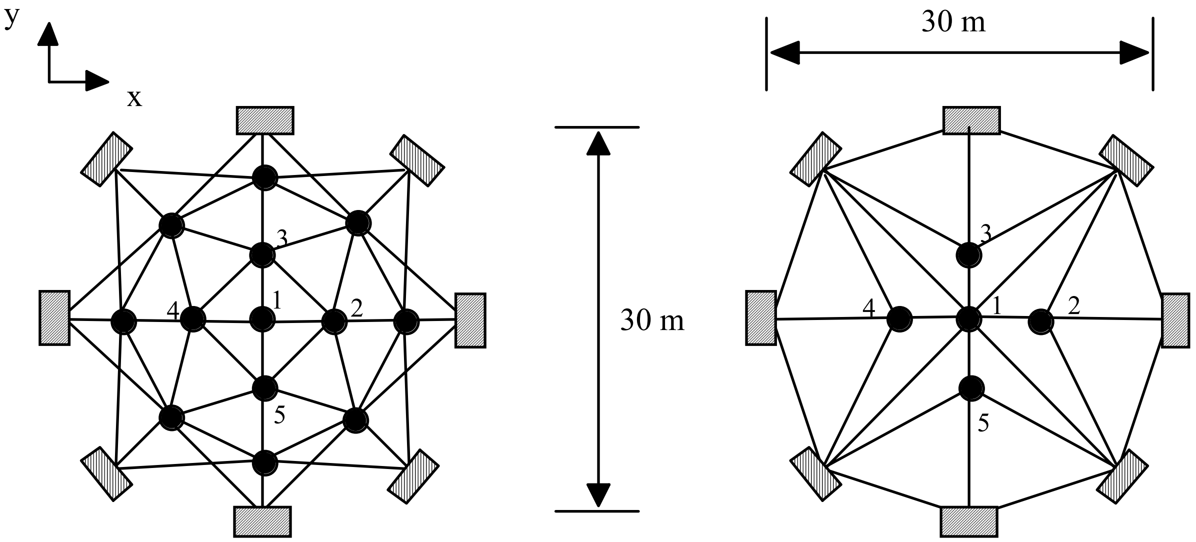
[3] Three constrained conditions were used: (1) linking all vertical coordinates of the joints and not allowing horizontal coordinates to change resulting in a weight of 65,482 N with a height of 9.25m, (2) allowing the horizontal coordinates to change in addition to the vertical coordinates resulting in a weight of 55,162 N and a height of 11.34m, and (3) constraining the height to 9.25m in addition to horizontal and vertical coordinate changes resulting in a weight of 56,241 N.

optimization in presenting solutions that may have secondary benefits not explicitly modeled in the optimization objective function.

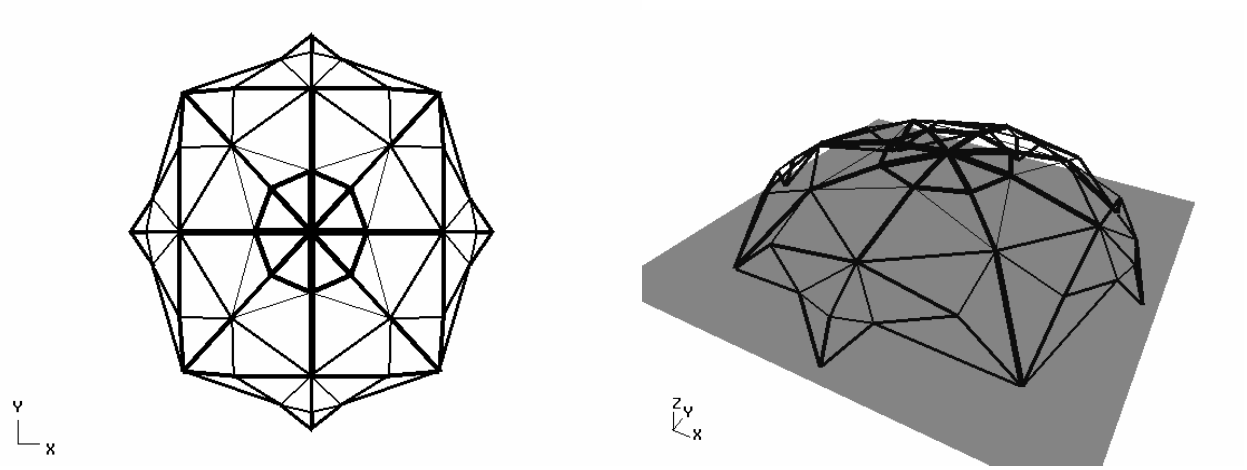
Asymmetric designs to the problem generated by shape annealing are shown in Figure 6.7 with weights of 65,129 N and 80,334 N respectively. It is interesting and somewhat expected that the most efficient design generated was asymmetric since the design can take advantage of the asymmetries to compensate for the asymmetric loading. Comparing the symmetric and asymmetric designs, both sets of designs provide the required function but have different visual effects. Since aesthetic value is partly dependent on designer interpretation, it is left to the designer to evaluate the relative importance between beauty and functional efficiency. This example illustrates that shape annealing is capable of designing comparable efficient solutions to traditional shape optimization problems under multiple loading conditions when well constrained, but, given more latitude, can generate visually interesting solutions that may be more suited to the problem specification.



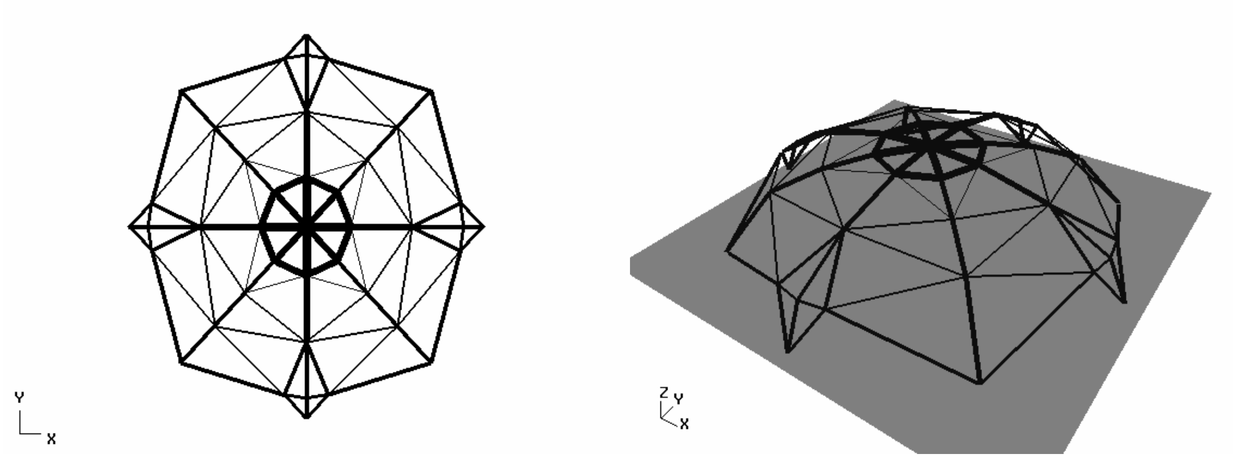
Shape Optimization Fixed Layout

Initial Shape Annealing Layout

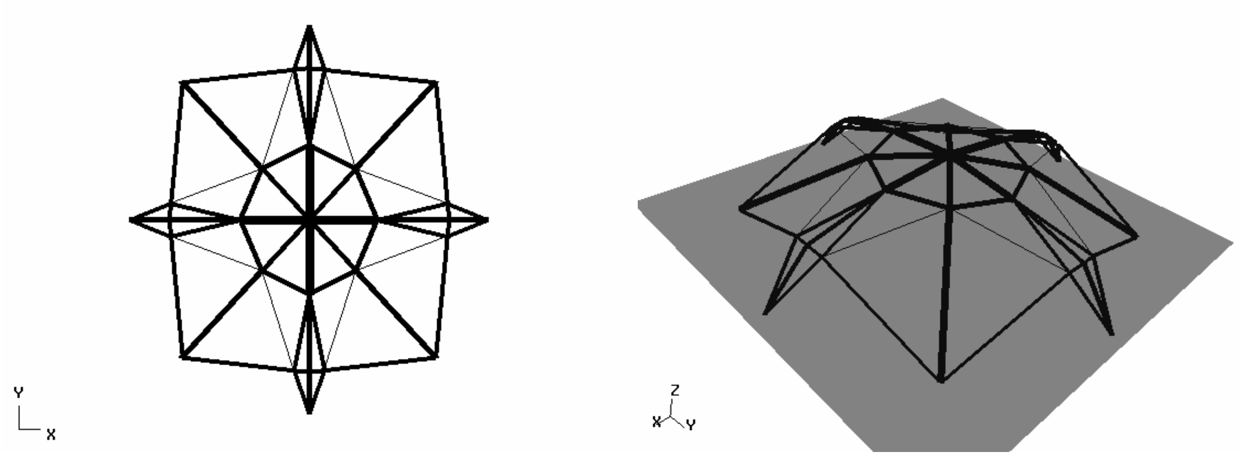
Figure 6.5: Initial layouts for a comparison to shape optimization



(a.) Weight = 66,757 N, Height = 7.62 m

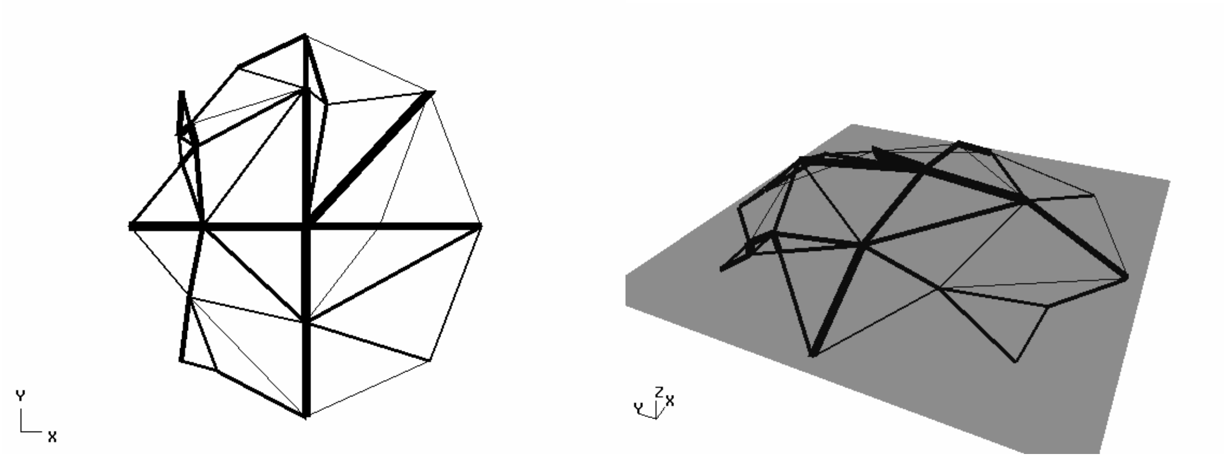


(b.) Weight = 67,679 N, Height = 7.45 m

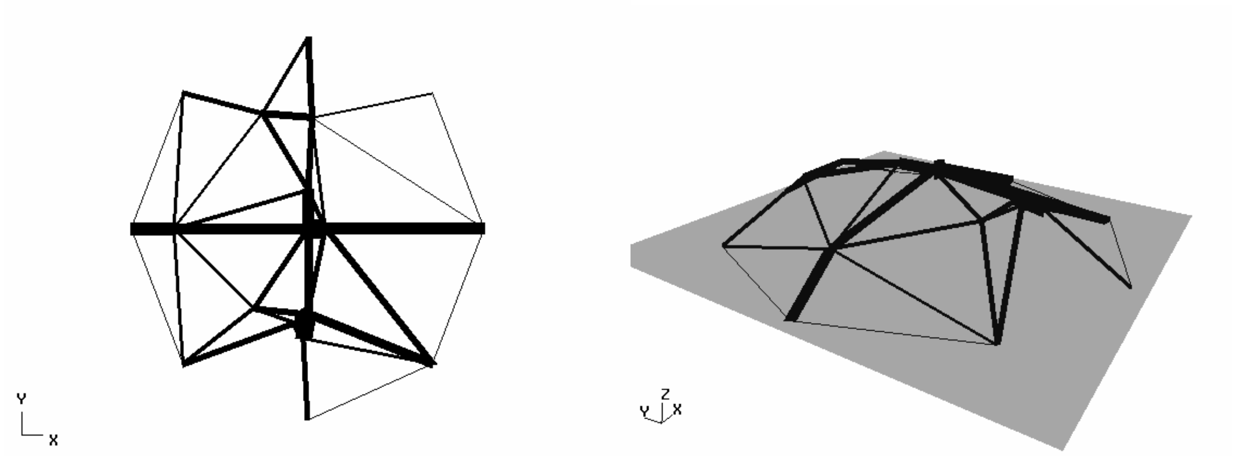


(c.) Weight = 68,503 N, Height = 6.33 m

Figure 6.6: Symmetric dome designs



(a.) 65,129 N, Height = 5.43 m



(b.) 80,334 N, Height = 5.31m

Figure 6.7: Asymmetric dome designs

6.1.2.2 Innovative Geodesic Dome Design

This section will explore applying geodesic patterns with shape annealing for both spatial and functional dome design.

6.1.2.2.1 Spatial Dome Design

The spatial breakdown of an ellipsoid for maximum enclosure space and minimum surface area will now be investigated. The purpose of this investigation is (1) to verify that the grammar, through random, iterative application of rules, is capable of generating geodesic-like spatial designs, and (2) to determine the achievable spatial limits of enclosure space and surface area using a maximum of fifty members in the layout. The problem dimensions are shown in Figure 6.8 where point 1 at the center of the structure is fixed at a height of 9.25 m thus defining an ellipsoid with a span, or circular base, of 30 m diameter and a height of 9.25 m. The design shown in Figure 6.9(a) has an enclosure space of 3555 m^3 and a surface area of 10.07 m^2 while the design in Figure 6.9(b) has an enclosure space of 3528 m^3 and a surface area of 9.77 m^2. Geodesic patterns can be seen in these designs as highlighted in Figure 6.9(a), thus verifying the ability of the dome grammar to generate geodesic-like dome designs under random application of rules.

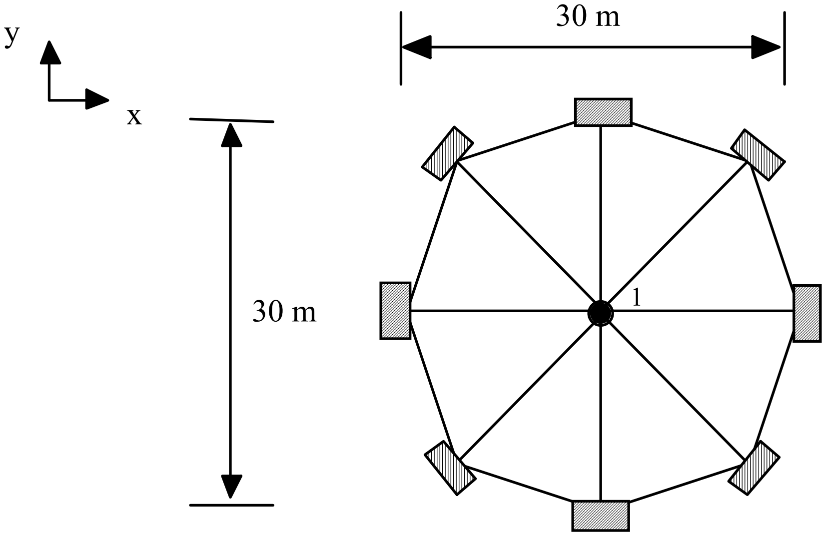
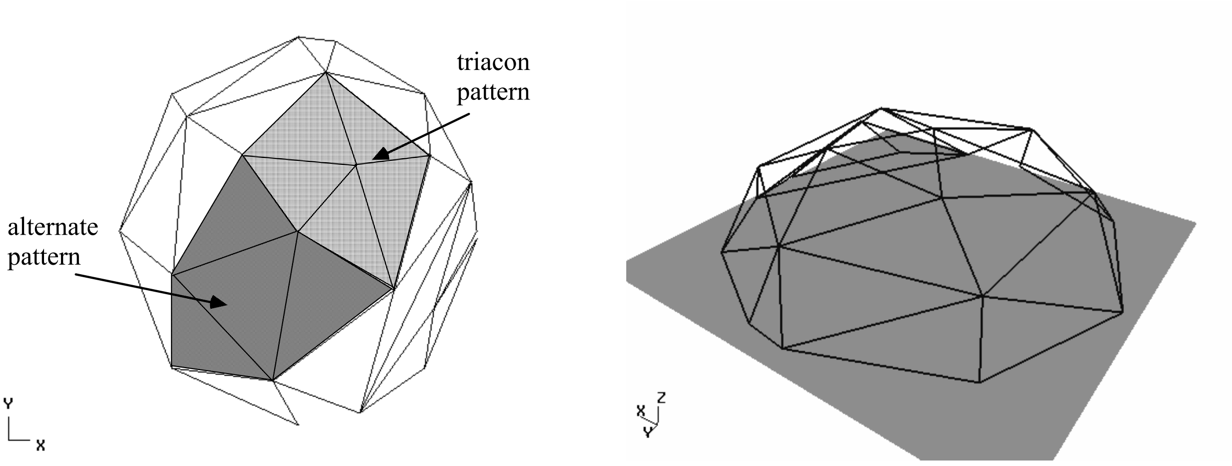
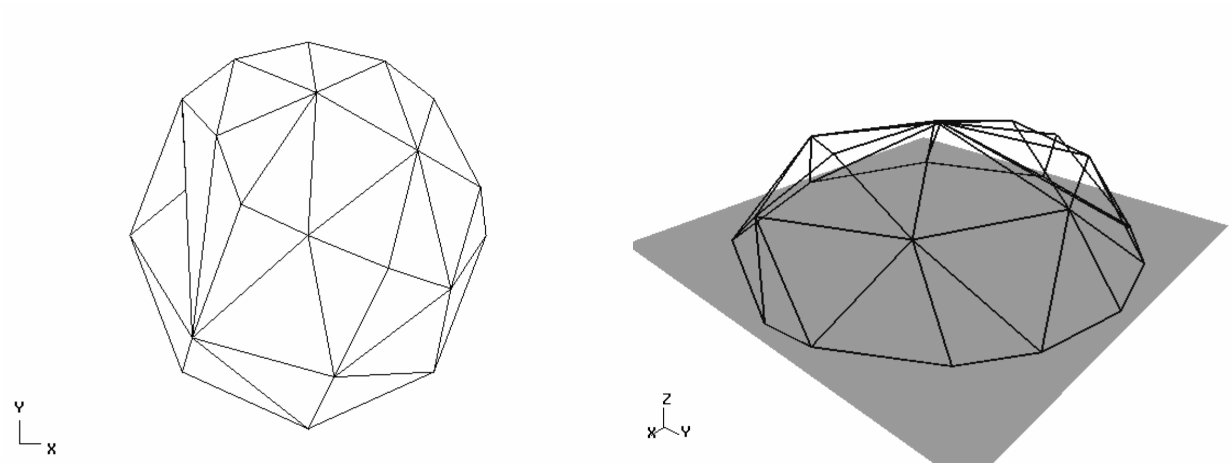


Figure 6.8: Initial shape for dome layout



(a)

Figure 6.9: Spatial layout of domes



(b)

Figure 6.9 (cont.): Spatial layout of domes

6.1.2.2.2 Spatial and Functional Dome Design

We will now apply the dome grammar to the design of domes for maximum efficiency, economy, utility and elegance. As discussed in Section 6.1 the purpose of a dome structure is to provide a maximum amount of enclosed space such as for an exposition structure or housing while providing low covering and energy costs by minimizing the surface area. We will now investigate how adding design goals to the optimization objective function influences the quality and appropriateness of the generated designs. The material properties used for these designs are listed in Table 6.3. The buckling limit is calculated using the Euler buckling formula, $P_{cr} = \frac{\pi Ea}{I}$. The initial layout for the problem is shown in Figure 6.8 where the two simultaneous loads applied are self-weight of structural members and -300,000 N applied at the center, joint 1, in the z-direction. A sequence of solutions will be shown in which each case expands the objective function to incorporate additional design goals as described in Table 6.5. The designs are shown in Figures 6.10 through 6.17 with the corresponding design metrics listed in Table 6.6. Costs are listed for each design shown in order to compare multiple designs from the same case. Convergence statistics for twelve designs generated for each case with standard deviations shown in parentheses are shown in Table 6.7. Since simulated annealing is a stochastic method that can get trapped in local optima it is standard practice to generate three designs and select the best design. Using this heuristic, a second set of statistics is shown in

Table 6.8 calculated from four sets of three designs using the same 12 designs generated for the statistics in Table 6.7.

Table 6.3: Material properties for dome designs (Figures 6.10-6.17)

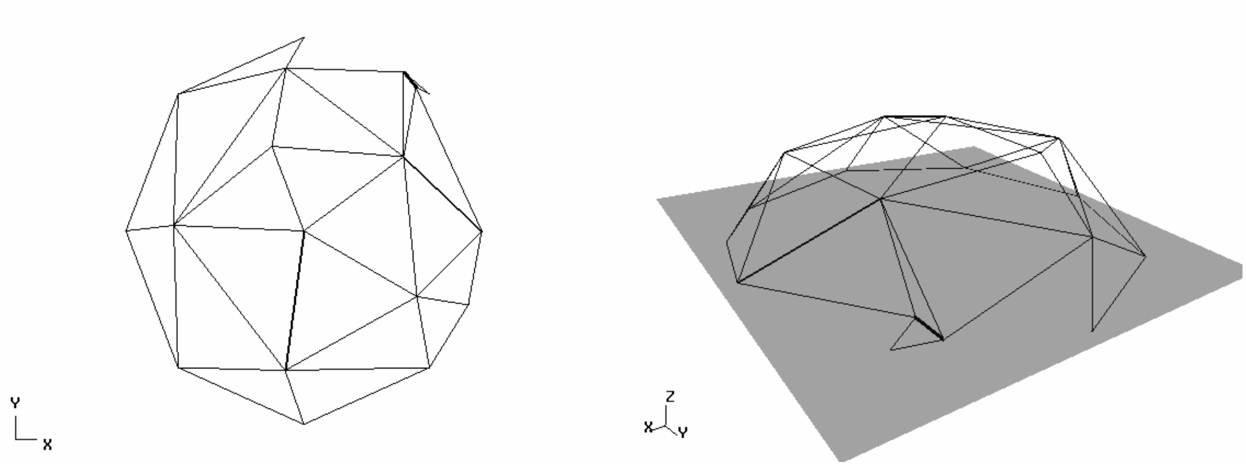
Material Properties	
modulus of elasticity, E	2.10 E^7 N/cm^2
allowable tensile stress	13,000 N/cm^2
allowable compressive stress	-10,400 N/cm^2
specific weight, υ	.00785 N/cm^3

Table 6.4: Method parameters for dome designs (Figures 6.10-6.17)

Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum group tolerance	3 cm^2
maximum area	none	minimum group tolerance	.6 cm^2
member areas	continuos/ discrete	number of iterations	170
minimum member length	.3 cm	number of designs per iteration	200
minimum angle between members	10º	space truss topology rules	1-7
intersections between members	not allowed	rule selection	static trajectories
maximum number of members	50	constraint violation normalization	none
member shape	tube, d/t=10		

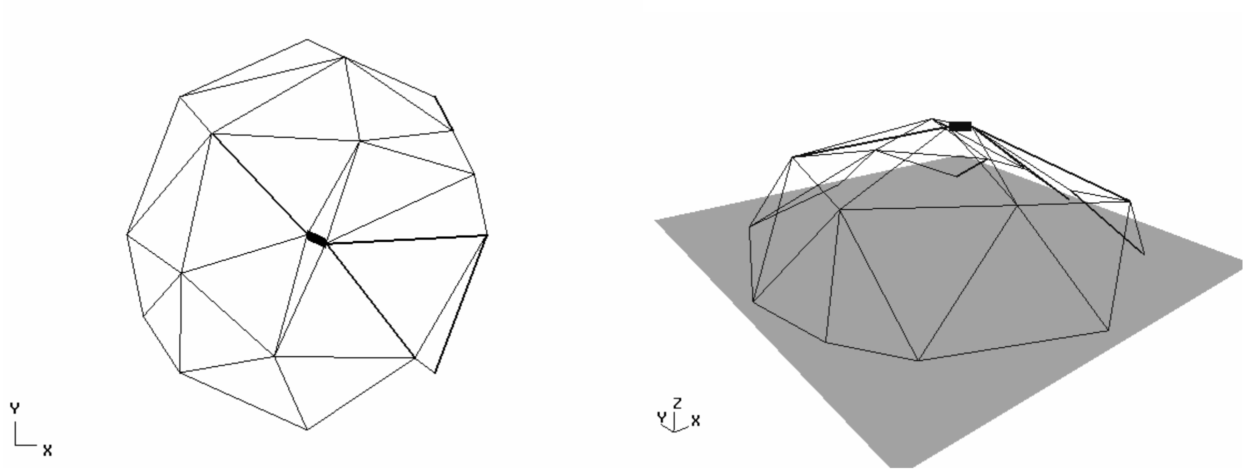
Table 6.5: Dome design cases

Case	Design Goals	Loading
1	utility = max. enclosure space economy = min. surface area	none
2	efficiency = min. weight utility = max. enclosure space economy = min. surface area	self-weight
3	efficiency = min. weight	self-weight and center applied load
4	efficiency = min. weight utility = max. enclosure space	self-weight and center applied load
5	efficiency = min. weight utility = max. enclosure space economy = min. surface area	self-weight and center applied load
6	efficiency = min. weight utility = max. enclosure space economy = min. surface area min. member area groups	self-weight and center applied load
7	efficiency = min. weight utility = max. enclosure space economy = min. surface area min. member length groups	self-weight and center applied load
8	efficiency = min. weight utility = max. enclosure space economy = min. surface area elegance = visual uniformity	self-weight and center applied load
9	efficiency = min. weight utility = max. enclosure space economy = min. surface area min. member area groups elegance = visual uniformity	self-weight and center applied load



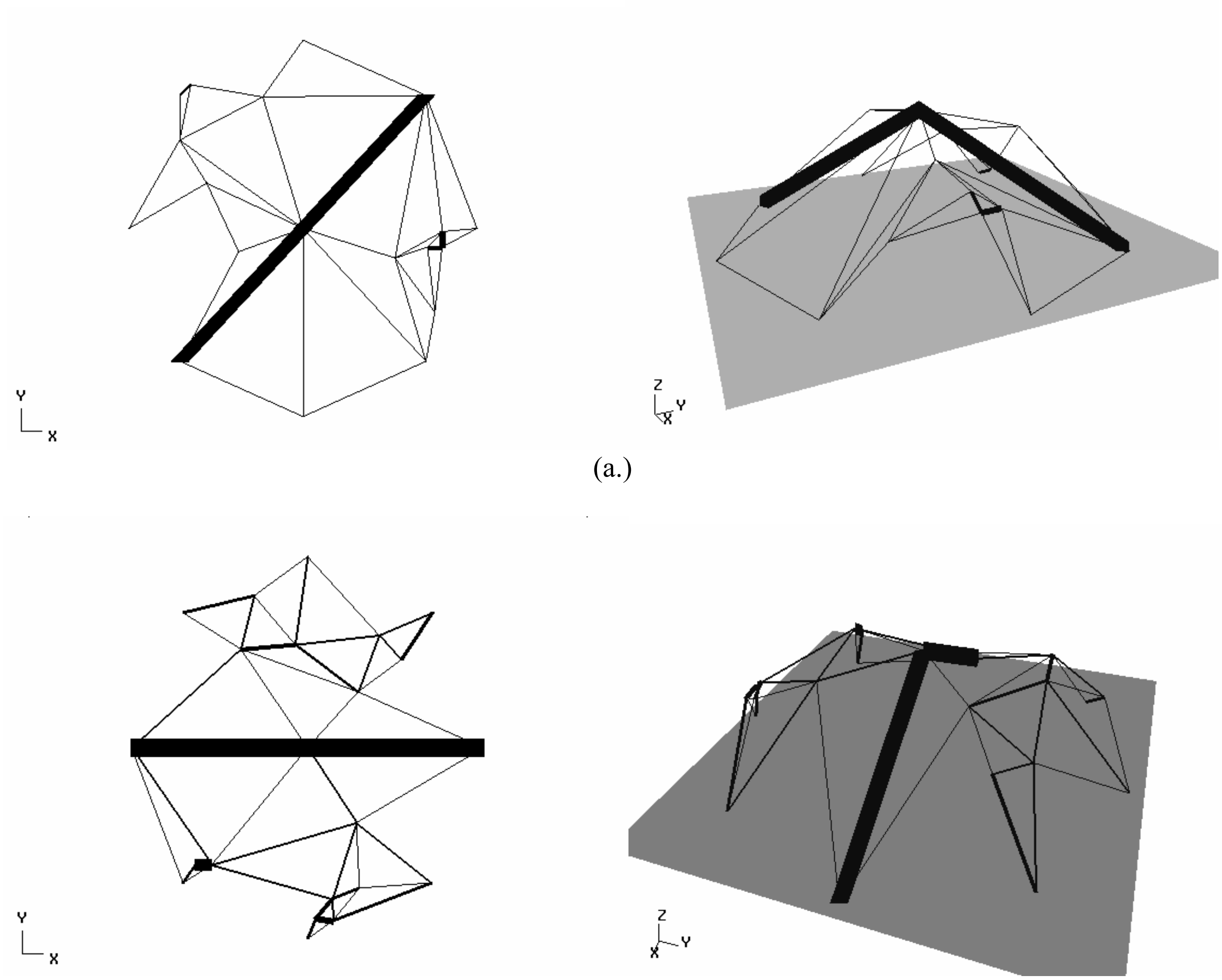
(a)

Figure 6.10: Dome design for self-weight



(b)

Figure 6.10 (cont.): Dome design for self-weight



(a.)

(b.)

Figure 6.11: Objective = weight

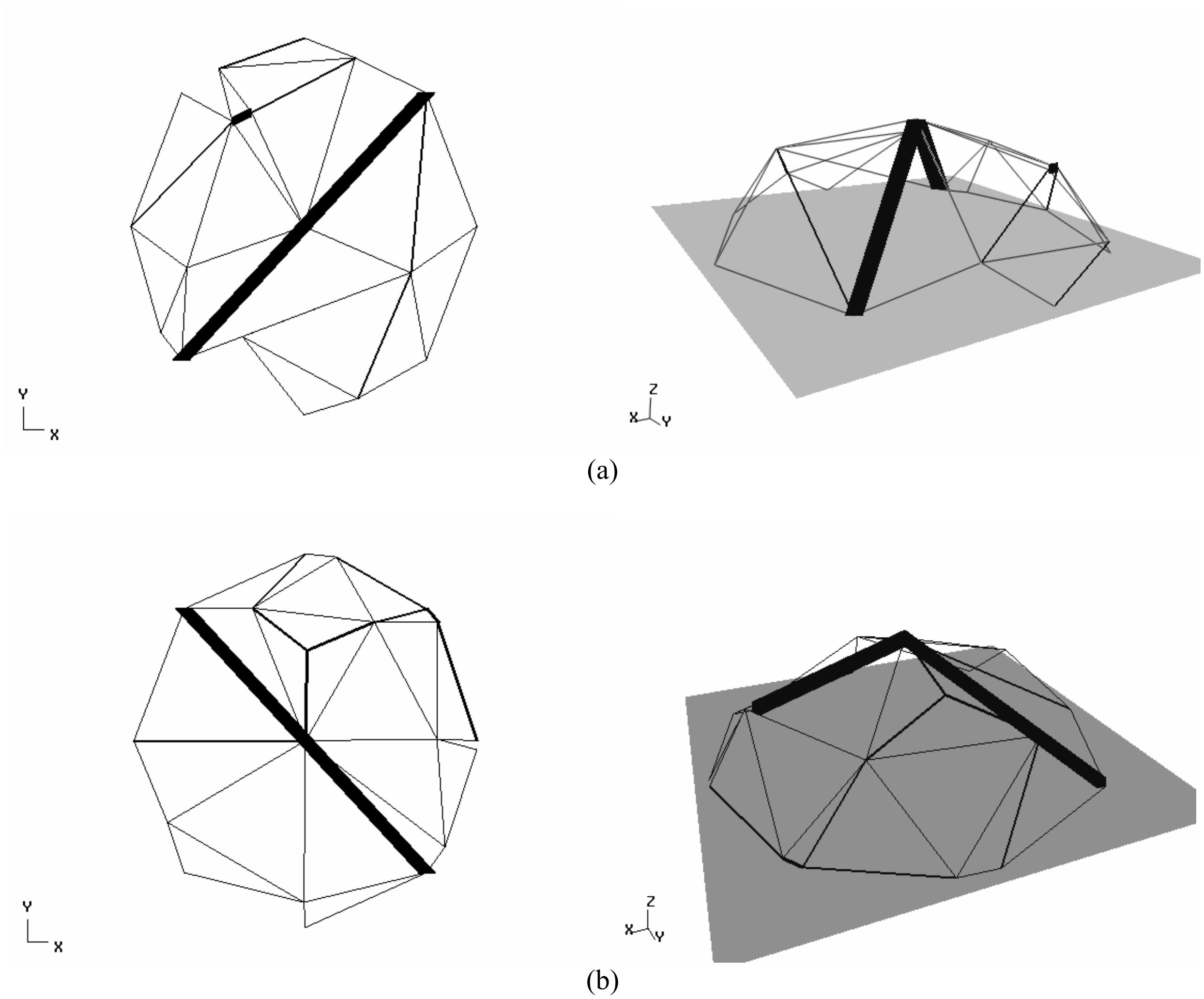
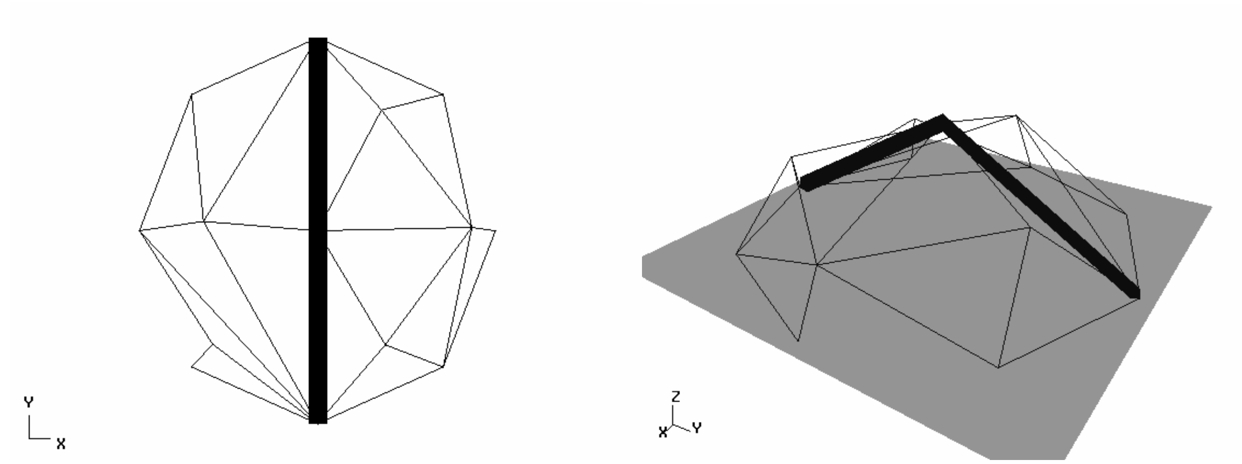
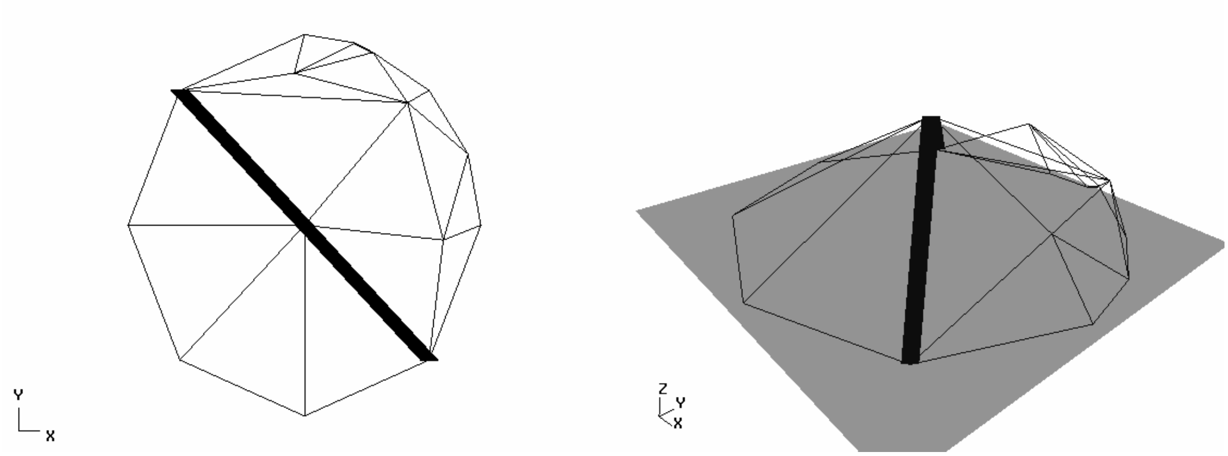


Figure 6.12: Objective = weight + (w_1/enclosure space)



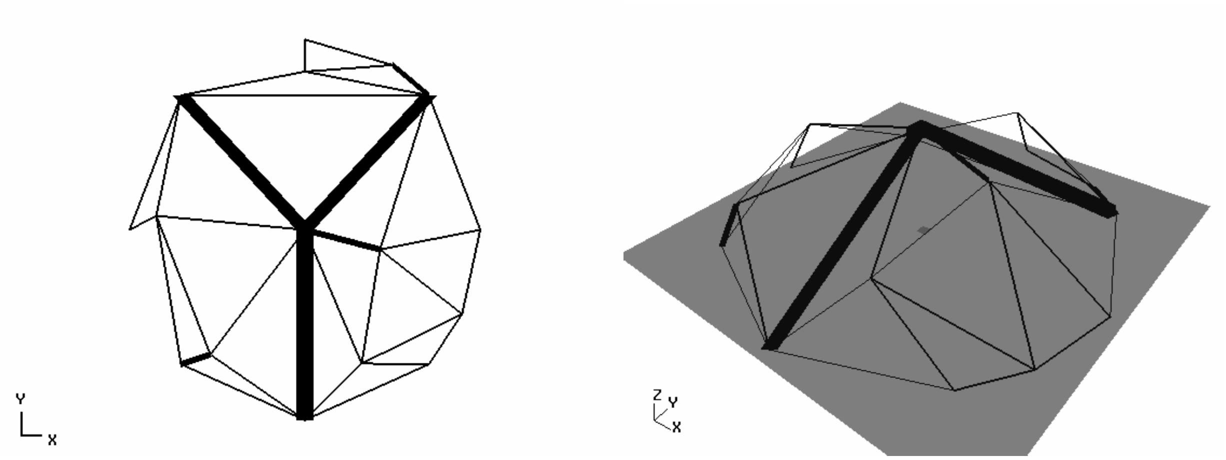
(a)

Figure 6.13: Objective = weight + (w_1/enclosure Space) + (w_2*surface area)

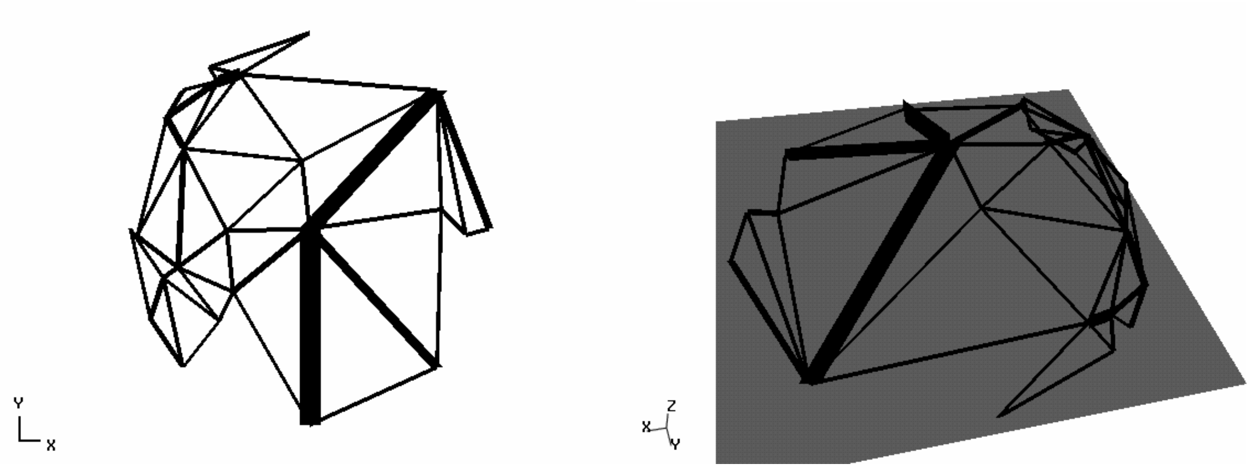


(b)

Figure 6.13 (cont.): Objective = weight + (w_1/enclosure space) + (w_2*surface area)

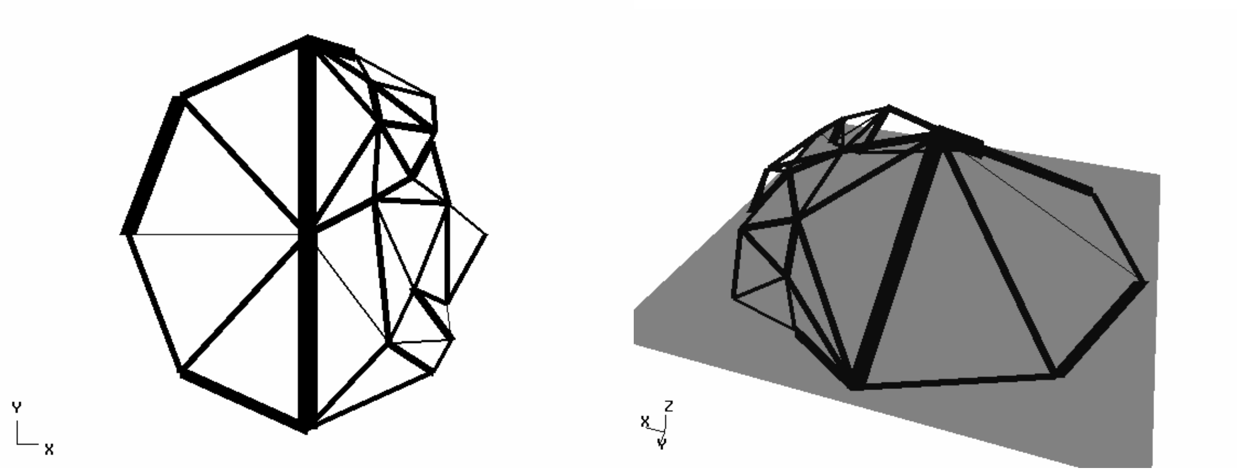


(a) 4 distinct cross-sectional areas

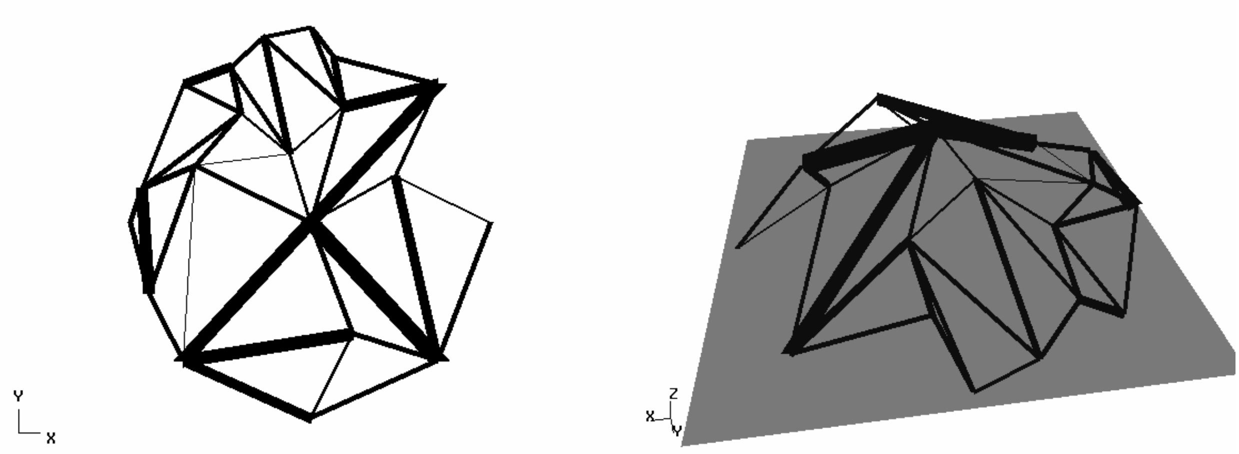


(b) 7 distinct cross-sectional areas

Figure 6.14: Objective = weight + (w_1/enclosure space) + (w_2*surface area) + group_penalty(area)

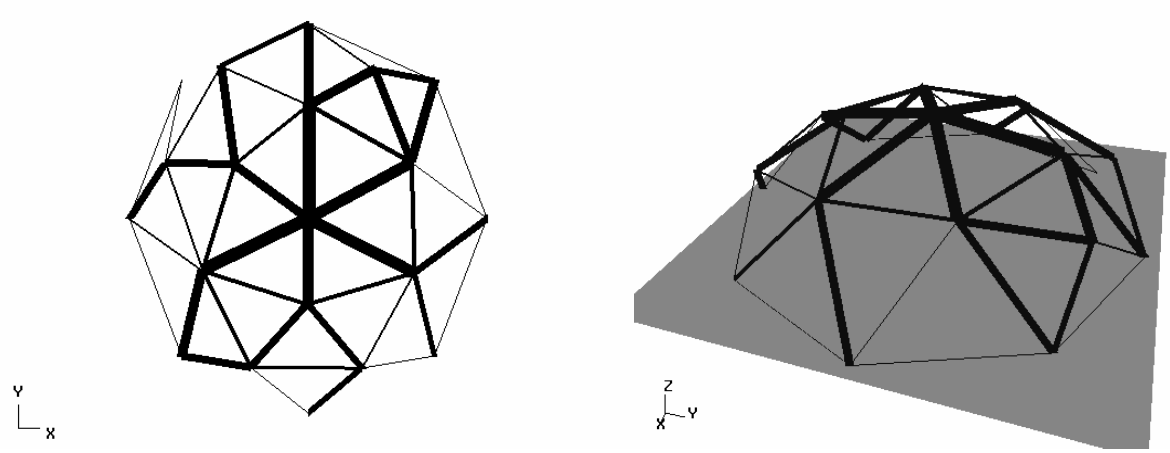


(a) 6 length groups



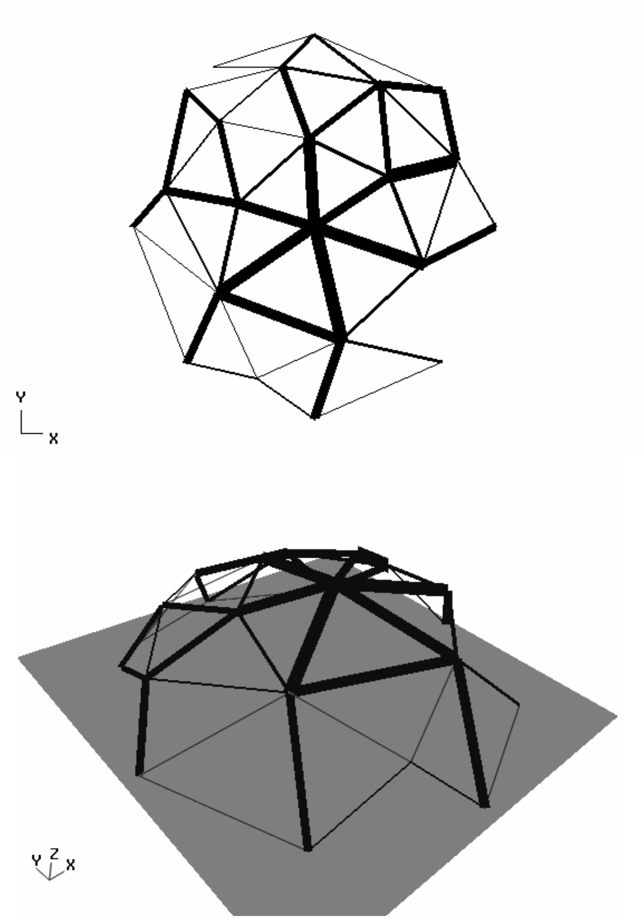
(b) 8 length groups

Figure 6.15: Objective = weight + (w_1/enclosure space) + (w_2*surface area) + group_penalty(length)



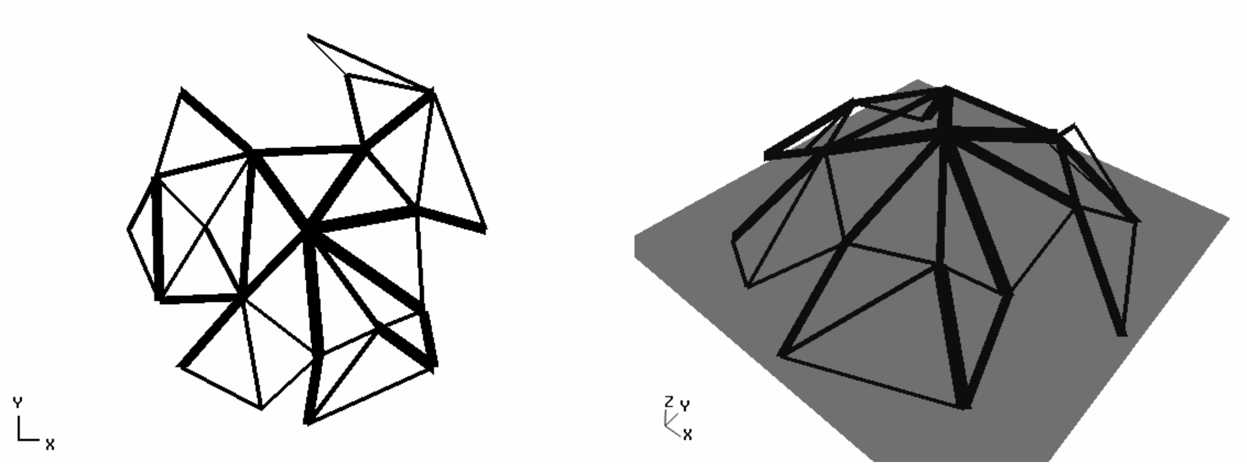
(a)

Figure 6.16: Objective = weight + (w_1/enclosure space) + (w_2*surface area) + σ_{length}

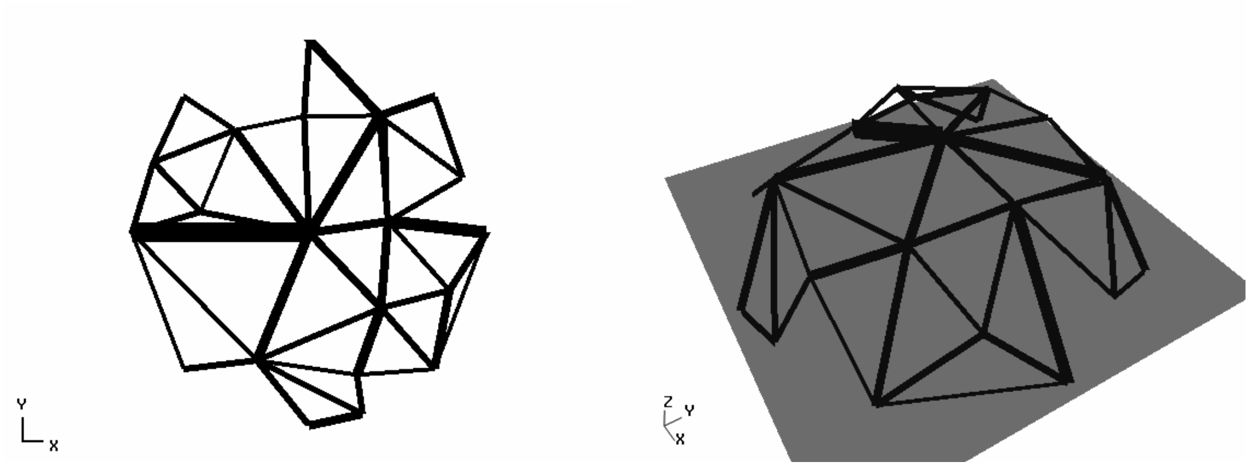


(b)

Figure 6.16 (cont.): Objective = weight + (w_1/enclosure space) + (w_2*surface area) + σ_{length}



(a) 7 distinct cross-sectional areas



(b) 8 distinct cross-sectional areas

Figure 6.17: Objective = weight + (w_1/enclosure space) + (w_2*surface area) + group_penalty(area) + σ_{length}

Table 6.6: Design Metrics for Dome Design (Figures 6.9-6.17)

Case	Figure	Objective to Minimize	Cost	Weight (N)	Enclosed Volume (m^3)	Surface Area (m^2)	# Groups
1	6.9 (a)	(w_1/volume) + (w_2∗surface area)	3741	N/A	3658	10.07	N/A
1	6.9 (b)	(w_1/volume) + (w_2∗surface area)	3789	N/A	3555	9.77	N/A
2	6.10 (a)	weight + (w_1/volume) + (w_2∗surface area)	4311	203	3101	8.82	N/A
2	6.10 (b)	weight + (w_1/volume) + (w_2∗surface area)	4626	405	2980	8.66	N/A
3	6.11 (a)	weight	6584	6584	1933	6.85	N/A
3	6.11 (b)	weight	7477	7477	1792	5.78	N/A
4	6.12 (a)	weight + (w_1/volume)	10591	6791	2641	8.34	N/A
4	6.12 (b)	weight + (w_1/volume)	10597	6889	2699	8.56	N/A
5	6.13 (a)	weight + (w_1/volume) + (w_2∗surface area)	10877	6449	2860	9.06	N/A
5	6.13 (b)	weight + (w_1/volume) + (w_2∗surface area)	11305	6538	2566	8.69	N/A
6	6.14 (a)	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(area)	18371	9343	2751	8.94	4
6	6.14 (b)	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(area)	33687	13940	2337	7.33	7

7	6.15 (a)	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(length)	32875	17984	2317	7.89	6
7	6.15 (b)	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(length)	43637	21211	2368	8.06	8
8	6.16 (a)	weight + (w_1/volume) + (w_2∗surface area) + σ(length)	29018	11203	3098	8.53	σ=1.36m
8	6.16 (b)	weight + (w_1/volume) + (w_2∗surface area) + σ(length)	30261	10756	2799	7.70	σ=1.50m
9	6.17 (a)	weight + (w_1/volume) + (w_2∗surface area) + σ(length) + group_penalty(area)	42489	14726	2277	6.46	8 σ=1.96
9	6.17 (b)	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(area) + σ(length)	43781	16889	2475	7.48	7 σ=2.48

Table 6.7: Dome Statistics for 12 Designs per Case

Case	Objective to Minimize	Weight (N)	Enclosed Volume (m^3)	Surface Area (m^2)	# Groups
1	(w_1/volume) + (w_2∗surface area)	N/A	3408 (149)	9.42 (.36)	N/A
2	weight + (w_1/volume) + (w_2∗surface area)	1407 (1301)	2733 (219)	8.02 (.5)	N/A
3	weight	11659 (3425)	1631 (337)	5.91 (1.06)	N/A
4	weight + (w_1/volume)	8983 (2046)	2588 (127)	8.34 (.45)	N/A
5	weight + (w_1/volume) + (w_2∗surface area)	9722 (2784)	2486 (145)	7.98 (.6)	N/A
6	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(area)	15826 (3269)	2312 (247)	7.37 (.86)	10 (4)
7	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(length)	22157 (4471)	2286 (153)	7.68 (.52)	13 (7)
8	weight + (w_1/volume) + (w_2∗surface area) + σ(length)	12137 (1381)	2844 (211)	8.26 (.53)	σ=2.01m (.48 m)
9	weight + (w_1/volume) + (w_2∗surface area) + group_penalty(area) + σ(length)	16901 (2833)	2455 (224)	7.48 (.80)	8 (1) σ=2.82m (.63m)

Table 6.8: Dome statistics for Best 1 of 3 Designs (four sets of three designs)

Case	**Objective to Minimize**	**Weight (N)**	**Enclosed Volume (m^3)**	**Surface Area (m^2)**	**# Groups**
1 (spatial layout)	(w_1/volume) + (w_2*surface area)	N/A	3545 (91)	9.71 (.28)	N/A
2 (self-weight only)	weight + (w_1/volume) + (w_2*surface area)	592 (353)	2912 (157)	8.42 (.4)	N/A
3	weight	8019 (1538)	1983 (223)	6.71 (.81)	N/A
4	weight + (w_1/volume)	6863 (367)	2713 (55)	8.60 (.29)	N/A
5	weight + (w_1/volume) + (w_2*surface area)	8536 (1558)	2559 (205)	8.29 (.67)	N/A
6	weight + (w_1/volume) + (w_2*surface area) + group_penalty(area)	14786 (4120)	2313 (340)	7.28 (1.41)	8 (3)
7	weight + (w_1/volume) + (w_2*surface area) + group_penalty(length)	18540 (2348)	2358 (119)	8.13 (.22)	7 (1)
8	weight + (w_1/volume) + (w_2*surface area) + σ(length)	11110 (293)	2965 (128)	8.46 (.57)	σ=1.77m (.41m)
9	weight + (w_1/volume) + (w_2*surface area) + group_penalty(area) + σ(length)	15542 (2416)	2547 (266)	7.85 (.63)	7 (1) σ=2.78m (.23m)

6.1.3 Discussion

Shape annealing has been shown to be capable of generating traditional solutions to structural design problems provided the design generation is properly constrained. When these constraints are removed shape annealing generates functional yet spatially innovative solutions for the same design problem. Comparing the solutions in Figure 6.6(a) and Figure 6.6(b), the topologies of the two designs are only slightly different. Previous work on the generation of transmission towers (see Section 5.2.2.1) that compared shape annealing of planar trusses to shape optimization solutions also resulted in a small number of distinct topologies. The solution in Figure 6.7(c) has a different topology but results in an increase in weight leaving the designer to decide which design is preferable. Although constraining the problem may lead to finding only a few distinct solutions, these solutions are still quite different from the standard geodesic forms. Constraining the problem generates solutions that a designer may expect, but the strength of this method is design exploration that allows the method to generate a variety of distinct, novel solutions. When the symmetry constraints are removed to allow for asymmetric designs, functionally feasible solutions with drastically different topologies are generated.

When the layout problem is not well constrained the design space increases, providing more possibilities of designs to explore. Semantics can be added to the generation process through both the optimization model and the grammar itself in order to increase the level of problem knowledge based on design goals that serve to focus the search for appropriate designs. Expanding the objective function to more accurately reflect the design problem reinforces good design decisions. For example, comparing the results in Table 6.8 for cases 3 and 4, adding the goal of maximizing volume not only increases the enclosure space but also decreases the weight implying that for this case these design goals cooperate. Design goals can also compete creating a tradeoff as illustrated when comparing cases 4 and 5. With the addition of minimizing surface area to the objective function the weight increases while the enclosure space decreases, but, the design goal of minimizing surface area was improved to only a small degree. This may indicate to the designer that the goal of minimizing surface area may not be necessary to explicitly optimize if the previous solutions were found to be satisfactory. It should be noted that small declinations of the objective values can also be attributed to the increase in the difficulty of the optimization problem from the addition of design goals. Adding economy to the objective function through member grouping in cases 6 and 7 creates further competition that has adverse effects on weight and enclosure space while improving the surface area. The design goal of elegance based on an aesthetic value determined from spatial uniformity creates designs that are visually more familiar and intuitive to the designer and additionally have the benefit of increasing the enclosure space at the expense of increasing the weight. Including all design goals, efficiency, economy, utility and elegance in case 9 results in an increase in weight when compared to case 7 that did not include elegance but an increase in enclosure space and a decrease in the surface area were gained.

When comparing the designs in Figure 6.13 to the designs in Figure 6.16 and the designs in Figure 6.14 to the designs in Figure 6.17 most engineering designers would tend to prefer the more uniform designs based on their sense of the relation between elegance and functional efficiency. The interpretation of designs is based on the level of knowledge and the intuition of the designer as well as their aesthetic values. A practical designer may look at the regular designs and automatically be able to interpret how the structure works while the functionality of the irregular patterns may be more difficult to comprehend. An advantage of computer generated designs is that functionality is based on analysis rather than intuition. For a visual designer the visual uniformity of designs may appeal to their sense of aesthetics while the irregular patterns may be seen as more unique with striking visual impact. The variance of designer intuition and preference motivates the need for semantics to be incorporated directly into the grammar in order to allow for further parametric control of the rules. In the designs presented, semantics have been used to ensure the generation of valid structural topologies and as global design goals to coerce the generation of desired designs. Semantics can also be used to control local aesthetic and functional properties of subdesigns. These local semantics can be formulated from the preferences and aesthetic values of the designer in order to explore the range of possible solutions within the designer's style.

Returning to the application of standard geodesic patterns with shape annealing, it can be seen that for case 1, spatial layout, case 2, layout under self-weight, and cases 8 and 9, functional layout for visual uniformity, geodesic patterns were approached. For functional design not including an aesthetic measure, although the random nature of the design process was not expected to generate these breakdowns exactly, local patterns of the modeled geodesic breakdowns can be seen with triacon patterns occurring more frequently than alternate patterns. Further control of aesthetics could also allow for designs that stylistically fall in between the traditional geodesic-like forms and the extreme asymmetric forms shown. This will be addressed in Sections 6.2 and 6.3.

6.2 The Golden Ratio System of Proportion

We have just seen in the design of domes that the incorporation of symmetry greatly affects the visual effects of designs with similar functional merit (compare the symmetric designs in Figure 6.6 with those in Figure 6.7). We also observed that the incorporation of a metric for visual uniformity brought visual order to the designs with less restriction on the design generation (compare the design in Figure 6.13 with those in Figure 6.16). While the metric for visual uniformity interpreted designs based on the use of a common shape, a linear proportional system, we will now demonstrate the use of a geometric proportional system, the golden ratio. Two means of implementing this proportional system in the generation of designs will be shown: (1) by adding syntax to the grammar rules that constrain the designs within the limits of the proportional system, and (2) by using the optimization to coerce the generation of designs that reflect the aesthetic model.

Similar to tensegrity structures, shown in the Chapter 5, the golden ratio (also called the golden section or the golden number) can be studied from different viewpoints as a mathematical phenomena or as a proportional system that is said to create harmonious order in a composition. It is this combination of mathematical and visual attributes that makes it particularly interesting for the layout of discrete structures. While the mathematical attributes of the golden ratio are indisputable the visual impact is questionable. Several studies have been performed on the aesthetic preferences of people in an attempt to prove the golden ratio as an aesthetically superior proportion but the results have been highly disputed and shown inconclusive. One explanation that seems appropriate for this work is that the golden ratio is a mathematical means to an aesthetic end and provides a simple example of the aesthetic goal of unity in variety (Schofield, 1958).

A great debate exists over who discovered the golden ratio and what structures have been designed using it as a proportional measure (also like tensegrity structures). The use of the golden ratio as a proportional system has been said to date back to the ancient Greeks who designed their temples according to this proportion and was again revived in nineteenth-century architecture as a geometrical system of proportion (Schofield, 1958). The golden ratio has been said to hold functional implications as Davinci related the golden ratio to the proportions of a "well-formed" human body, although similar drawings exist relating the human body to the system of Vitruvious and other harmonic proportional systems as well, (see Wittkower, 1988). From a mathematical standpoint, interesting applications of the golden ratio in geometry have produced the golden spiral and Penrose tilings (Grunbaum and Shephard, 1987; Smith, 1997).

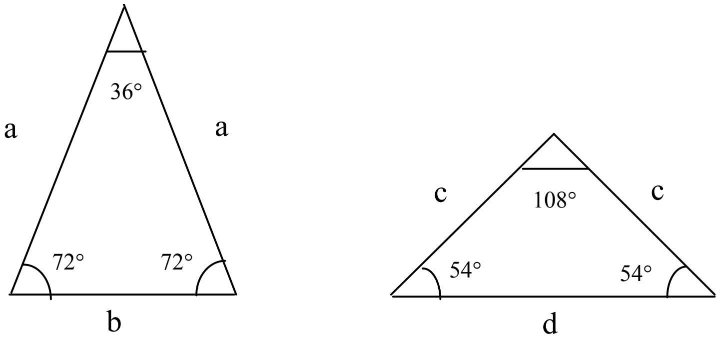
The golden ratio, denoted ϕ, has intriguing mathematical properties such as: $1/\phi = \phi - 1$ and $\phi^2 = \phi + 1$ and can also be related to the Fibinacci series such that the relation between any two numbers in the series approaches ϕ. Taking the Fibinacci series: 1, 1, 2, 3, 5, and 8 the ratios 1:1, 2:1, 3:2, 5:3, 8:5 oscillate around ϕ, 1.618...., while approaching ϕ as the series proceeds. Thus we can see that rather than coercing each line in a shape, or in the entire design as was done in the domes, there are multiple combinations of lengths all with a common base that can fit the aesthetic model.

An aesthetic measure using the golden ratio will now be formulated such that each shape in a design is interpreted based on its deviation from the golden proportions; the total aesthetic measure is then the summation of all deviations. Given a triangle with sides a, b, and c, an aesthetic measure is calculated for each shape in a design as:

$$\text{aesthtic measure } 1 = \left|\phi - \frac{b}{a}\right| + \left|\phi - \frac{b}{c}\right| + \left|\phi - \frac{a}{b}\right| \qquad \text{Eq. 6.1}$$

The aesthetic design goal is then to minimize the deviation from the golden ratio. While an explicit aesthetic measure of single shapes is calculated there is an implicit reflection of the relative proportions between adjacent shapes since a single line often lies in multiple shapes. The number of alternative geometric patterns that will meet the aesthetic design goal is large considering the allowance for multiple ratios of lengths to satisfy the golden ratio. We can see that the golden ratio aesthetic model provides for a much different aesthetic interpretation of designs than the global uniformity model used for dome design in Section 6.1.

The minimum aesthetic measure from Equation 6.1 is ϕ-1, where the relations of the sides are: b:a = ϕ and b:c = ϕ making a:b = 1 and create the golden triangle (Smith, 1997). The golden triangle is an iscocales triangle such that the ratio of a:b is ϕ resulting in base angles of 72° and a top angle of 36° (see Figure 6.18(a)). In this work the golden triangle is defined by the proportion of lengths regardless of the angles they form since this allows for greater freedom in pattern generation since two types of golden triangles are now valid (see Figure 6.18).



(a) golden triangle type a (b) golden triangle type b

Figure 6.18: Description of golden triangles

An interesting attribute of a golden triangle is that by bisecting one of the larger angles in a golden triangle two new golden triangles are formed (see Figure 6.19). While this attribute has been used to create the golden spiral[4], since both resulting shapes are golden triangles by proportional relations either shape can be subdivided such that two new golden triangles are formed, see Figure 6.19.

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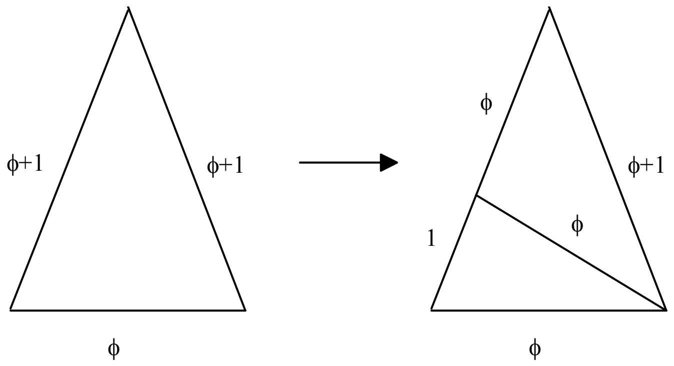
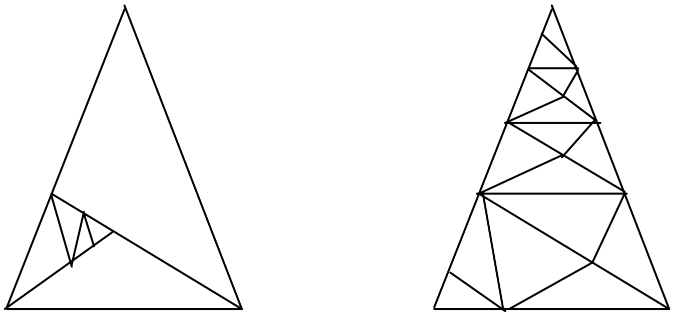
[4] This is the same relation as holds when creating a golden spiral from a golden rectangle.



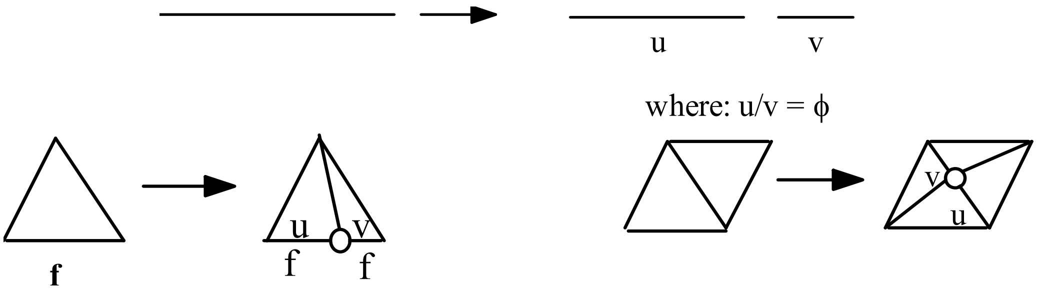
Golden Section



Golden Spiral Golden Pattern

Figure 6.19: Creating new golden triangle patterns

The golden section can be used to modify the shape grammar division rules, rules 1 and 8 from Figure 3.3, such that rather than bisecting the exterior line, the line is now divided into two segments, u and v, such that the ratio of segments, u/v, equals the golden ratio, ϕ. Reflected in the division modification rules, rules 1 and 8 from Figure 3.3, they now take the form:

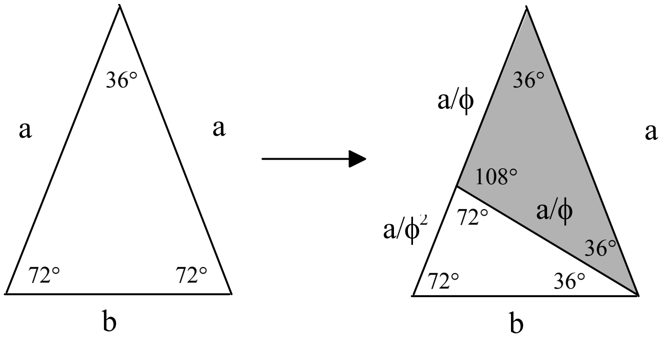


rule 1

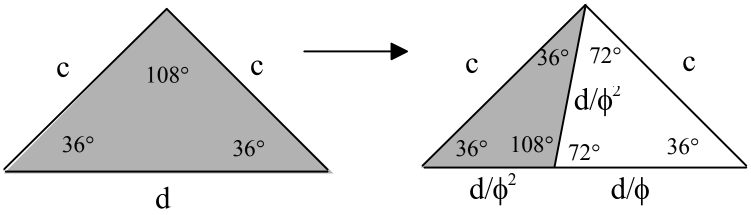
rule 8

Figure 6.20: Golden section rules

The reverse rules are unaffected by this new division. If the golden triangle is to be strictly adhered to, additional syntax is required in the grammar rules resulting in two syntactically different versions of rule 1 shown in Figure 6.21; both divisions result in one type a (white) and one type b (gray) golden triangle. Through the addition of syntax to the grammar rules, a dynamic grid that reflects the golden proportions is formed upon which the members of the structure can lie.



rule 1a



rule 1b

Figure 6.21: Syntax for strict golden section rule

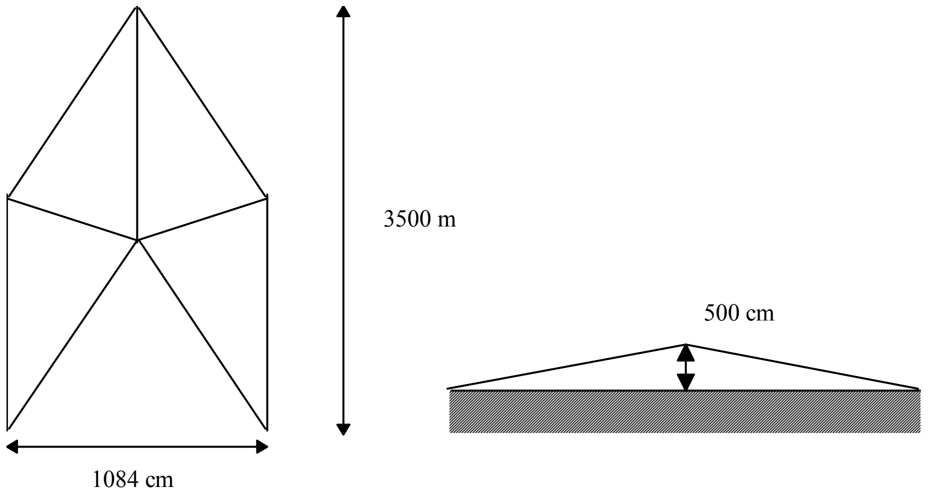
Next we shall explore the patterns that can be generated using the golden section as a proportional system within shape annealing. In order that the golden triangle is strictly adhered to, an initial layout that consists of golden triangles is required. One such shape, a common pattern found in Penrose tilings (Smith, 1997), is shown in Figure 6.22. The center point is elevated such that this structure could be used as a novel entrance way. The design objective is to minimize mass while meeting stress and buckling limits defined by the material properties in Table 6.9. The load on the structure is self-weight. Allowable displacement is calculated relative to the connected members such that a joint displacement causes less than 2° of rotation of a member about the opposite joint. The method parameters are listed in Table 6.10. Two different patterns generated with shape annealing and comprised of golden triangles are shown in Figure 6.23(a) and 6.23(b).[5]

Table 6.9: Material Properties for golden ratio study (Figures 6.23 and 6.25)

Material Property	
modulus of elasticity, E	2.067 E^7 N/cm^2
allowable tensile stress	14,880 N/cm^2
allowable compressive stress	-14,880 N/cm^2
specific gravity, ρ	.00785 N/cm^3

Table 6.10: Method parameters for golden ratio study (Figures 6.23 and 6.25)

Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum number of members	150
maximum area	none	number of iterations	170
member areas	discrete	number of designs per iteration	300
minimum member length	15 cm	space truss topology rules	1-9
minimum angle between members	10º	rule selection	Hustin
intersections between members	not allowed	constraint violation normalization	yes
member shape	tube		



Top View

Elevation

Figure 6.22: Golden triangle layout problem specification

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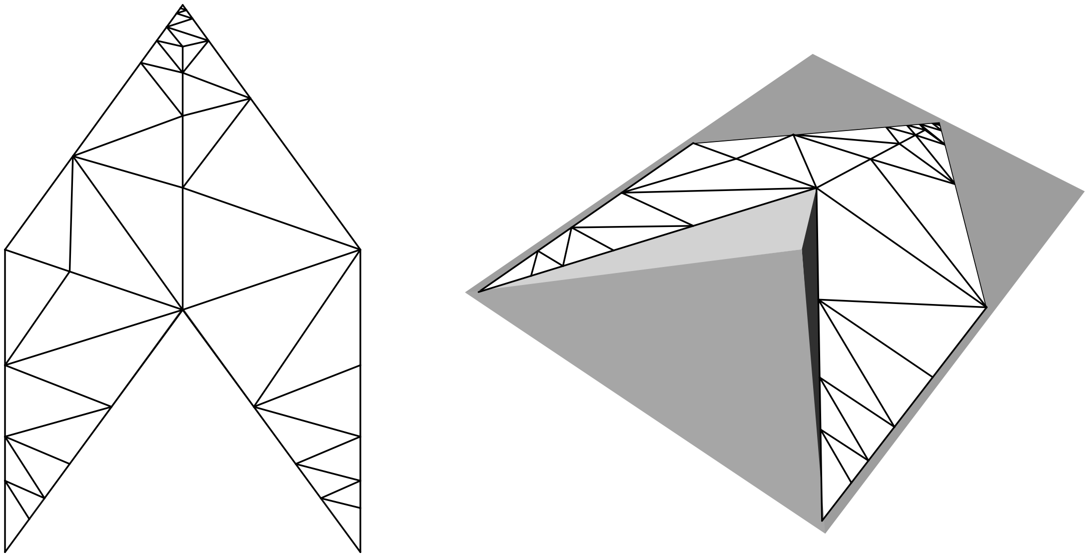
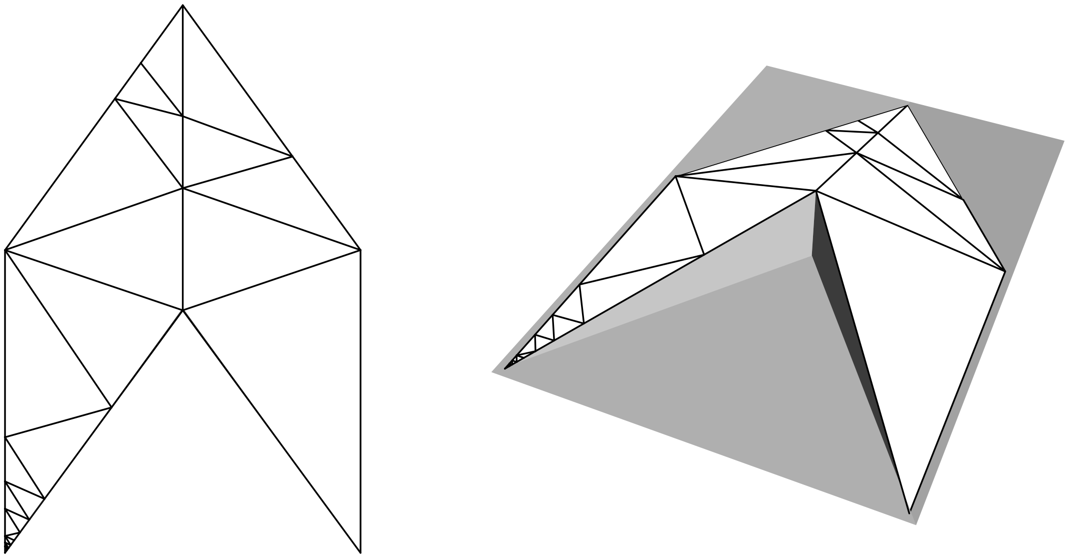
[5] The width of the lines represents the diameter of the members uniformly scaled with the geometry of the figure.



(a) mass = 371 kg



(b) mass = 353 kg

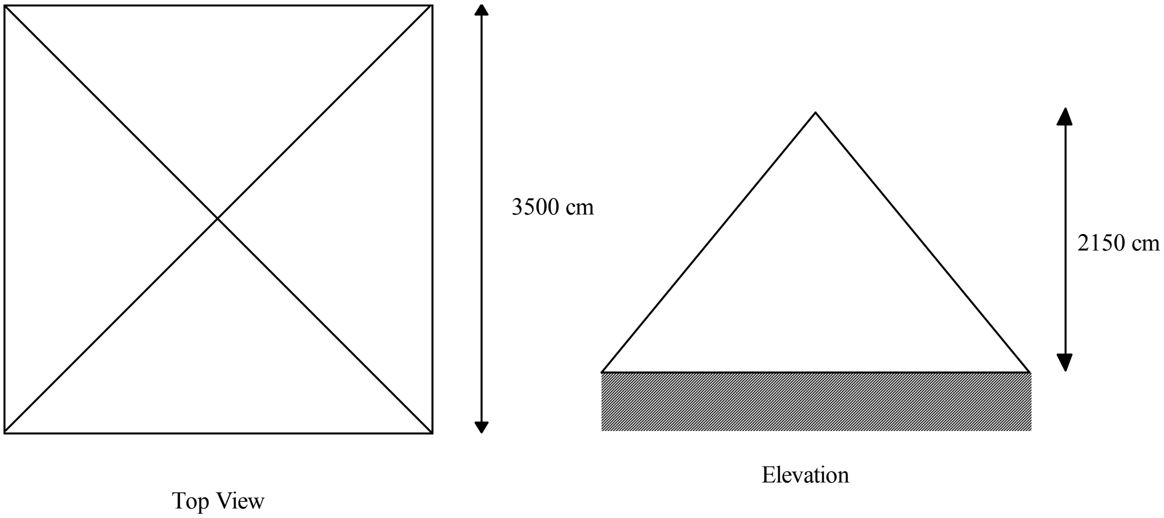
Figure 6.23: Constrained golden triangle design

Since there are limited practical applications of a structure comprised of golden triangles a general approach using the golden triangle as the aesthetic model rather than the golden ratio used in Equation 6.1 will now be explored. Through a combination of the syntax free golden section rules from Figure 6.20 and calculating an aesthetic measure for a shape based on the deviation from the golden triangle, shapes can be coerced to the golden triangle by optimizing the aesthetic design goal. Considering a shape with sides a, b, and c where b: c is less than b:a a second aesthetic measure is calculated as:

$$\text{aesthtic measure2} = \left|\phi - \frac{b}{a}\right| + \left|1 - \frac{b}{c}\right|. \qquad \text{Eq. 6.2}$$

This relation will be demonstrated on the layout of a pyramid inspired by I.M. Pei's design for the glass pyramids at the Louvre in Paris (Eggen and Sandaker, 1995) that has a golden proportion between base length and height (Figure 6.24). The material specifications and method parameters are the same as the constrained golden triangle example (Figure 6.22) and are listed in Table 6.5 and Table 6.6 respectively. The aesthetic measure is calculated using the planar projections of shapes. Comparing this design, Figure 6.25, with the designs in Figure 6.23 although the strict golden section pattern is not followed local patterns of golden triangles can be seen within the design. The question of which implementation is better, constraining the layout to a proportional geometric system or coercing desired geometric proportions through the optimization, must be answered by a designer's individual preferences and design objectives. If a conventional design is sought then constraining the proportions used in the design will explore designs within the limits of the proportional system. Conversely, if a novel solution is desired that reflects the proportional system but does not strictly adhere to it then it is better to model the desired aesthetics rather than constrain the space of design alternatives. The important point is that both aesthetic design objectives can be formulated within the shape annealing method.

(a) Pyramid at the Louvre, architect: I.M. Pei (Eggen and Sandaker, 1995)



(b) Pyramid problem specification

Figure 6.24: Pyramid design problem

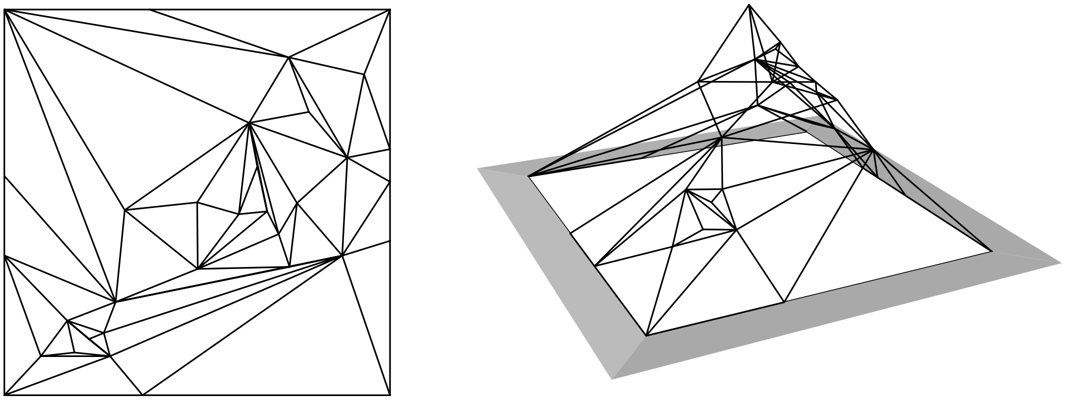


Figure 6.25: Pyramid design with coerced golden ratio proportions; mass = 4,428 kg

6.3 Roof Design

The application of space trusses to the design of roofs can provide for large open enclosure spaces unencumbered by intermediate supports. The application of domes as well as different geometric shapes, a vaulted barrel and a conoid will be explored for roof designs. The examples presented use dimensions from existing structures: the Oasis Swimming Pool, in Swindon, England designed by Roper IBG (Orton, 1988), the Galleria Vittorio Emanuele II in Milan designed by architect G. Mengoni (Eggen and Sandaker, 1995), and an octagonal airplane hanger in New York designed by civil engineer C. Sacre (Sacre, 1996). The design of each of these structures was modeled using a discrete space truss that supports the roofing material[6]; glass in the case of the pool and gallery and steel in the case of the airplane hanger. Since roofs are often designed for visual effects, the model of elegance using the golden ratio proportional system from Section 6.2 will be used. In each case the loading on the structure is determined from self-weight of the structural members and a surface load that is a combination of the weight of the roofing material and a snow load. Allowable displacement is calculated relative to the connected members such that a joint displacement causes less than 2° of rotation about the opposite joint.[7]

For roof design, rather than using a design goal to maximize enclosure space as was done for dome design in Section 6.1, a constraint is placed on the design such that the space is always enclosed since a roof with holes does not meet the functional requirement. In order to constrain

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[6] The surface load on each shape in a design is distributed to the nodes of the shape based on the lengths of members in the shape.

the design to enclose the specified space, additional syntax was added to grammar rule 1 from Figure 3.1 that divides an exterior edge in the design, which in the case of roof design is always on the perimeter of the roof. Instead of allowing the new point to move freely it is constrained to move along the perimeter of the defined roof boundary according to the line on which the division occurred; see Figure 6.26. For instance, if the point divides a vertical line the constrained point will only move vertically, while if the point divides a diagonal line the constrained point will move according to the line equation of the line that was divided. This additional syntax will ensure the generation of roofs without holes in order to meet the design problem specification.

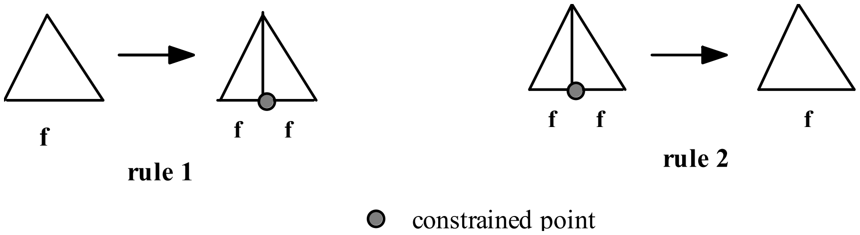

Figure 6.26: Constrained point rule syntax

6.3.1 An Aluminum and Glass Dome Roof Design

The first essay in roof design is a dome that is used to cover a pool based on the roof of the Oasis Swimming Pool, in Swindon, England designed by Roper IBG (Orton, 1988); see Figures 6.28(a) and 6.28(b). This is a practical application of the previous dome investigation in Section 6.1 for a larger scale design problem where two times the number of members were used (100 compared to 50) and the structure is constrained to fully enclose the prescribed space. The problem specification for a Schwedler type dome will be used differing from the geodesic domes in that it is a statically determinant structure that forms a compression ring around the base of the dome. The generation of a simple Schwedler dome using the grammar from Figure 3.3 is shown in Figure 6.27. The problem specification is shown in Figure 6.28(c) with a span of 4513 cm and a height of 914 cm. The material specifications are listed in Table 6.11, the method parameters are listed in Table 6.12, and the surface loading is listed in Table 6.13. The design shown in Figure 6.29 was generated for efficiency and visual uniformity. Comparing this design with that in Figure 6.13 we can see that now the design fully encloses the space while the coercion of the structure to a spherical enclosure arises from the surface load applied (the structure wants to minimize the surface area) and the projection onto a prescribed ellipsoid. This design also

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[7] The width of the lines for all roof designs was uniformly scaled with the geometry of the figure which

achieves a closer approximation of an ellipsoid since more members can be used in the breakdown. It is interesting to note that the impact of the uniformity design goal is not as significant as was seen in Figure 6.16 partly because the points on the perimeter are constrained to remain there and party due to the increase in the space of possible designs due to doubling the number of allowable members.

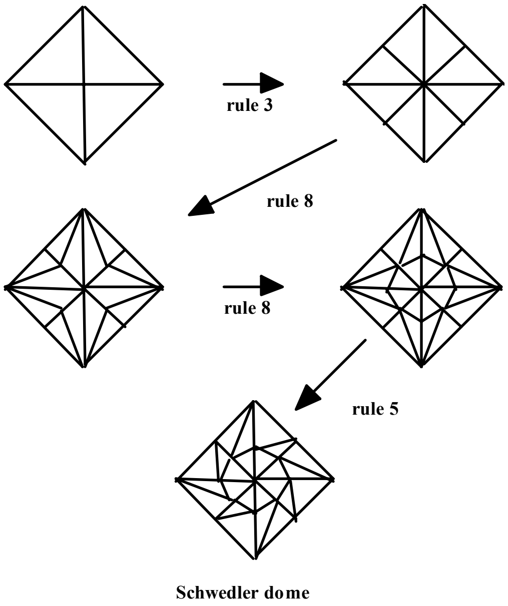

Figure 6.27: Generation of Schwedler dome using the grammar in Figure 3.3

Table 6.11: Material properties for dome roof design (Figure 6.29)

Material Property	*Aluminum*
modulus of elasticity, E	$7.1\ E^6\ N/cm^2$
allowable tensile stress	$21{,}000\ N/cm^2$
allowable compressive stress	$-21{,}000\ N/cm^2$
specific gravity, ρ	$.0027\ N/cm^3$

Table 6.12: Method parameters for dome roof design (Figure 6.29)

Method Parameter		Method Parameter	
minimum area	$.01\ cm^2$	maximum number of members	150
maximum area	263	number of iterations	170
member areas	continuous	number of designs per iteration	300
minimum member length	15 cm	space truss topology rules	1-9
minimum angle between members	10º	rule selection	Hustin
intersections between members	not allowed	constraint violation normalization	yes
member shape	tube. d/t=10		

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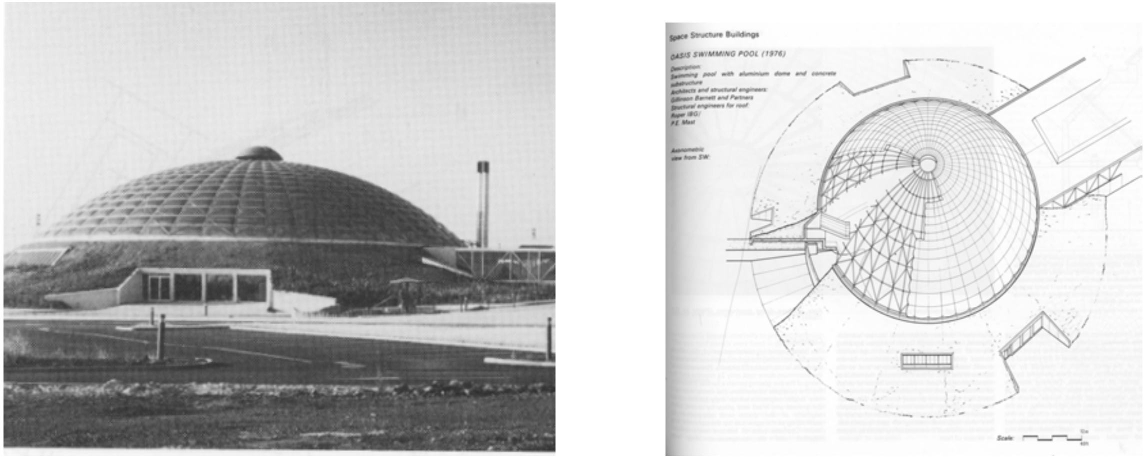
results in the same width line for all members.

Table 6.13: Loading for dome roof design (Figure 6.28)

Surface Load	
snow	.1920 N/cm^2
1/2” plate glass	.0321 N/cm^2
Total Load	.2476 N/cm^2



Space Structure Buildings

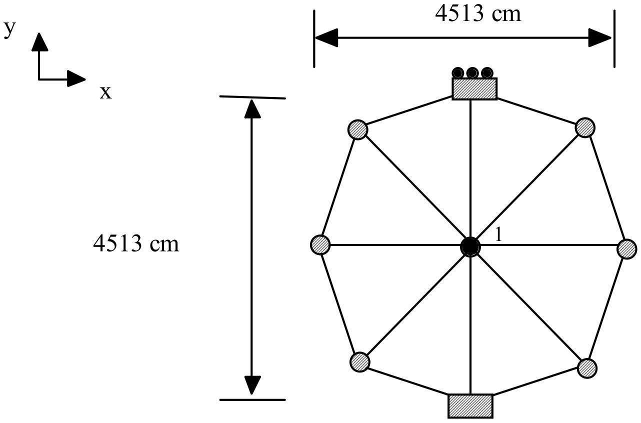
OASIS SWIMMING POOL (1976)

Description:
Swimming pool with aluminium dome and concrete substructure
Architects and structural engineers:
Gillinson Barnett and Partners
Structural engineers for roof:
Roper IBG/
P.E. Mast

Axonometric
view from SW

(a) Oasis swimming pool (Orton, 1988)

(b) roof design: Roper IBG (Orton, 1988)



(c) dome problem specification

Figure 6.28: Dome roof design problem

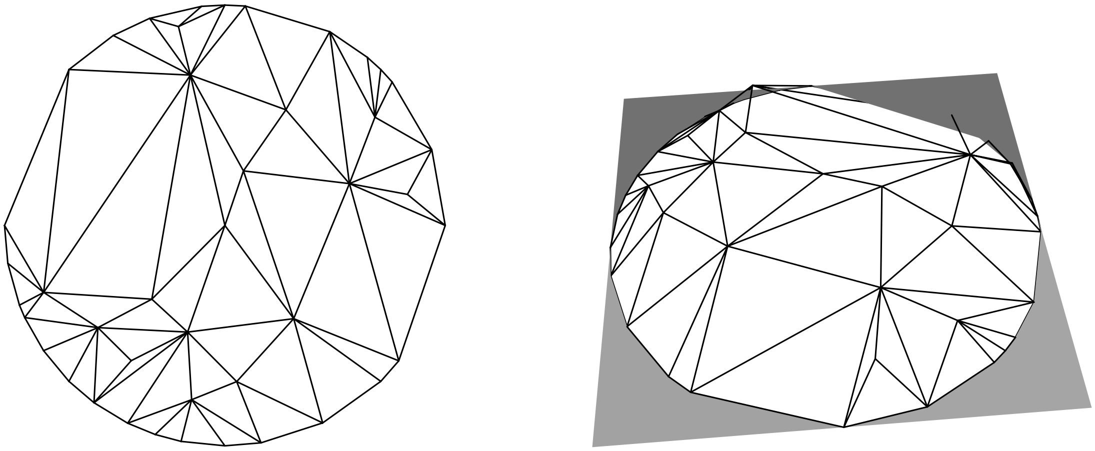


Figure 6.29: Dome roof design; mass = 12,858 kg

6.3.2 A Steel and Glass Gallery Roof Design

A gallery is an example of a glazed space that makes use of a glass ceiling to provide natural lighting to the enclosed space. In this example the layout is based on the 14.5m passages of the Galleria Vittorio Emanuele in Milan that has a glass vaulted barrel roof (Eggen and Sandaker, 1995); see Figure 6.30(a). For this example, a 14.5m by 14.5m module has been arranged in golden proportions such that the overall ratios are 2:3:5, see Figure 6.30(b). The material properties are listed in Table 6.14 with the method parameters in Table 6.15. The surface load from the glass roofing and an estimated snow load as 75% of the maximum suggested for typical urban areas (Schodek, 1980, p.91) are listed in Table 6.16. Two designs are presented in Figure 6.31: the first, Figure 6.31(a) uses the aesthetic measure of visual uniformity from Section 6.1, and the second Figure 6.31(b) uses the aesthetic measure of the golden proportion from Equation 6.1. The two designs, although similar in mass, have very different visual impacts due to the difference in aesthetic models. This effect is increased in the perspective views of the two designs since the aesthetic measure was calculated from the actual proportions of the shapes rather than a planar projection as we saw in the previous golden section examples (Figures 6.23 and 6.25).

Table 6.14: Material properties for gallery roof design (Figure 6.31)

Material Property	
modulus of elasticity, E	$2.067\ E^{7}\ N/cm^{2}$
allowable tensile stress	$14{,}880\ N/cm^{2}$
allowable compressive stress	$-14{,}880\ N/cm^{2}$
specific gravity, ρ	$.00785\ N/cm^{3}$

Table 6.15: Method Parameters for gallery roof design (Figure 6.31)

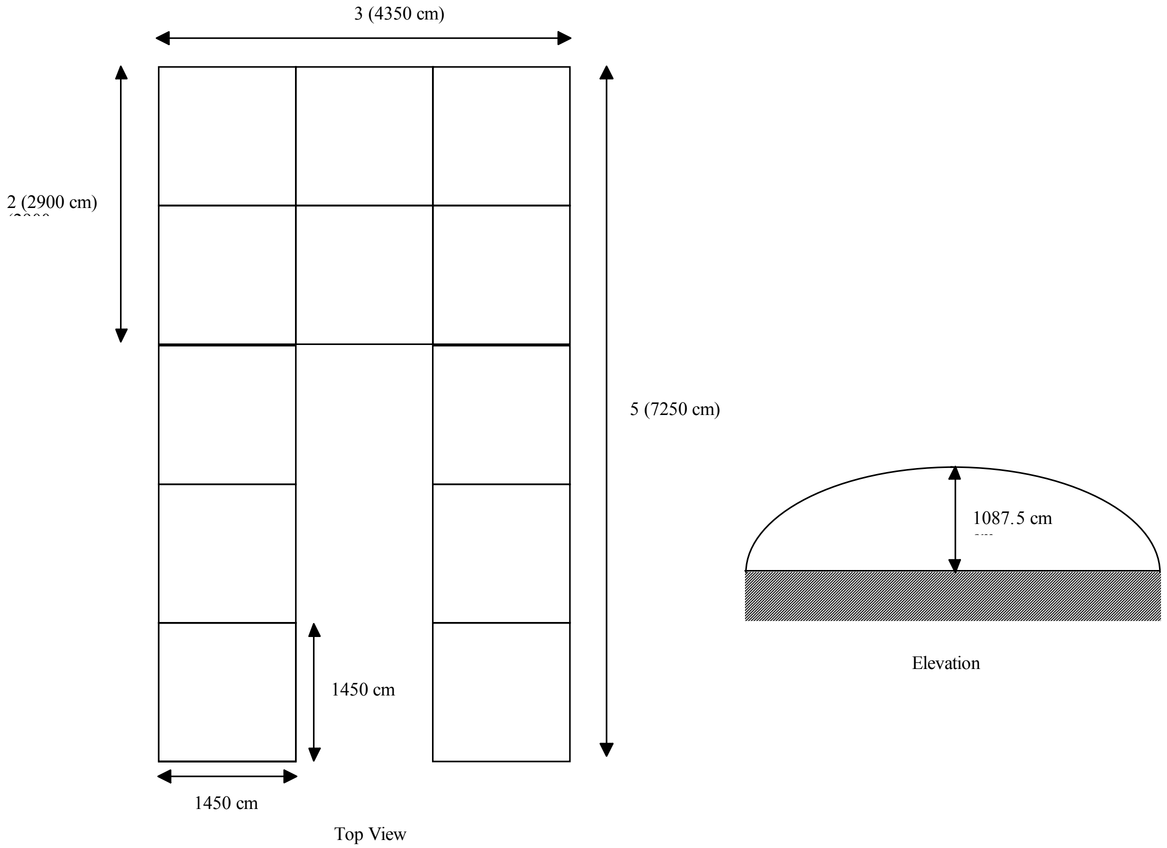
Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum number of members	150
maximum area	none	number of iterations	170
member areas	discrete	number of designs per iteration	300
minimum member length	15 cm	space truss topology rules	1-9
minimum angle between members	10º	rule selection	Hustin
intersections between members	not allowed	constraint violation normalization	yes
member shape	tube		

Table 6.16: Loading for gallery roof design (Figure 6.31)

Surface Load	
snow	.2155 N/cm^2
1/2" plate glass	.0321 N/cm^2
Total Load	.2476 N/cm^2

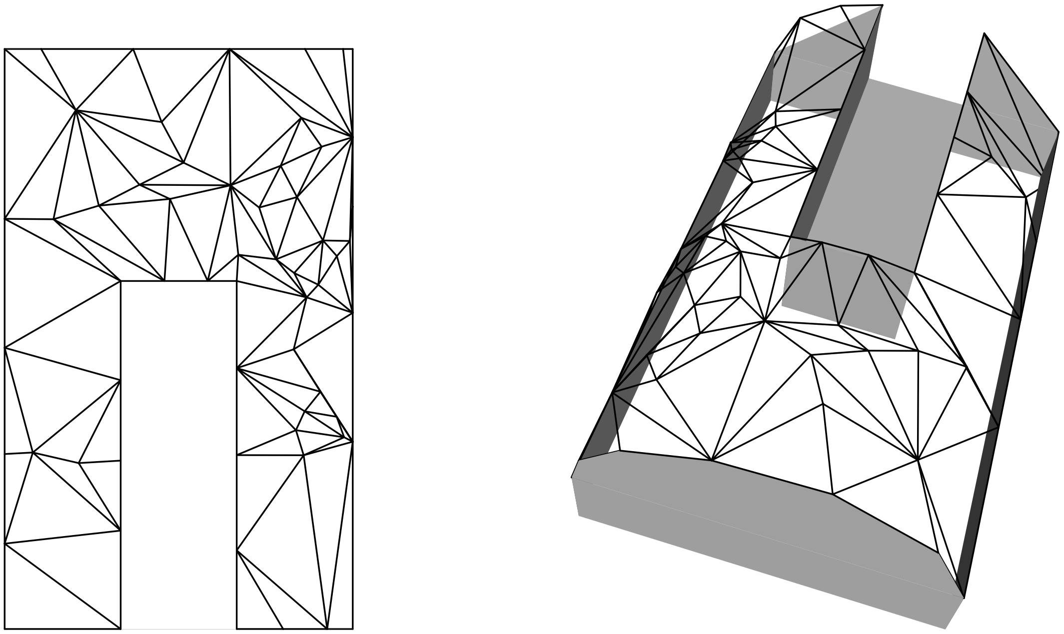
(a) Passages of the Galleria Vittorio Emanuele,

architect: G. Mengoni (Eggen and Sandaker, 1995)

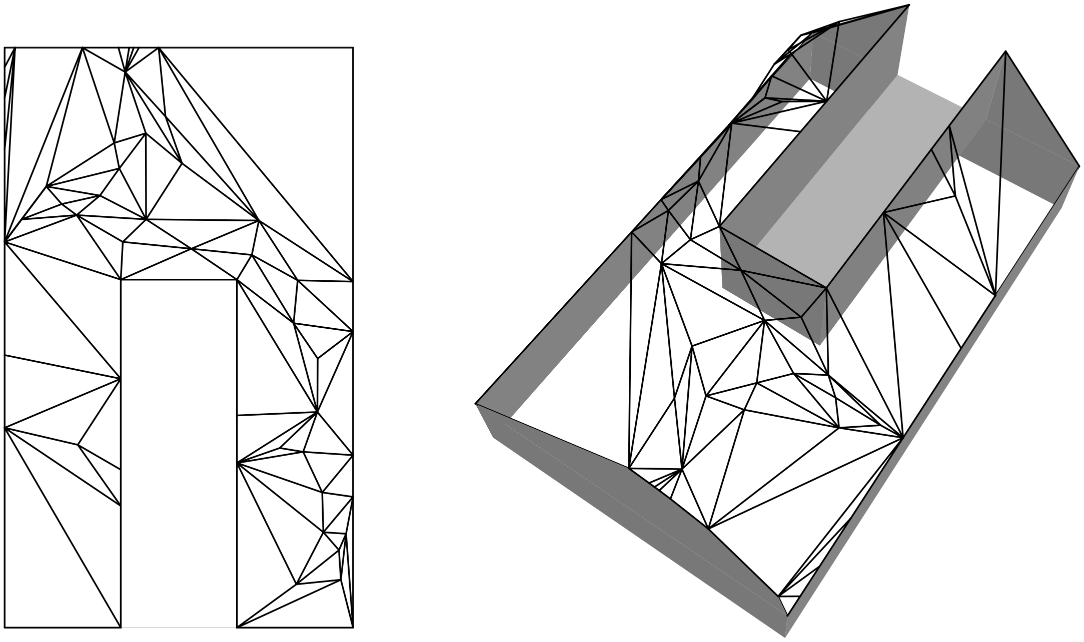


(b) gallery roof problem specification

Figure 6.30: Gallery Design problem



(a) uniformity layout; mass = 24,017 kg



(b) golden proportion layout; mass=27,280 kg

Figure 6.31: Gallery designs

6.3.3 An Octagonal Airplane Hanger Design

An airplane hanger is also a good example of a large space that requires enclosure without the use of intermediate columns. The model for this example is the design of an octagonal airplane hanger in New York designed by civil engineer C. Sacre (Sacre, 1996) where the objective of using an octagonal perimeter was to decrease costs when compared to a square perimeter. This example demonstrates the capability of the method to design a structure that covers a complex boundary shape and takes advantage of using a double-curvature roof shape to reduce the roof size while providing the desired enclosure space. The specifications of the hanger were such that the difference in clearance needed at the nose of the airplane and at the tail, which is drastically higher, was 1128 cm, see Figure 6.32. To take advantage of this, a conoid was prescribed over the perimeter to provide high clearance only on one end of the hanger. This is different than the design by Sacre who used a sequence of steel planar trusses at varied heights to support a sloping roof. The structure was designed using the loading and material specifications reported by Sacre (1996): ASTM A500 Gr. B steel tubes were used for the discrete members with the material properties listed in Table 6.17 whereas the surface loading from the roofing material and the prescribed snow load are listed in Table 6.19. The method parameters are listed in Table 6.18. Three designs are presented in Figure 6.33; the first, 6.33(a) is designed for pure efficiency, the second, 6.33(b), uses the aesthetic measure of visual uniformity from Section 6.1, and the third 6.33(c) uses the aesthetic measure of the golden proportion from Equation 6.1. Comparing the designs shown, imposing a design goal of uniformity works counter to the efficiency of the structure. since the surface is complex, while the golden ratio model provides for more latitude in dimensions that satisfy the aesthetic model and thus works with the efficiency of the structure.

Table 6.17: Material Properties for airplane hanger design (Figure 6.33)

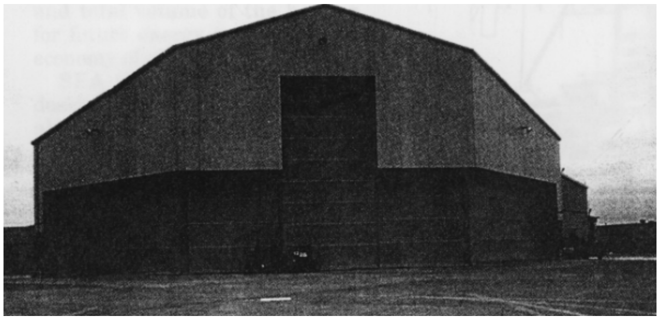
Material Property	
modulus of elasticity, E	2.067 E^7 N/cm^2
allowable tensile stress	19,016 N/cm^2
allowable compressive stress	-19,016 N/cm^2
specific gravity, ρ	.00785 N/cm^3

Table 6.18: Method parameters for airplane hanger design (Figure 6.33)

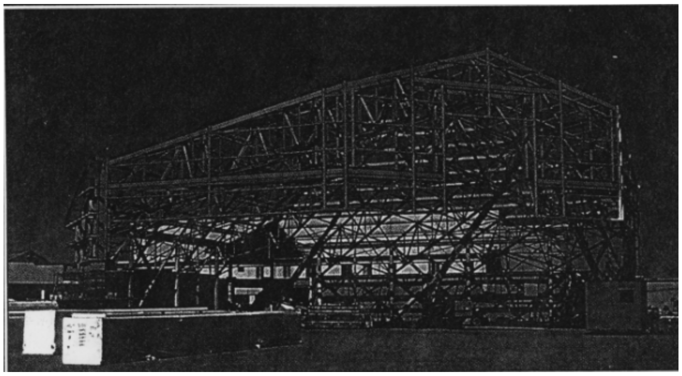
Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum number of members	150
maximum area	263 cm	number of iterations	170
member areas	continuous	number of designs per iteration	300
minimum member length	15 cm	space truss topology rules	1-9
minimum angle between members	10º	rule selection	Hustin
intersections between members	not allowed	constraint violation	yes
member shape	tube, d/t=10	normalization	yes

Table 6.19: Loading for airplane hanger design (Figure 6.33)

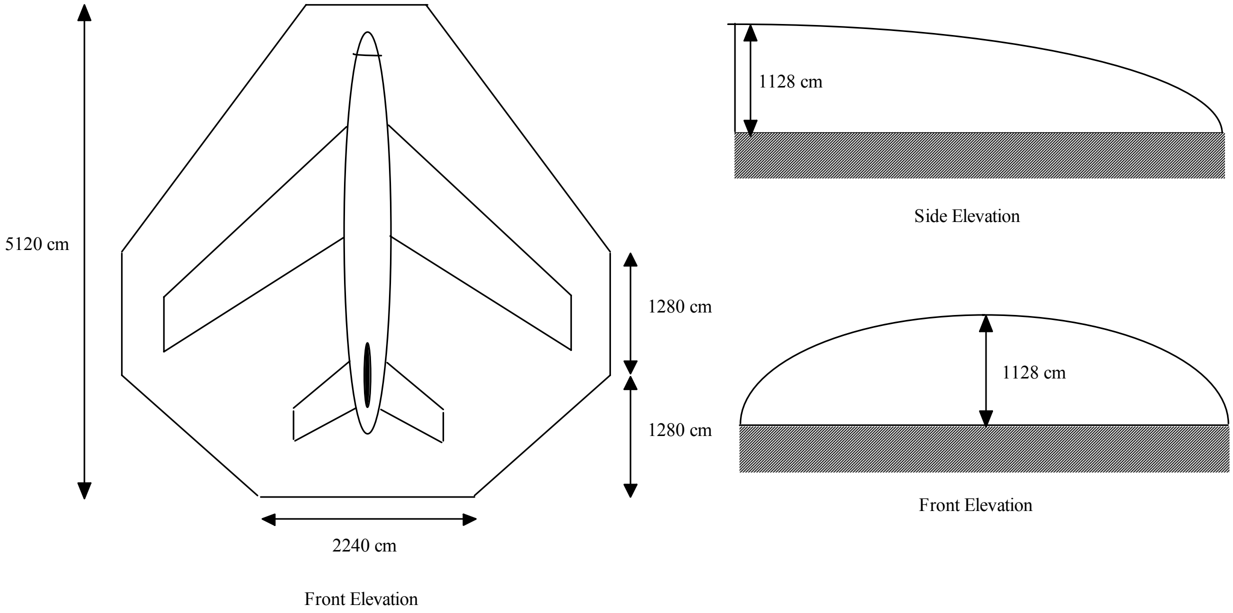
Surface Load	
snow	.1676 N/cm^2
roofing	.0192 N/cm^2
Total Load	.1868 N/cm^2



(a) Octagonal airplane hanger, civil engineer: C. Sacre (Sacre, 1996)

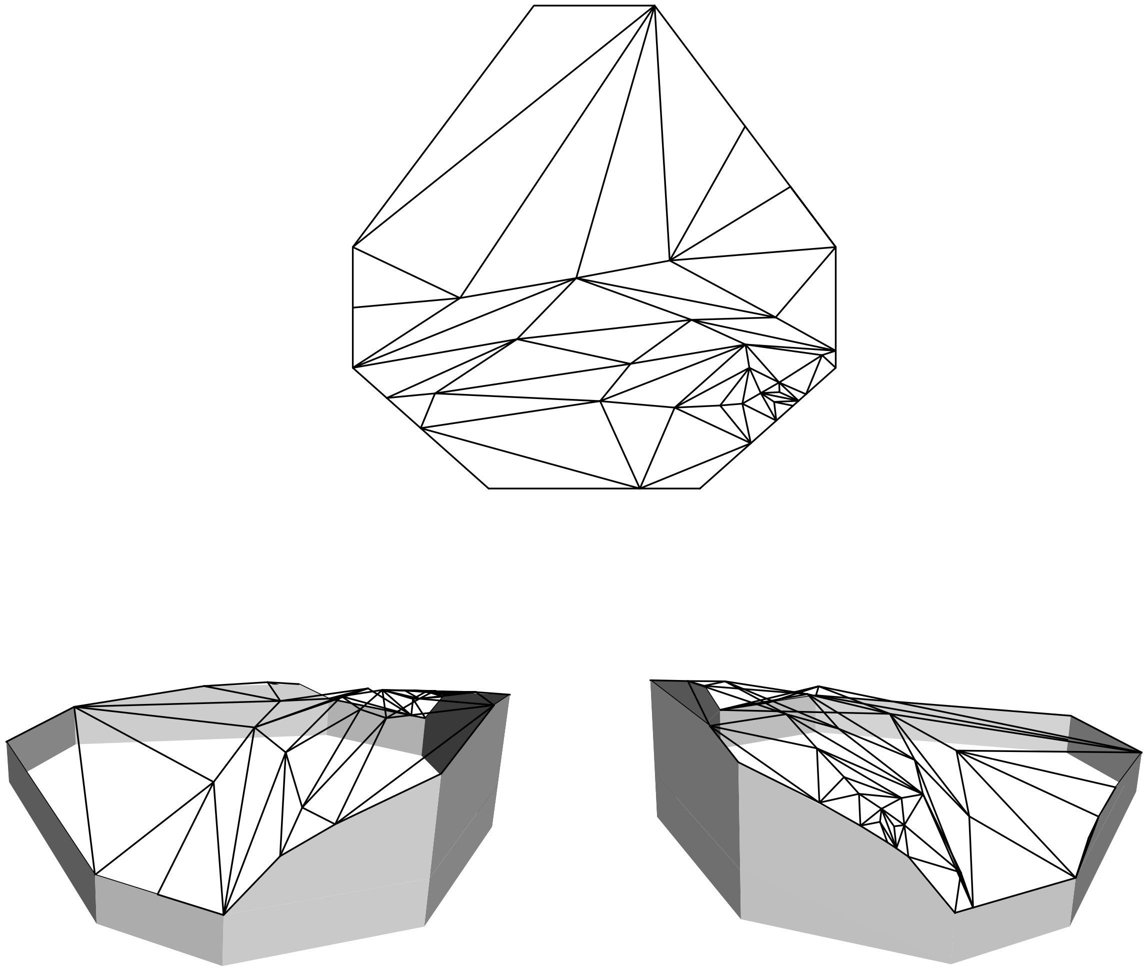


(b) side view of airplane hanger



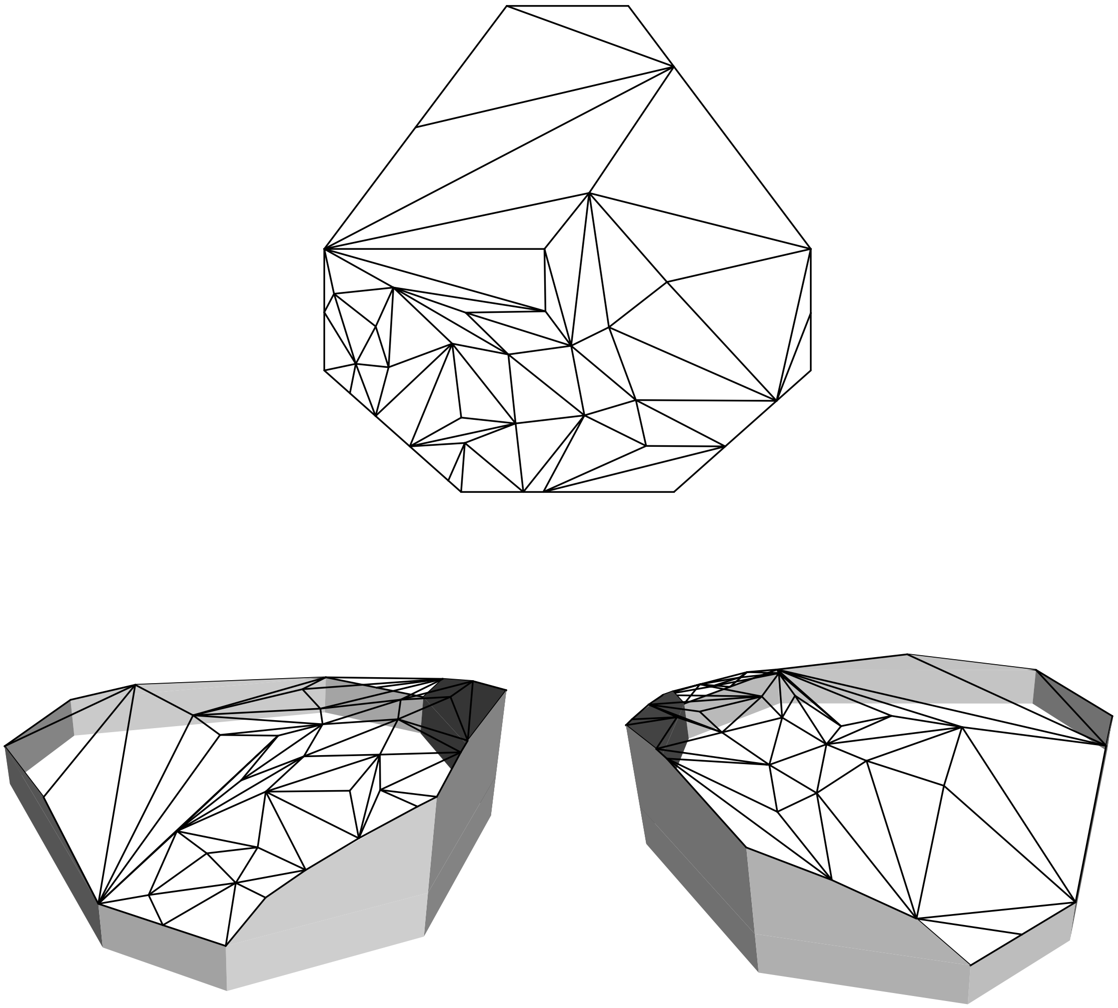
(c) airplane hanger problem specification

Figure 6.32: Octagonal airplane hanger design problem



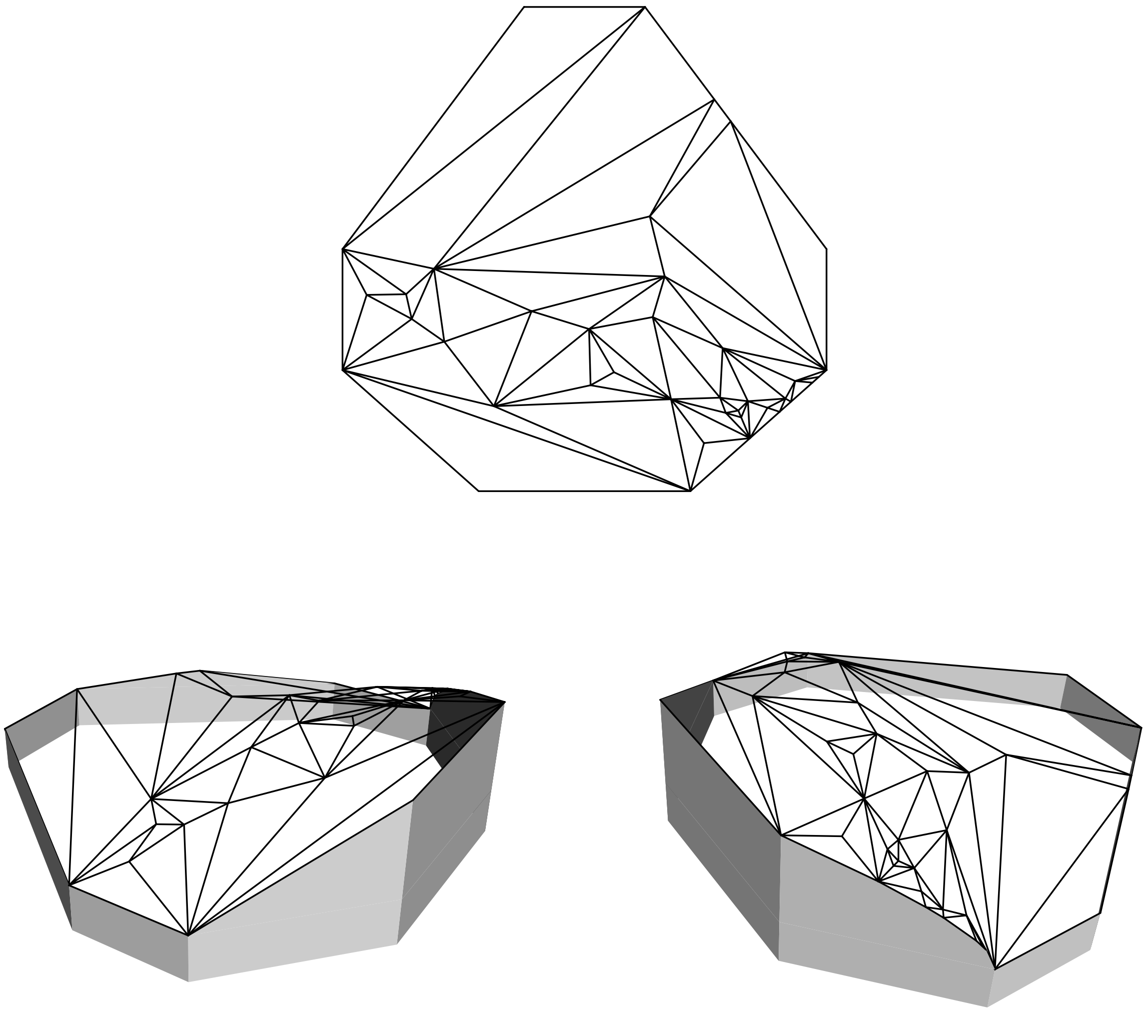
(a) Hanger design for functional efficiency; mass = 41,133 kg

Figure 6.33: Octagonal airplane hanger designs



(b) Hanger design for visual uniformity; mass = 69,909 kg

Figure 6.33(cont.): Octagonal airplane hanger designs



(c) Hanger design with golden proportions; mass = 31,725 kg

Figure 6.33(cont.): Octagonal airplane hanger designs

6.4 Summary

Essays of space trusses were shown for a range of design applications that present various styles for solving a given structural problem when granted the latitude for design exploration. The derivation of the space truss shape grammar rules was shown based on standard rules for geodesic dome design and applied to the design of both traditional and novel domes thus illustrating the capability of shape annealing to generate three-dimensional geodesic-like dome designs. An additional proportional system using the golden ratio was added to provide for a geometric proportional system rather than the model of uniformity used in the dome examples. This geometric system was seen to be particularly useful in the proportioning of space truss designs for polygon roof perimeters since it allowed the structure to adapt to the varying dimensions of the shape. Two scales of problems were also shown to demonstrate that the shape annealing approach is not limited to the design of idealized problems.

7. METHOD ASSESSMENT

A study of roof truss designs conceived by human designers and those generated by shape annealing will be presented. The purpose of this study was to assess the capabilities of the shape annealing method in (1) meeting the needs of designers with varying design intent and (2) presenting novel structural forms that expand the range of designs considered by designers in the conceptual design stage. This study was carried out for the design of an exposed roof truss in the aquatics area of the University Center at Carnegie Mellon University. In addition to the study, an advanced computer tool for structural design based on the shape annealing method will be discussed to exploring how shape annealing can support structural designers.

7.1 Comparing Shape Annealing to Designers

In the essays of structures shown in previous chapters, shape annealing has been presented as a method for the configuration of optimally directed discrete structures. This study will assess the capabilities of the shape annealing method as an effective aid to structural designers through a comparison of roof truss designs proposed by six structural designers, three civil engineers and three architects, to designs generated by shape annealing. The study consists of three parts: (1) an analysis of the design rationale used by the participating designers to identify important design goals in conceptual structural design, (2) an exploration of design models implemented in shape annealing that reflect the goals identified to generate appropriate design alternatives, and (3) a comparison of the resulting shape annealing designs to those of the participating designers. Comparing shape annealing to human designers will allow us to evaluate the capabilities of shape annealing in modeling a range of practical design goals and generating structural forms that would not be conceived by human designers. The purpose in generating multiple design alternatives is to expand the number and range of designs considered by the designer in the conceptual design stage.

In order for the designs to be purposeful to a designer they must be appropriate to the design task and suit a designer's style. Since design preferences and goals change during the design of one structure and also among different structural design problems, a computational tool that can adapt to different design goals and problems is beneficial in suiting the varied requirements of these structural design problems. The generality of shape annealing makes it an appropriate and effective tool for the dynamic nature of structural design by generating a variety of conceptual designs that reflect the desired design objectives and satisfy problem constraints.

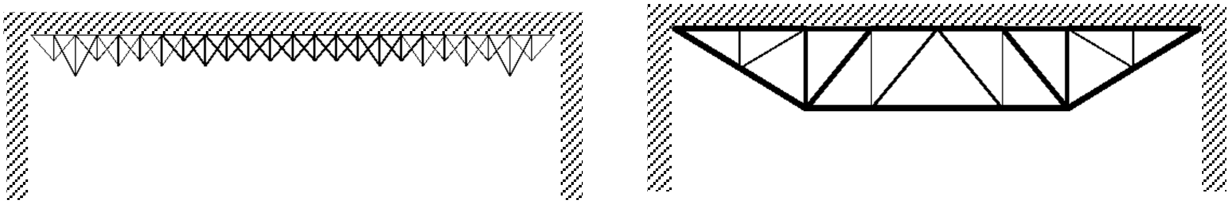
The intent of this study is not to conclude that either humans or computers can design more efficient trusses but rather to compare attributes of designs conceived by both means, allowing us to explore the requirements of an effective computational tool for conceptual structural design. This design study was deliberately conducted such that individual designers were allowed to apply their own design rationale and style to the problem rather than being constrained to a specified design goal or set of goals. This approach was taken so that designers could explore the problem in a natural way since specifying a common metric of design goals for both architects and civil engineers would be artificial. Although this approach does not allow us to make quantitative comparisons, qualitative comparisons can be made about the design goals that various configurations achieve and the different ways that designs can achieve the same goal. These qualitative comparisons can then be used to illustrate strengths and weaknesses of both computer-based design and the human design process allowing us to identify where shape annealing could best aid the designer.

This study was performed to explore the capabilities of shape annealing as an aid to structural designers with different preferences in expanding both their creative ability and problem insight. Design and design perception is based on a combination of insight, intuition and experience. In structural design, each designer has a different style that is founded in their knowledge of functional forms and their viewpoint of the importance of functional efficiency, economy and beauty and the relation among them. Following from the civil engineers' quest to control forces, they are primarily focused on functional efficiency and clarity as well as construction costs, or economy, resulting in preferences for conventional, uniform structural forms without ornamentation, or non-functional material. Civil engineers tend to also find beauty in structures that allude to natural forms since they are perceived to be structurally efficient. Conversely, an architect's attempt to control space results in primary design goals of artistic expression and visual impact with only a secondary goal of functional efficiency. Architects tend to have more variation in their aesthetic values since they are not always founded in functional efficiency alone allowing for greater latitude in the structural forms that are considered in the design process. In order to be effective, a computational method must support these varying preferences and design rationales used in the structural design process.

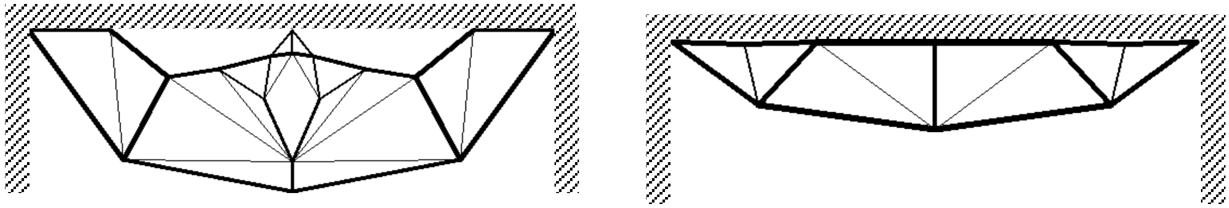
Varying intentions and preferences also play a role in design perception that has been evident in the reactions of different designers to the designs generated by shape annealing. While regular, symmetric designs please civil engineers, architects are unenthused. On the other hand, a civil engineer's preference for the rhythm of conventional designs makes them cringe at the highly asymmetric designs while the visual impact of the asymmetries intrigue architects. These conflicting opinions stem from the different roles the two types of designers play in structural design. A civil engineer creates structural form to control physical effects, or forces, while an architect seeks to control the space to be used by people (Billington, 1983). The design of a balanced structure can be achieved through an appreciation of both functional efficiency and visual appeal. Just as a conventional truss is unappealing to an architect for its lack of attention to visual impact, a novel structural form that lacks attention to physical laws is as outrageous to a civil engineer. The shape annealing method will be shown to be capable of aiding both types of designers in generating structures that are derived from functional efficiency yet through the stochastic nature of the method allow aesthetic design goals to be satisfied as well.

A common structural design application for both civil engineers and architects is the design of buildings. Considering truss design in buildings, trusses are primarily used as structural support mechanisms designed purely for utilitarian purposes since in most cases the truss is not visible. The example used in this study considers the design of an exposed roof truss allowing the designers to explore both the functional and visual components of the structural problem. While a civil engineer still may decide to use a more conventional form viewing the design problem as one of function alone, the architect is given the latitude to view the problem from the visual side while keeping the functional aspect of the problem as an aside.

To illustrate the range of structural forms that can be designed for a roof truss, a sample of trusses are shown in Figure 7.1. For this study, an architect submitted the design in Figure 7.1(a) while the more conventional design shown in Figure 7.1(b) was submitted by a civil engineer. Meeting the design intent of both types of designers, shape annealing was used to generate designs that suit both designers' preferences and are shown in Figure 7.1(c) and (d). The details of these designs will be presented later.



(a.) design proposed by an architect (b.) design proposed by a civil engineer



(c.) novel shape annealing design (d.) conventional shape annealing design

Figure 7.1: Illustrative comparison between human designers and shape annealing

First, the study will describe the specifications of the roof truss design problem and present the solutions submitted by the designers along with their design rationale to identify significant design goals. Shape annealing will then be used for two purposes: (1) to optimize the geometry of the submitted designs, both shape and sizing, and (2) to generate alternative topologies for a series of optimization models that reflect the designers' objectives. Optimizing the fixed topologies proposed by the designers will explore the relationship between structural efficiency and form for traditional truss layouts while the generation of alternative topologies will illustrate the range of layouts appropriate to the design problem. The shape annealing designs will then be compared to the human designers' designs to evaluate the advantages and disadvantages of both, as well as the integration of the shape annealing method in the structural design process. Finally, a critique of the shape annealing designs by the participating designers will be discussed to explore the value of the generated designs and their capability in expanding the problem insight of designers.

The configuration of trusses with the shape annealing method is quite different from the process generally used by a designer. Rather than rely on knowledge of standard truss configurations, shape annealing uses topology modification rules to add and remove members based on principles of simple truss design. Shape annealing is a form-driven strategy where the structural form is modified independent of the behavioral implications. The function of a design then follows the generated form and a design is selected in the optimization based on the functional evaluation alone. Even though the optimization is independent of the structural form, the stochastic nature of the method as well as the tradeoffs among design goals lead to the generation of spatially innovative structures. The use of aesthetic models seen in Chapter 6 but was not considered in this study.

The advantage of computer-based conceptual design with shape annealing is that the resulting design is a direct derivative of the allowable forms described by the shape grammar and the design goals formulated in the optimization model. While preconceived notions and numerous competing design issues constantly influence a designer, computational design supports unbiased exploration for optimally directed functional forms that reflect the set of modeled design goals. In the current shape annealing implementation the focus is on the generation of conceptual designs so detailed design considerations such as fabrication, maintenance and durability are not included in the design evaluation and are left for the designer to assess. But, any important criteria that can be articulated can be included in the computational analysis. While shape annealing is not intended to replace the designer, the goal is an effective tool to enhance designer capabilities by presenting alternative concepts that would be difficult to conceive by hand. Additionally the generation of multiple design concepts that are optimally directed and satisfy the design goals and constraints of a given problem specification could spark creativity and lead to new insights about the design problem.

7.1.1 Problem Statement

The design problem used in this study is based on the open truss system found in the aquatics area of the University Center at Carnegie Mellon University. The following specifications present the information given to the designers. The problem posed entails designing a truss system to support the roof using the simplified model shown in Figure 7.2. The placement of the structure in the building is such that the structural system is exposed and visible from both inside the room and through windows in a dining area that overlook the room (Figure 7.3). The current design uses eight identical, standard inverted Warren trusses to support the roof (Figure 7.4). In this study, the designers were asked to either submit one design to be used for all eight trusses or different designs that could be interspersed with the assumption that lateral cross bracing between trusses would be provided. The designers were also asked to keep the proposed structure below the roofline to eliminate the effect a new design could have on the form of the exterior roof. An assumption was made that the applied load is uniformly distributed across the length of the truss and is attached at the points where the roof and the truss meet. The walls at the two ends of the truss are assumed to be load bearing. The applied load was calculated from a combination of the prescribed dead (weight of the roof) and live (snow and rain) loads given by the structural designer of the current truss system. The material used for the truss was structural steel ASTM-A36 with young's modulus, E, 206,700 MPa, mass density, ρ, 7850 kg/m^2, and allowable stress in both tension and compression, σ_a, 111.6 MPa (.45 σ_y).

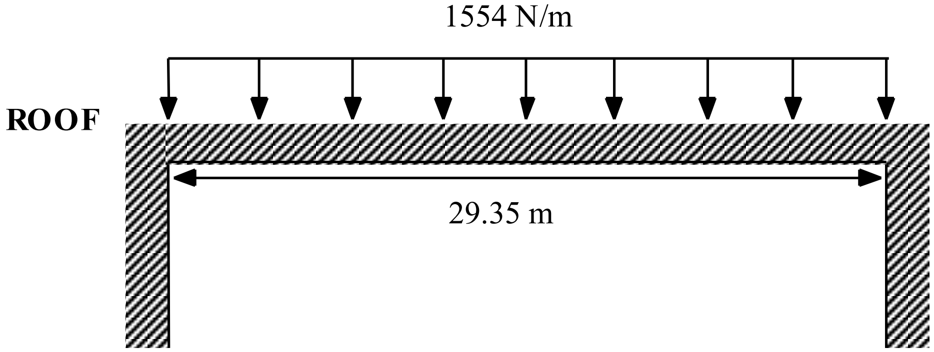


Figure 7.2: Roof truss problem specification

Figure 7.3: University Center aquatics area


Figure 7.4: Roof truss system

7.1.2 Designs Proposed By Human Designers

This section will present the roof truss designs submitted by the six participating designers, three architects and three civil engineers. The participants were given the problem statement described in Section 3 and asked to submit a design along with their assumptions and the criteria used when formulating their solution. The designers were not told to design a truss for minimum mass but were left to interpret the problem statement and impose their own design goals and personal style, although it was mentioned that eight trusses would be needed. The designers were also told that the submitted design could be shape or size optimized if so desired. While no designer wanted their design shape optimized since this could drastically change the appearance of a design, all of the architects wanted their design sized indicating their role as creators of form and not function. Conversely, the engineers submitted their designs sized and did not request size optimization perhaps since they wanted to keep the particular shape member they had chosen or wanted a uniform size member to be used. While shape annealing is capable of generating designs with a limited number of distinct sizes (see Chapter 5) this option was not considered in this study.

The designs submitted are summarized in Table 7.1 along with a description of each designer's rationale for selecting the proposed structural form. All six designs that were submitted are based on familiar truss styles: three are variations of Warren trusses, two are variations of Pratt trusses while one design is a tensegrity (tensional-integrity) truss. The design rationale listed identifies important design goals in the conceptual phase of structural design for the participating designers and fall into four categories: efficiency, economy or fabrication and building costs, elegance, and durability. Comparing the designs submitted by civil engineers to those submitted by architects, while the civil engineers submitted traditional, utilitarian designs, the architects submitted designs motivated by the visual impact of the structure from either the inside or the outside the building. The architects' original drawings are shown in Figure 7.5 through 7.7 to best illustrate the visual nature of their designs.

Since efficiency always plays a role in the design rationale, either as a primary or secondary goal, all but one of the submitted designs were optimized for efficiency as an illustrative measure in using shape annealing to indicate the level of structural efficiency of a proposed layout. One design, the design submitted by Architect 2, could not be optimized since it consists of multiple materials and the design rationale would be considerably altered if a uniform material were imposed. The optimization model used in shape annealing minimized mass, or material cost, subject to behavioral constraints on stress and Euler buckling. Additionally, a geometric obstacle was placed below the designs at approximately one-sixth the span, chosen by relaxing the heuristic suggested by Civil Engineer 3, to constrain the depth of the truss. Two design variable cases were considered: (1) varying member size only, and (2) varying member size and the planar location of joints not attached to the roof. The results of the optimization are shown in Figure 7.8. The design submitted by Architect 1 was modified to include a top structural member, rather than a non-structural membrane as specified, to make the design stable. This alteration results in a transformation of the design type from a tensegrity truss to a Fink truss but maintains the design intent of horizontal compression members and crossing tension members.

The shape optimization of the submitted designs reveals two observations: (1) all trusses become a bowed string shape and (2) a Warren truss is more efficient than a Pratt for the specified loading. It is interesting to note that the one Pratt truss submitted by Civil Engineer 2, while the best size optimized design of the ones submitted, converts to a Warren truss when shape optimized. Thus, if a horizontal lower chord is desired a Pratt is an efficient choice, but given the latitude for depth variation, a Warren will be a more efficient truss. Shape optimization was also useful in indicating that, for efficiency, too many members were used in the design by Civil Engineer 1. Integrated into the design process this exploration of the relation between structural efficiency and form could provide feedback to the designer concerning their assumptions of the level of efficiency of conventional topologies for the problem at hand. When shape optimized for efficiency the submitted designs resulted in only two distinct forms: a bowed string Warren and a bowed Fink. While shape annealing can be used for shape and sizing optimization the advantage of the method that will be illustrated is the generation of design alternatives to standard layouts that reflect the design goals of the participating designers.

Table 7.1: Summary of Submitted Designs

Designer	**Truss Style**	**Design Rationale**
Civil Engineer 1	Warren	• assumed horizontal top chord • considered depth variation or bowed string truss unattractive • finds the rhythm of a Warren truss attractive • prefers 60° inclination angles • prefers square tubes for attractiveness and durability • prefers uniform tubes that only vary the inside dimension • considered fabrication of joints
Civil Engineer 2	Pratt	• designed for behavior and efficiency
Civil Engineer 3	Warren with verticals	• used a rule of thumb that the length of upper chords should be 6-12 feet • considered practical depths to be 1/8-1/12 span • chose 45° diagonals to keep joints aligned • desired an even number of panels • reduced the number of connections by removing the bottom verticals • noted that the fabrication of joints effects the choice of member sizes
Architect 1	tensegrity	• derived from Buckminster Fuller's tensegrity (tensional-integrity) patent • noted fabrication considerations
Architect 2	Pratt	• changed problem specification from an interior roof truss for a flat roof to an exterior roof truss • used a rule of thumb for the depth = 1/ 24 span but increased for ease of detailing • used a combination of steel and glass for even light distribution • considered weather protection since the truss is now on the exterior of the building
Architect 3	Warren	• chose an arched design to allow for a shallower depth in the center to improves the site line from the windows that overlook the pools

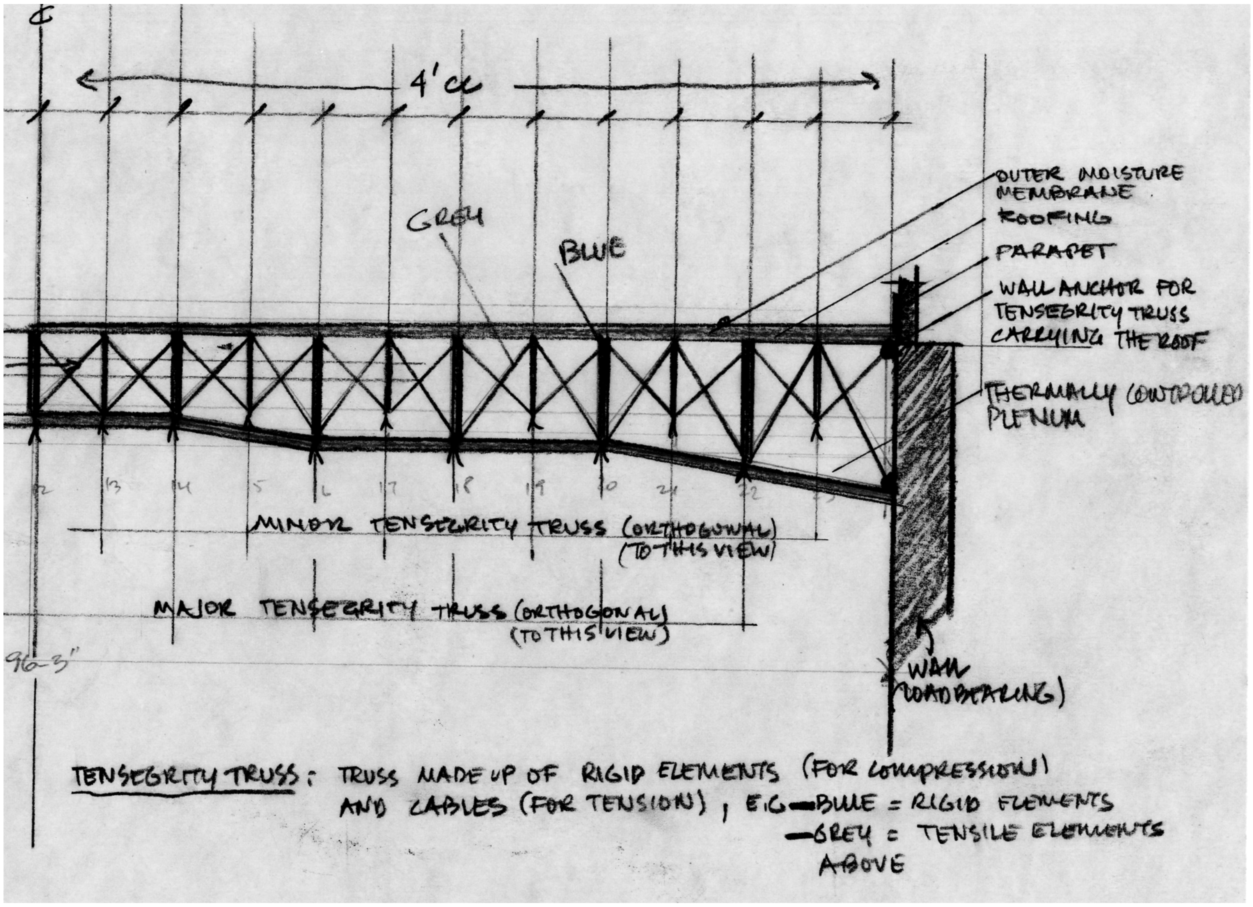


Figure 7.5: Architect 1

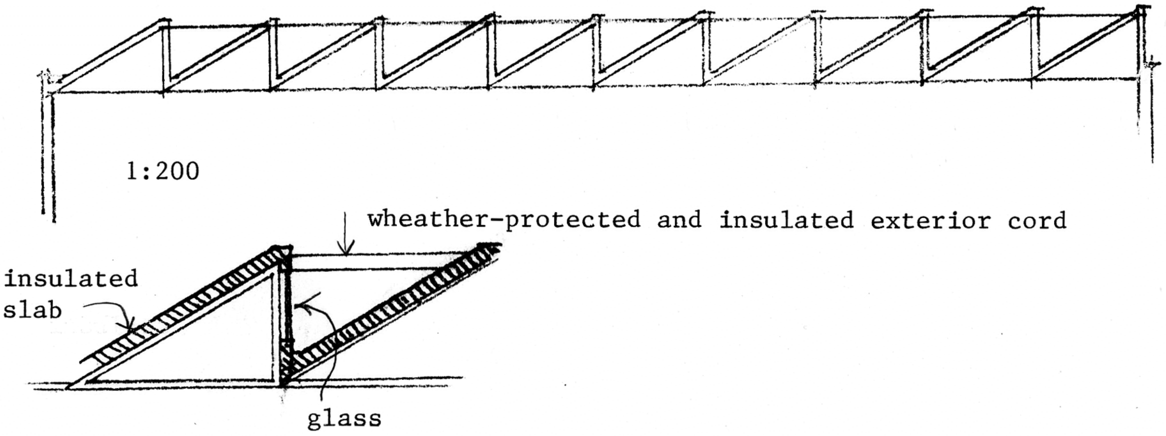


Figure 7.6: Architect 2

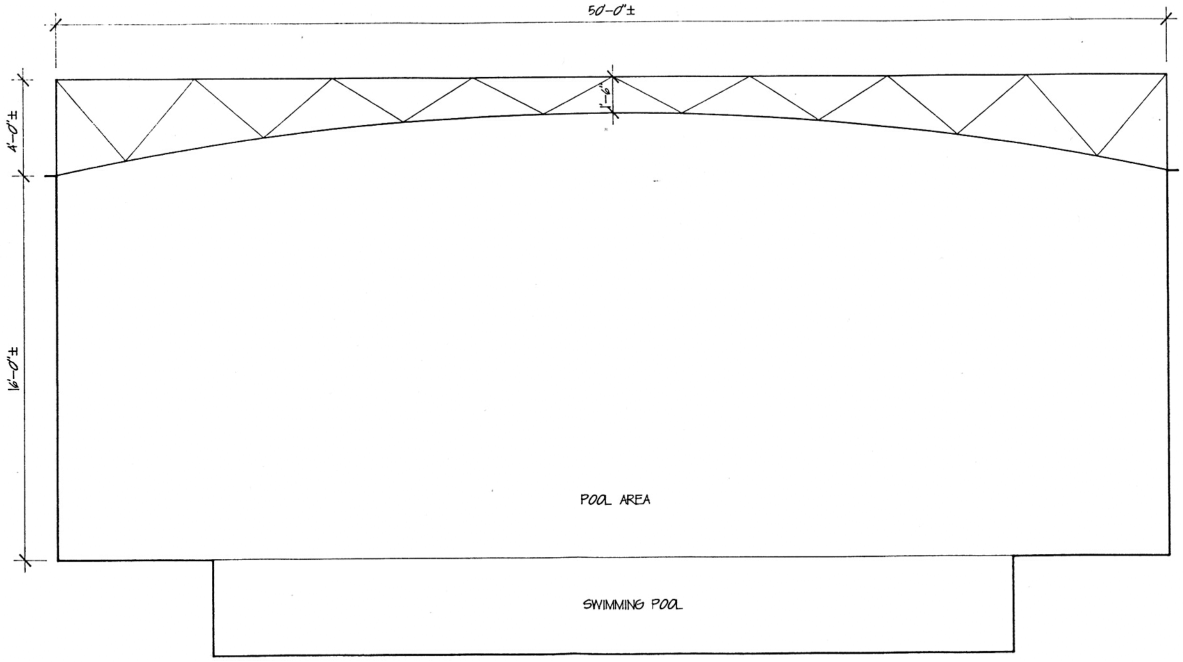
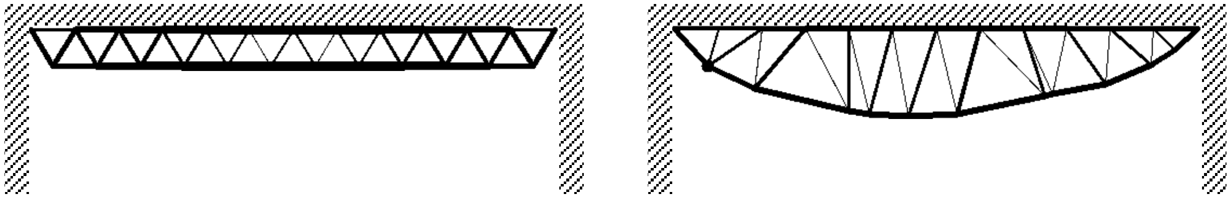


Figure 7.7: Architect 3

Size Optimized Design **Size and Shape Optimized Design**

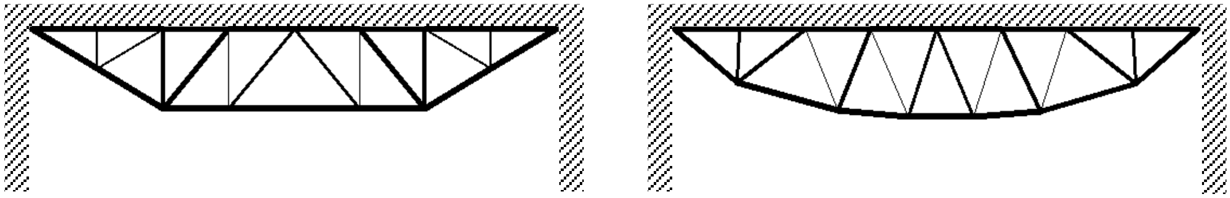
Civil Engineer 1



bowed string Warren: mass = 1,728 kg

Warren: mass = 2,736 kg

Civil Engineer 2



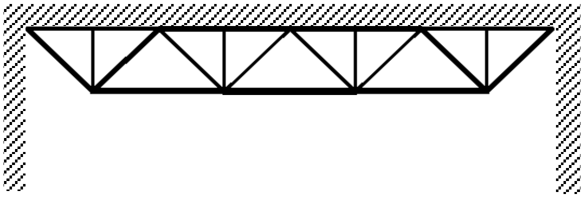
bowed string Warren: mass = 1,636 kg

Pratt: mass = 1,932 kg

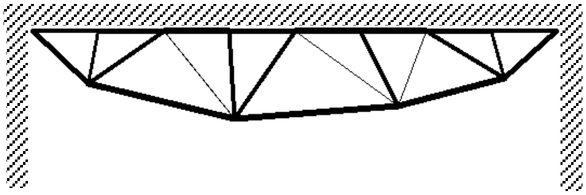
Figure 7.8: Shape and sizing optimization of the designers' layouts

Size Optimized Design **Size and Shape Optimized Design**

Civil Engineer 3

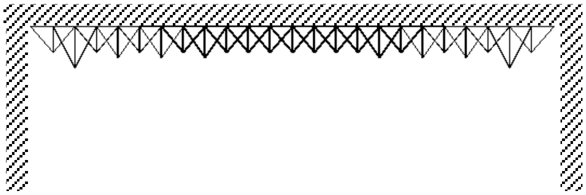


Warren with verticals: mass = 1,947 kg



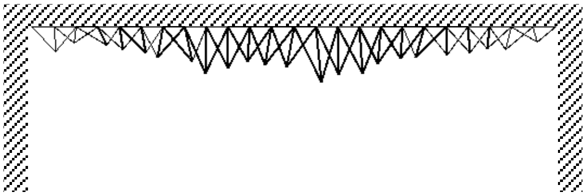
bowed Warren with verticals: mass = 1,680 kg

Architect 1
(line dimension reduced to illustrate detail)



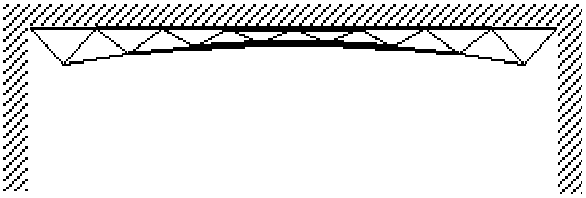
Fink: mass = 15,872 kg

(line dimension reduced to illustrate detail)

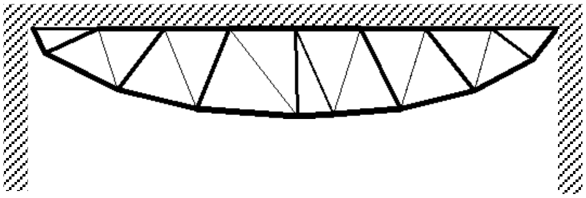


bowed Fink: mass = 17, 900 kg

Architect 3:
(line dimension reduced to illustrate detail)



arched Warren: mass = 9,002 kg



bowed string Warren: mass = 1,680 kg

Figure 7.8(cont.): Shape and sizing optimization of the designers' layouts

7.1.3 An Essay Of Roof Trusses Generated With Shape Annealing

We have seen how designers have approached the design of an exposed roof truss and will now explore the capabilities of shape annealing in generating novel solutions to the same structural problem.[1] Using the design criteria detected in the design rationale of the human designers, six different optimization models of the design problem were formulated. Shape annealing was then used to generate design alternatives for each of the six models to form a range of design styles that reflect the designer preferences and also adhere to the functional constraints of the problem. The material specifications and constraints given in Section 7.3 were used along with a specification for a round tube member with a ratio of diameter to thickness equal to 10 for all models with continuous sizing.

While all structures generated took into account structural efficiency, additional design considerations included in some designs were economy, imposed by discrete sizing and uniformly spaced load points, a constraint on the depth of the truss and a requirement that the structure be symmetric. The first optimization model is a symmetric truss with three load points, two of which can move horizontally along the roofline (Figure 7.9). Additional joints generated in the design that contact the roof are also loaded. The second optimization model places a geometric constraint below the structure to constrain the depth of the structure using a guideline noted by Civil Engineer 3 that the depth of the truss should be approximately one-eighth the span (Figure 7.10). The third optimization model relaxes this constraint to allow more physical design space for the truss while still constraining the depth (Figure 7.11). The fourth optimization model adds the constraint of discrete member sizes chosen from standard gauge sizes for circular tubes (Figure 7.12). Since the previous designs contain a limited number of non-uniformly spaced points of attachment to the roof, a second loading model that uses seven uniformly spaced fixed points along the roofline was also considered. Equal spacing between load points is advantageous so that roof decking can be purchased in a uniform size, as noted by Civil Engineer 3. Solutions for this loading model were generated for both the asymmetric (Figure 7.13) and symmetric (Figure 7.14) cases and with a constraint on depth for both the asymmetric (Figure 7.15) and symmetric (Figure 7.16) cases. Where appropriate, the names of conventional trusses that a design alludes to are noted.

The designs shown in Figure 7.9 through 7.16 comprise an essay of design alternatives that meet the functional requirements of the problem specification and satisfy design issues presented by the designers. The generation of similar quality design alternatives can be seen in Figure 7.9 where three drastically different topologies are shown with the lightest design being only 3.3% heavier than the heaviest design. Among the designs generated some expected and some new observations can be

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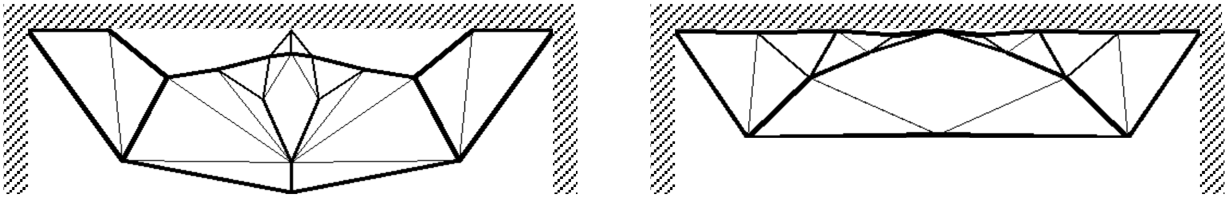
[1] A novel or innovative structure in this study is defined as a structure that is not a conventional truss layout (Warren, Pratt, Baltimore, etc.).

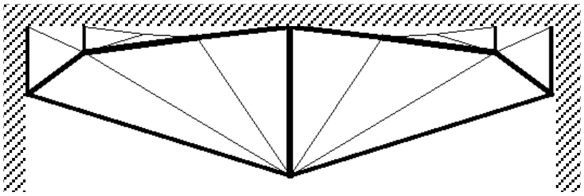
made that lead to new insights about the form-function relation for this design problem. If the depth of the truss is not constrained the design moves away from the roof line and can become very deep (Figure 7.9a). When the depth of the truss is constrained the mass increases, the designs are simpler and they approach conventional Warren truss designs (Figure 7.10). This effect decreases as the depth is less constrained allowing for more design variation and lighter structures; compare the designs in Figure 7.10 with a shorter depth to those in Figure 7.11. Introducing gauge sizing in addition to constraining the depth of the truss not only substantially increases the mass of the truss but also produces the simplest structures (Figure 7.12). Fixing the load points at seven locations results in a lighter design; compare the design in Figure 7.14a to Figure 7.9a and the design in Figure 7.16a to Figure 7.11a. An interesting observation can be made when comparing the asymmetric designs in Figure 7.13 to the symmetric designs for the same model in Figure 7.14; the best asymmetric design is only 2% heavier than the best symmetric design. When the depth is constrained the best design found is asymmetric (Figure 7.15a) and is 3.1% lighter than the best symmetric design (Figure 7.16a). This indicates a possibility that an asymmetric design could be a global optimum.

Since the shape annealing method is not guaranteed to produce an exact global optimum, a wide range of designs that are quantitatively near a global optimum can be found. This latitude in exploration allows for the generation of novel forms that are optimally directed. Although symmetric designs exist that are optimum for symmetric loading within a restricted problem formulation (Kirsch, 1993) it is unclear that asymmetric designs cannot, in general, be optimal. For this reason, and that global optimality is not guaranteed, if perfectly symmetric designs are desired they can only be ensured when symmetry is imposed in the design generation. Highly asymmetric designs as well as symmetric designs have been presented for arch and truss problems in Chapter 5. While these designs may not be the exact optimal solution they serve to provide design alternatives of near optimal forms.



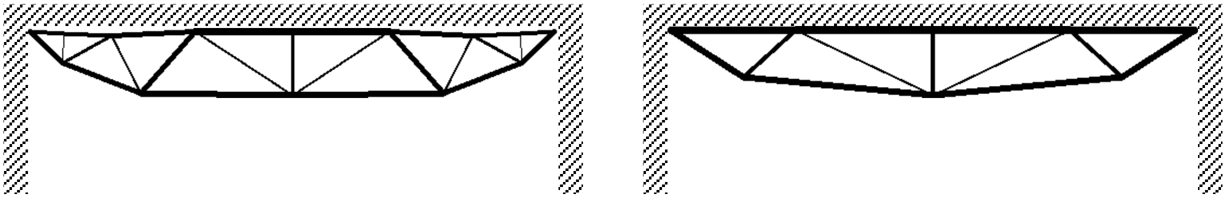
(a.) mass = 1,417 kg

(b.) mass = 1,459 kg (tied arch)



(c.) mass = 1,468 kg (kingpost)

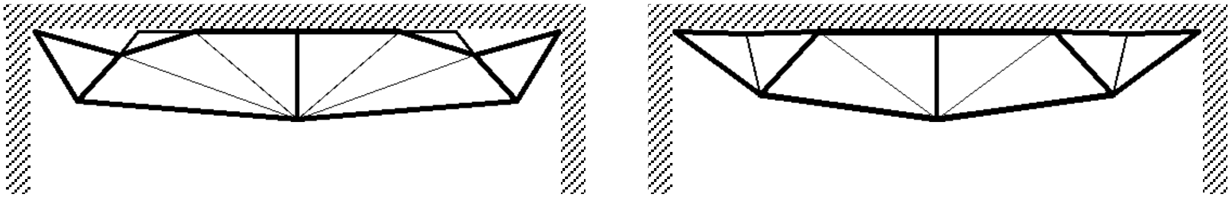
Figure 7.9: Symmetric roof truss layout



(a.) mass = 1,737 kg (Warren)

(b.) mass = 2,173 kg (Warren)

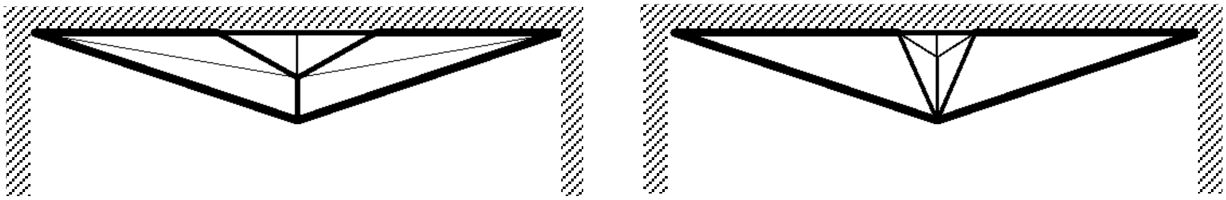
Figure 7.10: Symmetric roof truss layout for depth = 1/8 span



(a.) mass = 1,663 kg (kingpost)

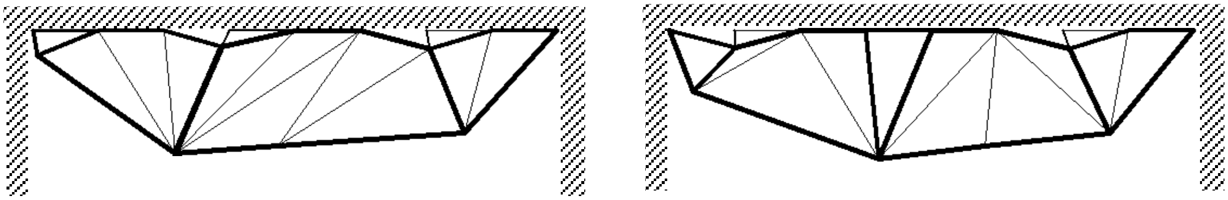
(b.) mass = 1,675 kg (Warren)

Figure 7.11: Symmetric roof truss layout for depth = 1/6 span

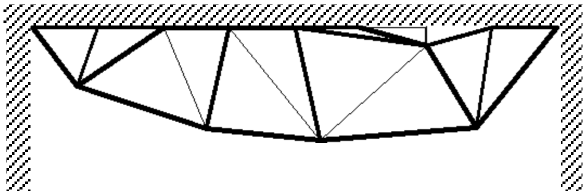


(a.) mass = 2,674 kg (kingpost) (b.) mass = 2,681 kg (kingpost)

Figure 7.12: Symmetric roof truss layout for depth = 1/6 span with gauge sizing

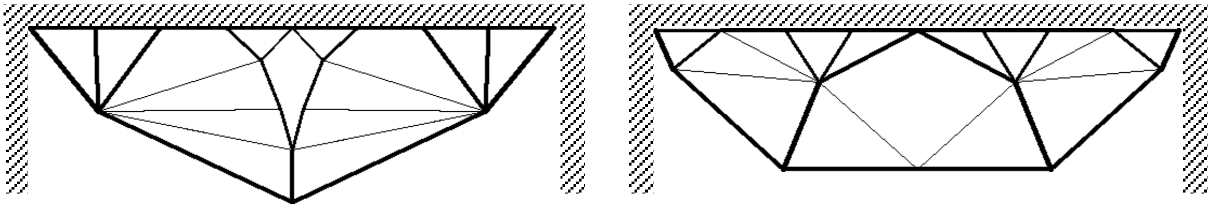


(a.) mass = 1,415 kg (b.) mass = 1,465 kg



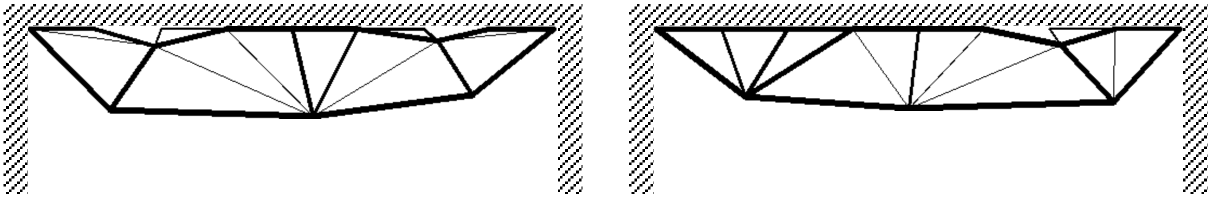
(c.) mass = 1,598 kg

Figure 7.13: Roof truss layout for 7 fixed load points



(a.) mass = 1,387 kg (b.) mass = 1,416 kg

Figure 7.14: Symmetric roof truss layout for 7 fixed load points



(a.) mass = 1,515 kg

(b.) mass = 1,749 kg

Figure 7.15: Roof truss layout for 7 fixed load points and depth = 1/6 span

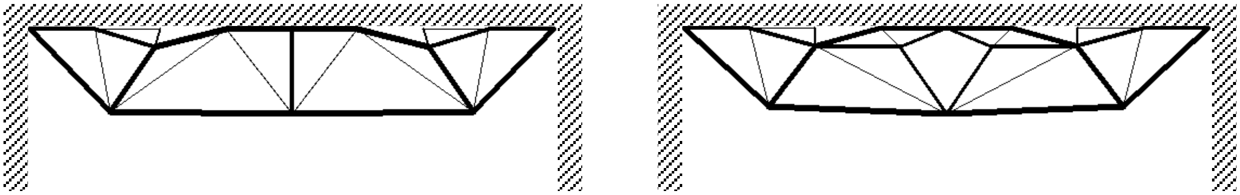


Figure 7.16: Symmetric truss layout for 7 fixed load points and depth = 1/6 span

Comparing the relation between efficiency and novelty of the forms generated by shape annealing, as constraints were added the forms became progressively more conventional until, at the extreme, the generation of kingpost designs found in Figure 7.12. Standard truss designs like the Warren truss and the Pratt truss have been around since the 19th century and were developed without the aid of computer analysis making symmetric and regular forms easier to analyze. This exploration of solutions shows that given the constraint of depth, which is often a practical consideration, the most efficient forms are also conventional. When the constraints are relaxed, with the aid of computer-based design, solutions can become innovative without requiring the designer to have an intuitive ability concerning the function of complex geometry. A comparison of these innovative styles to the conventional designs will now be discussed.

7.1.4 Comparing Shape Annealing Designs To Human Designs

Qualitative comparisons between the designs conceived by human designers and those generated by shape annealing will now be made. The shape annealing designs can be compared to the civil engineers' designs for their level of functional efficiency and economy as well as expression of functional clarity. Compared to the optimized designs from the civil engineers (Figure 7.8) shape annealing presents similar conventional solutions in Figure 7.10 and Figure 7.11b that exhibit functional efficiency and clarity. Additionally, an unconventional configuration was found (Figure 7.11a) that provides the same level of structural efficiency but uses a variation on a traditional kingpost design. In contrast, even though the layouts submitted by the engineers were different, when shape optimized only one efficient style resulted, the bowed string Warren. While Civil Engineer 1 was aware that a bowed shape would be more efficient, he did not like the aesthetics of this shape and opted for a horizontal bottom chord instead. However, a bowed shape is not the only efficient shape for this problem. Shape annealing generated non-traditional topologies that are not bowed and remain efficient (see Figure 7.9b and 11a). Comparing the designs purely for structural efficiency, the primary design goal of Civil Engineer 2, the shape optimized designs in Figure 7.8 compared to the shape annealing designs in Figure 7.11 have comparable efficiency. But, comparing the designs in Figure 7.8 to those in Figure 7.15 and 7.16 more efficient, yet unconventional, solutions are found.

Civil Engineer 3 considered issues of economy in his rationale through a desire for an even number of equally spaced support points and a limited number of joints. While it is intuitive that an even number of load points would lead to a conventional layout there are alternative layouts for this loading case that shape annealing illustrates with the designs presented in Figure 7.13 through 7.16. The designs shown in Figure 7.16 illustrate the additional practical considerations of a constrained depth and a symmetric truss while still not adhering to a conventional layout. Designs with a very limited number of joints were also generated, even though this was not an explicit design goal, as seen in Figure 7.12. These comparisons illustrate that shape annealing can generate designs that appeal to a civil engineer's desire for rhythmic, conventional layouts as well as present new solution types that posses desired design characteristics.

Comparing the shape annealing designs to those submitted by the architects for their visual expression, the shape annealing solutions achieve innovative styles while also providing functional efficiency. While visual expression is a subjective design characteristic, in truss design an innovative style, or a non-conventional truss configuration, is considered to be visually expressive. The subjective nature of this evaluation will be explored later in the designers' critiques. While architects are less concerned with efficiency, it is always advantageous to achieve both the primary (form) and secondary (efficiency) design goals simultaneously. Attempting to impose structural efficiency on the Architects' designs results in highly inefficient structures that also loose the intent of the design; note the degeneration of the design by Architect 3 when optimized (Figure 7.8).

In order to assess the shape annealing designs for functional clarity, spatial intrigue and general usefulness, the designers were asked to critique the shape annealing designs. This portion of the study also served to indicate a designer's receptiveness to and intrigue of non-conventional structural forms. As expected, the civil engineers perceived the designs primarily in terms of functional efficiency and clarity as well as the fabrication costs that would be entailed in constructing the designs. The civil engineers remarked that the complex geometry of the designs shown in Figure 7.9 and Figure 7.13 were not designs that would be conceived by hand due to the lack of functional clarity and the complexity of analysis required. Some engineers also noted that the depth of the unconstrained trusses shown in Figure 7.9 was too great. One civil engineer remarked that while he preferred the more conventional solutions in Figure 7.10 if he were not as set in his ways he might opt to pursue one of the more unconventional designs. Another civil engineer liked the prospect of asymmetric designs that had similar levels of efficiency as the symmetric designs and although this made him think critically about his assumptions concerning the relation between optimal designs and symmetry he would not prefer them. It was interesting to find that one civil engineer preferred the innovative designs that he could find functional clarity in such as that in Figure 7.14a where he associated the design to a “natural form”.

The architects’ perceptions of the designs were based on their assessment of the visual interest and uniqueness achieved by each alternative. As expected, they were not as concerned with the functional aspects of the designs but rather the expression a design could make. Two architects preferred the asymmetric designs for their uniqueness and expressiveness. One architect favored the extremes of the designs and singled out the design in Figure 7.9a for its uniqueness and the design in Figure 7.11b for its minimalist expression. It was also noted that the designs without a horizontal top chord could be used to create interesting roof contours. In contrast to the engineers, the architects were not adverse to using the deeper trusses in Figure 7.9, 7.13 and 7.14 and were intrigued by the effect that would be created when looking through the structure from the upper windows. While the evaluation of structures for visual goals is very subjective some commonalties were found. Most designers were intrigued by the design in Figure 7.12b for its simplicity but unconventional form. Also, there is some common ground between the two types of designers; the symmetric innovative designs tended to please architects and civil engineers alike.

It was noted by many designers that there are considerations (fabrication, maintenance, durability) that go into a practical design, some of which were noted in the design rationale portion of Table 7.1, that were not modeled explicitly in the shape annealing method. Some of these considerations could be incorporated in the problem formulation, such as the spacing between load points, or as design goals, such as minimizing the number of connections in the structure. For this particular problem, roof truss design, the cost of connections is not as great a factor as in larger scale designs such as bridges, and thus connection cost was not considered. Since the purpose of the current shape annealing implementation is to provide conceptual design alternatives and not perform detailed designs, many of these considerations are left for the designer to evaluate when assessing the set of generated designs. While we did not include design issues of manufacturing and construction, any important criteria that can be articulated can be implemented in the optimization.

This study has presented a look at a simple structural design problem that would generally be considered routine and explored alternative design concepts. An additional extension to the problem is the design of eight different trusses, all with the same function but different spatial form, to support the roof. This variation of the design problem turns a utilitarian structure into one that adds interest to the surrounding space. Although it was mentioned to the designers that eight trusses would be needed to support the roof, no designer submitted more than one design. While there is much expense involved in a designer conceiving multiple structures that serve the same purpose, shape annealing can easily generate a range of similar quality solutions. Once the problem is modeled, the only costs of generating alternative solutions with shape annealing are in computation time as well as further assessment by the designer. Thus, it is conceivable that eight different trusses could be used to support the roof and perhaps provide for maximum visual impact.

7.1.5 Design Study Conclusions

Structural design is moving towards the design of intricate configurations that are only conceivable with the aid of computers and integrated manufacturing techniques. The intent of shape annealing is not a structural design tool that replaces the designer but rather one that aids the designer by providing new possibilities for structural forms that may enhance their creativity and insight. This study has presented an investigation of designs conceived by structural designers, without the use of computers, and the design goals that were used in their conception. These design goals were then modeled in the shape annealing method and used to generate both conventional and novel, functionally efficient structures. The generation of both types of structures makes it possible to satisfy both the architect's preference for visual impact and the engineer's preference for functional efficiency and clarity. The results of this study illustrate that shape annealing is capable of generating multiple, spatially innovative solutions to a standard truss design problem that efficiently achieve the design goals of conventional truss styles. The resulting essay of structural forms provides design alternatives for the designer to investigate further to create a detailed design that takes into account considerations such as fabrication, maintenance and durability.

The advantage of computer-based conceptual design is that a design is not restricted to intuition based on knowledge of standard forms but rather can generate innovative forms based on the evaluation of the imposed design goals. The disadvantage is that design goals are often difficult to model computationally. While human designers through their intuition, experience and knowledge can very quickly come up with a satisfactory solution it is this knowledge that often hinders them from moving freely within the space of design alternatives. Thus, the combination of computational methods for the design of novel yet functional forms and the human designer to assess these forms in the context of more extensive design goals can make an effective design team that is creative in designing innovative, feasible structures.

7.2 A Computer Tool for Structural Design

In the study presented we saw that the shape annealing method allowed for the generation of designs that ranged in visual form from known solutions submitted by the designers, when properly constrained, to unique, innovative forms when the design space was unconstrained. An important feature of shape annealing is the ability for topological and geometric design exploration that is either constrained and unconstrained. While computer tools exist for shape optimization of a discrete structure, there are no known computer tools that explore discrete topologies without using a predefined grid. Given that this method would extend the state-of-art of structural design tools the larger question is: how could a tool for conceptual structural design, such as this, be incorporated in the structural design process? An interactive tool that works with the designer rather than full automation seems to be the best approach to developing an effective design tool.

A computer tool based on shape annealing to assist the designer in determining innovative discrete structures within the confines of practical design constraints is proposed. The following are key features of the proposed tool:

- a GUI (graphical user interface) for configuring the problem specification: specifying loads, supports, geometric obstacles, materials, discrete sizes, and constrained boundaries,
- a means of specifying an objective function from a defined set of design goals and setting their individual weightings to reflect a designer preferences,
- a base grammar for classes of structures, and
- a GUI for integrating new grammar rules.

An example of an interface for shape annealing developed at MIT by Mitchell and Smith is shown in Figure 7.17 and 7.18. Figure 7.17 shows the interface that a designer would use to specify new grammar rules, while, Figure 7.18 shows the general interface that displays the rules used in the design process and the current design that is updated throughout the annealing process.

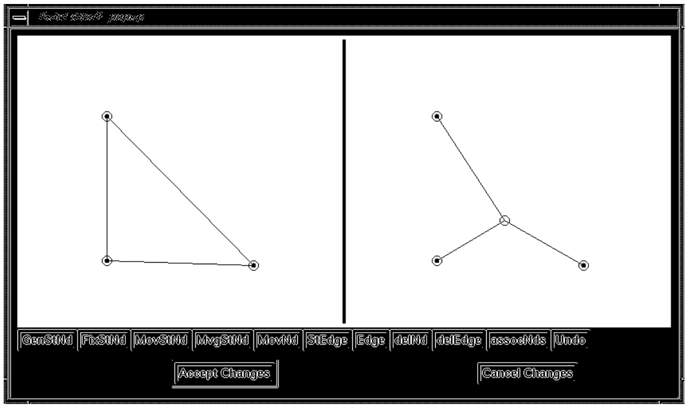


Figure 7.17: Grammar interpreter (courtesy of W. J. Mitchell and E. Smith)

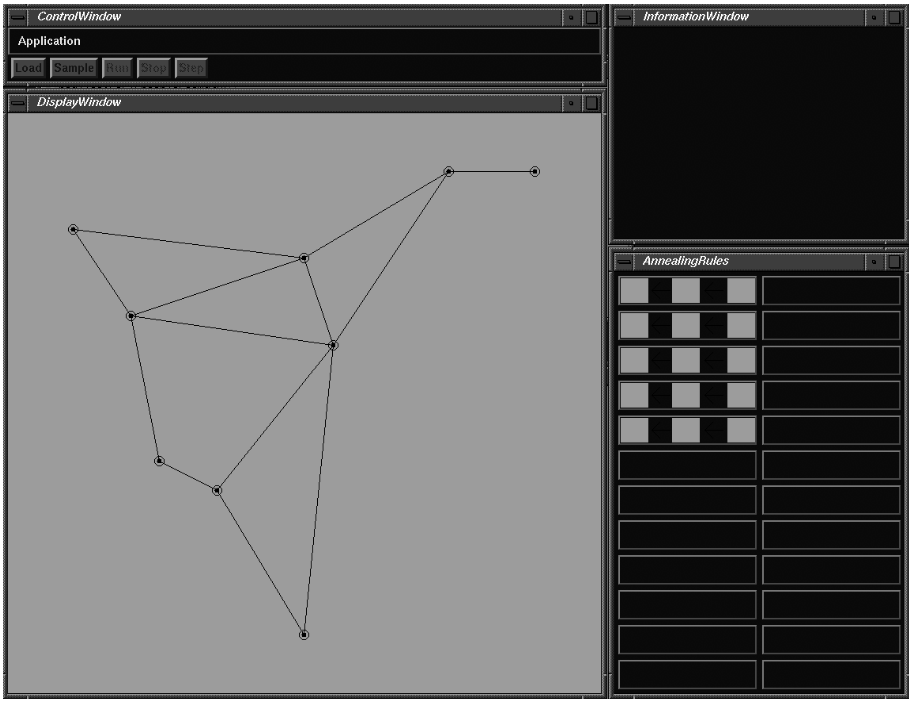


Figure 7.18: GUI for shape annealing (courtesy of W. J. Mitchell and E. Smith)

Let's return to the problem of designing a roof truss presented in Section 7.1 and investigate how a designer could use this tool to explore alternative design solutions. A designer would start by specifying the problem description including a geometric description of the problem and the required loading. Next, the design goals and the designer's preferences for their weighting would be defined. The designer would then specify an initial design, this could be either a simple connection of points from the problem specification or a layout that a designer wants to start the process from. Once the problem specification and the initial design are defined, the designer would select from a set of base rules defined for the structural class of the design problem and, if desired, add new rules to this set to specify their individual style. Once all the components of the method were specified the shape annealing process would start. At this point it would be up to the designer to decide whether to let the design generation be fully automated or interactive. The interactive mode could be used if the designer did not like the way the design generation was progressing. The designer could interrupt the process, apply grammar rules manually to observe the effects of individual rule applications, modify the set of rules accordingly, and when satisfied start the process again. This would not interrupt the optimization process since the optimization progresses as a series of design transformations, whether manual or automated.

Considering the proposed design system at an interactive level, such a tool would provide for insight into the functional and behavioral consequences from the application of a single grammar rule. This would also allow the designer to assist in the design generation which would enhance the designer's understanding of the resulting solution and consequently the likelihood of accepting the generated design as a feasible alternative. Used as an automated system, the method would be allowed to generate an essay of designs to be evaluated by the designer. If none of the resulting designs suited their requirements a designer could either modify the optimization model or the grammar rules and generate a new essay of designs that reflects the modifications made. Alternatively, components from the essay of designs could be merged to suit a designer's preferences and needs and then used as an initial design.

It was mentioned in Section 3.1 that the initial design has little effect on the final design. This was done intentionally such that no knowledge of an appropriate solution was needed to start the design process and also to minimize the bias of the initial design allowing for more innovative solutions. However, the initial design could also be used to check the quality of a known design against alternatives generated by shape annealing. Since grammar rules are reversible the design process can always return to the initial design, especially if it is the best design. In addition, parallel to the simulated annealing algorithm, a downhill search is performed that tracks the best design found. If the initial structure were to be the best design considered in the entire generation process then this would be evident at the end of the design process. So, the shape annealing algorithm can be used to generate innovative topologies, check known solutions for their relative quality compared to other solutions in the design language, and provide further modifications for the improvement of known solutions.

7.3 Summary

In this chapter we explored the applicability of the shape annealing method in aiding the designer through a comparison between designs proposed by human designers and those generated by shape annealing. This case study explored how computational tools, and in particular the shape annealing technique, can support design from multiple perspectives. A computational tool based on shape annealing was outlined along with the implications of what such a tool can provide to engineering and architectural designers in practice.

8. CONCLUDING REMARKS

A summary will be made of the work presented and the resulting contributions. In addition, further extensions of the shape annealing method, the application of the method to an additional network topology optimization problem, land use and transportation, and an evaluation of the method's capability for design innovation will be discussed.

8.1 Dissertation Summary

The shape annealing method, a design technique that combines a generative grammar with stochastic optimization, has been applied as a grammatical approach to the design of discrete structures. Two shape grammars, a planar truss grammar and a single-layer space truss grammar, have been developed that model the form-function relation in truss structures to define a language of discrete structures. In order to generate purposeful designs from this language, an optimization model was formulated that incorporates the structural design goals of efficiency, economy, utility and elegance. This computational model for structural design was then used within simulated annealing, a stochastic optimization method, to search the language of discrete structures for solutions that meet the design objectives and requirements. Applying a grammatical approach to structural design has resulted in the generation of structural essays that present sets of designs that explore design issues in a particular structural design domain. Since multiple designs will often satisfy a set of design goals, varying design styles can be generated within a structural essay. Essays have been presented for planar trusses, towers, pseudo-tensegrities, and single-layer space trusses in the form of domes and complex roof shapes. The purpose in generating structural essays is to provide the designer with a set of functionally feasible and optimally directed designs that explore the relation between form and function in the context of the design application. Essays of structural designs also provide alternative design styles that could enhance the creativity of the designer in conceiving a novel design solution.

8.2 Contributions

The four main contributions of this work are:

1. a grammatical approach to structural design,
2. a design method for the generation of essays of innovative, discrete structures that reflect practical design goals,
3. a stochastic, discrete method for structural topology, shape and sizing optimization, and

4. a proof-of-concept of shape annealing as an effective method for design configuration problems.

8.3 Method Extensions

The shape annealing method was shown to be capable of generating optimally directed structures for a range of truss design problems. Further extensions and improvements can be made to all components of the method: shape grammar, design optimization model, optimization technique and analysis.

8.3.1 Grammar

The grammar for the design of discrete structures was developed such that it defined an infinite space of alternative design topologies. Further extensions to the grammar will discuss the language definition by the grammar, parametric rules that reduce the design language to include only topologies with specified parametric relations, and the addition of new structural elements in the grammar to define languages of structures for different classes of structures. An example of a frame structure will be shown.

8.3.1.1 Defining a Language of Discrete Structures

The purpose of the grammar is to define a language of designs, or in this case a language of truss structures. The goal in formulating a grammar is to fully define all design objects of interest in the language by embodying knowledge about how designs can be generated through combinations of shape transformations. This suggests that a purpose of the shape grammar is to model the designer's intention. In other words, if a desired topology cannot be configured from the grammar, that is the design does not exist in the language, then it is not the case that the optimization model does not find that topology optimal but rather that it does not exist within the language of designs that is being explored. Two means of testing that a grammar models the designer's intentions are (1) to generate by hand standard or known configurations using the grammar and (2) to change the grammar rules and observe the effects on the design generation. Section 6.1 presented an application of the space truss grammar rules to generate standard geodesic patterns by hand. We will now investigate the effects of modifying the rules in the shape grammar using a simple example.

Considering the two rule pairs for planar design, rules 1 and 2, and rules 3 and 4 from Figure 8.1, truss designs can be configured using all rules, including rule 5, or using only one rule pair at a time. The problem specification for this example is shown in Figure 8.2, with materials specifications shown in Table 8.1 and method parameters shown in Table 8.2. Using all rules, 1 through 5, results in the design shown in Figure 8.3 with a mass of 2023 kg. Limiting a design to be generated from only the divide rule pair, rules 1 and 2, results in the design shown in Figure 8.4 with a mass of 1802 kg, whereas, only using the add rule pair, rules 3 and 4, result in the design shown in Figure 8.5 with a mass of 3056 kg. All designs shown were the best designs generated from a total of six designs.

Table 8.1 Material Properties for Figures 8.2 to 8.4

Material Property	*Steel*
modulus of elasticity, E	6.88 E^6 N/cm^2
allowable tensile stress	14,880 N/cm^2
allowable compressive stress	14,880 N/cm^2
mass density, ρ	.00785 kg/cm^3

Table 8.2 Method Parameters for Figures 8.2 to 8.4

Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum number of members	50
maximum area	730 cm^2	number of iterations	170
member areas	continuous	number of designs per iteration	200
minimum member length	15 cm	planar truss topology rules	1-5
minimum angle between members	1°	rule selection	Hustin
intersections between members	not allowed	constraint violation normalization	yes
member shape	solid rod		

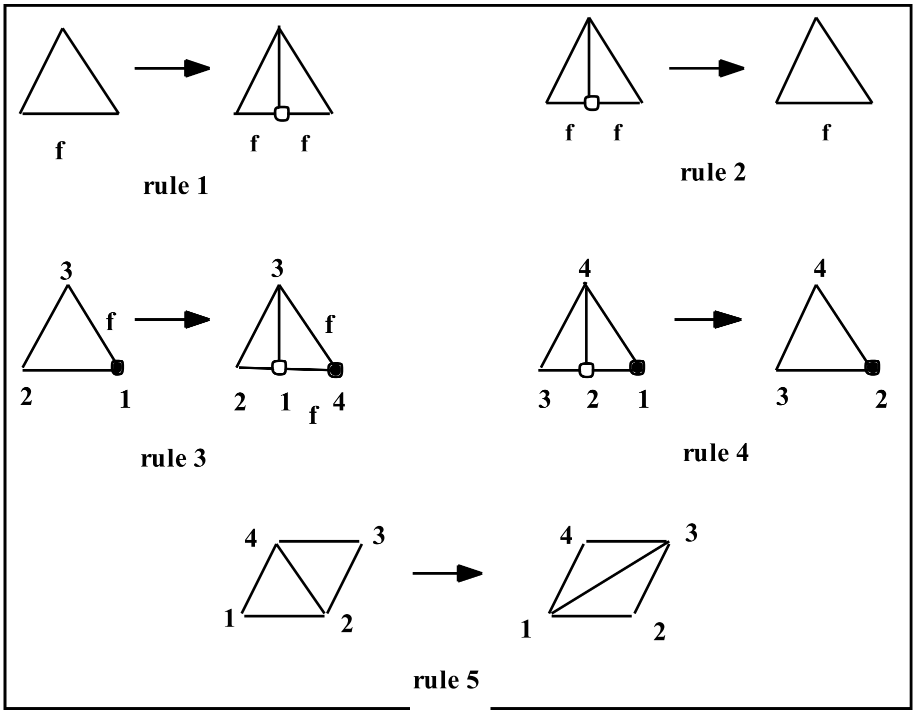


Figure 8.1: Planar truss topology rules (redrawn from Figure 3.1)

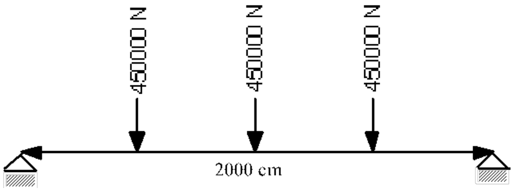


Figure 8.2: Indeterminant truss problem specification

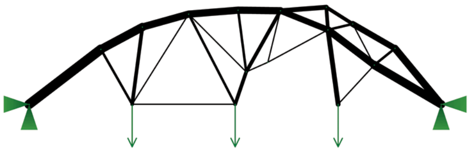


Figure 8.3: Truss design generated using all rules; mass = 2023 kg

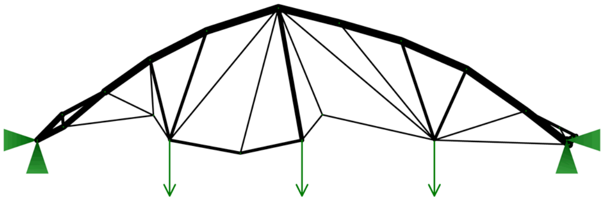


Figure 8.4: Truss design generated using rules 1 and 2; mass = 1802 kg

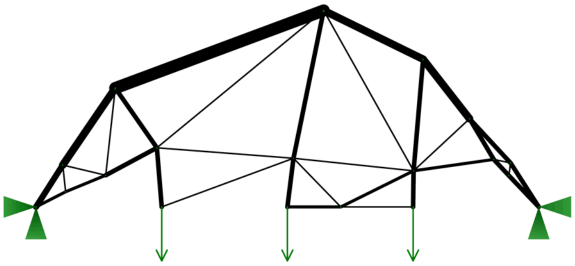


Figure 8.5: Truss design generated using rules 3 and 4; mass = 3056 kg

Comparing the two designs in Figure 8.4 and 8.5 we can instantly recognize the topological differences in the shapes. While the shape in Figure 8.4 is much like a tied arch, the shape in Figure 8.5 is a combination of truss patterns and an arch. Comparing all three designs, the best design, Figure 8.4, was found using only the divide rule.

Separating the application of grammar rules is a good means of testing the individual effects of rules and their impact on solving the design problem. It is difficult to foresee the combined effects of different grammar rule applications, especially when combined with shape optimization. However, effective grammars can be written for classes of structural design using the diagnostic tools investigated here: generating standard known forms by hand and comparing designs generated from different rule pairs.

From this example, it can be seen that the topology rules do affect the definition of the language of designs that can be generated. While the number of topologies that can be generated by the rules in Figure 8.1 is large, the grammar is not inclusive of all possible truss topologies. For instance the shape shown in Figure 8.6 with two intersecting members without a joint cannot be generated from the current grammar. If this shape is not of interest, then the grammar fully models the designer’s intention. But, if this shape is sought, then the grammar would be incomplete since no resulting designs could contain this shape. Investigations into further rule formulation in order to generate all topologies of interest could be made.

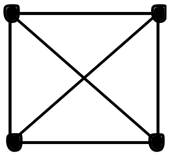
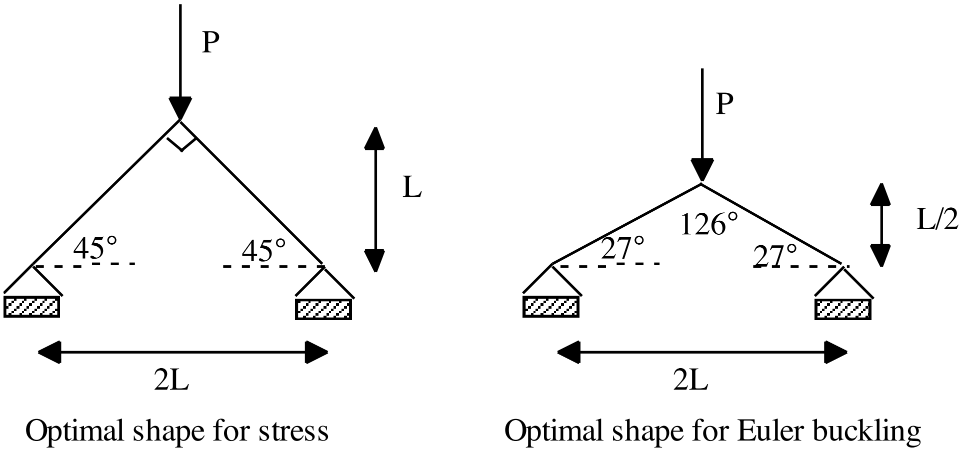


Figure 8.6 Truss topology that cannot be generated from the rules in Figure 8.1

8.3.1.2 Parametric Grammar Rules

A second aspect of the grammar that can be explored is the use of parametric grammar rules to reduce the defined language of designs based not on topology considerations but rather on geometric requirements. Two mechanisms exist within the shape annealing method for controlling the form of the generated designs: syntax and semantics. Syntax is used to place constraints on the geometric relations of grammar rule transformations. Semantics are used to control the calculated quality of a design to reflect preferences of particular design attributes. Both control mechanisms were used in the application of the golden ratio proportional system by using rule syntax to constrain the design generation to designs composed of golden triangles or by preferring designs in the semantics model that were closer in proportion to the golden ratio. While syntax and semantics have been used to control geometry for visual purposes, they can also be used to control geometry for behavioral purposes. Incorporating parametric knowledge in the grammar will now be considered in further detail.

This example will explore form-function relations based on the behavior of a structural shape. Considering a simple determinant truss with a single applied load, we can determine analytically the optimal geometric proportions of that shape for both a fully stressed design and a critical design based on Euler buckling, see Figure 8.7. For the least weight design of the shape in Figure 8.1, the optimal angles between the horizontal and the inclined members are 45° for a shape with considering only stress and 27° for a shape that incorporates buckling. This simple example will be used as an estimate for the geometry of a functionally efficient shape. Another estimate could be provided by the minimum suggested skew angle in a finite element model that is generally about 30°, similar to the 27° in the buckling shape.



Optimal shape for stress

Optimal shape for Euler buckling

Figure 8.7: Analytic optimal shape of a two bar truss design considering stress and Euler buckling constraints

Incorporating the relation between the geometry (form) of a shape and its corresponding function in the design generation can take two paths, as a fixed constraint in the design generation or as a grammatical rule for shape improvement. A simple rule can be formulated for shape improvement that finds a shape in the design with an angle less than the desired minimum angle and then rotates one member that forms this angle such that the new angle is equal to the minimum desired angle (see Figure 8.8). One drawback to this rule is that while the shape that is acted upon has the desired angle the effects on the connecting shapes are unknown. The optimization can be used to assess the effect on the entire design and filter out the undesirable transformations.

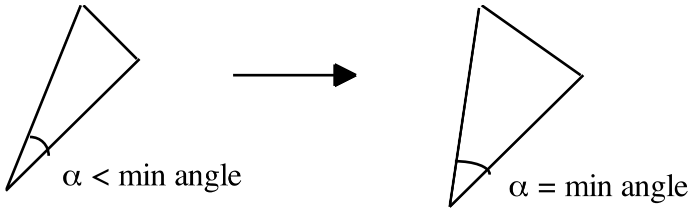
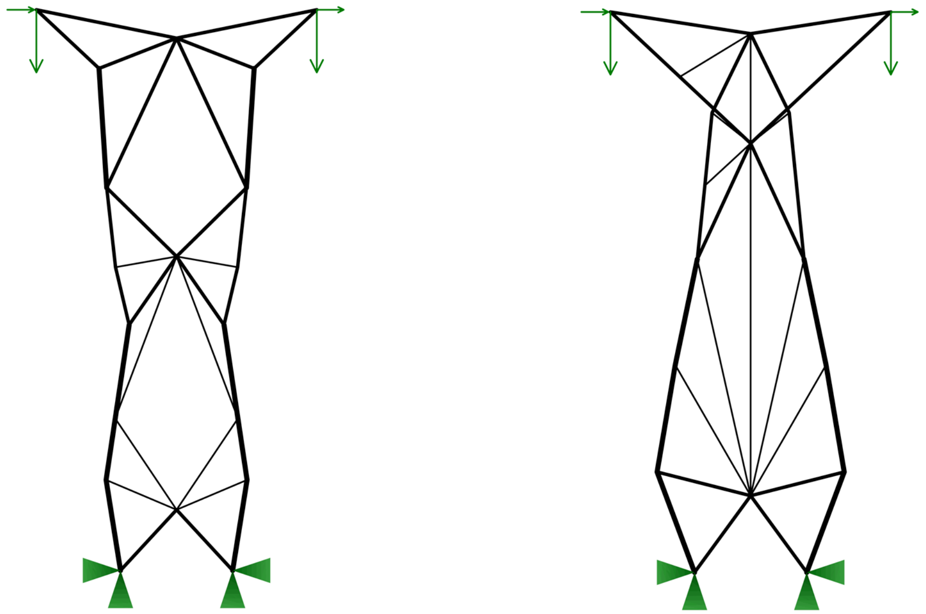


Figure 8.8: Shape improvement rule

To illustrate this point we will return to the example of the design of a transmission tower shown in Section 5.2.1. Without using form-function knowledge in the design transformations results in the designs shown in Figure 8.9 with masses of 874 kg and 850 kg. Imposing form-function knowledge by fixing the minimum angle between members in a shape to 30° results in the designs shown in Figure 8.10 with masses of 895 kg and 938 kg; the mass has increased.

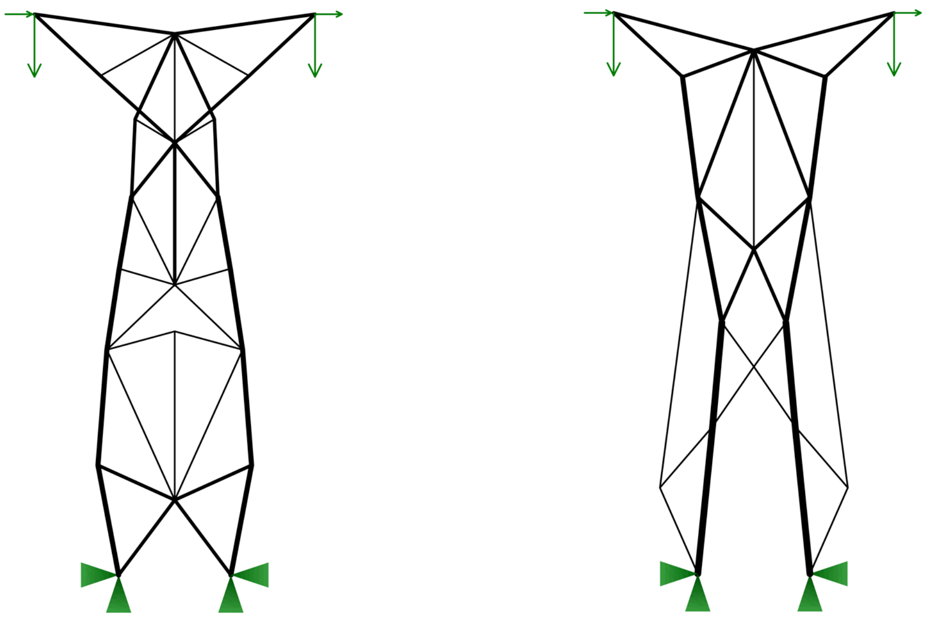
Using the shape improvement rule with a minimum desired angle of 30° results in the designs of Figure 8.11 where the lighter design shown in Figure 8.11(a) with a mass of 954 kg contains shapes that do not meet the minimum angle (note the shape that is attached to the loads) while the heavier design in Figure 8.11(b) with mass of 1077 kg meets the desired minimum angle in all shapes. In each case the designs shown are the best two designs from a total of ten designs generated.



(a) mass = 850 kg

(b) mass = 874 kg

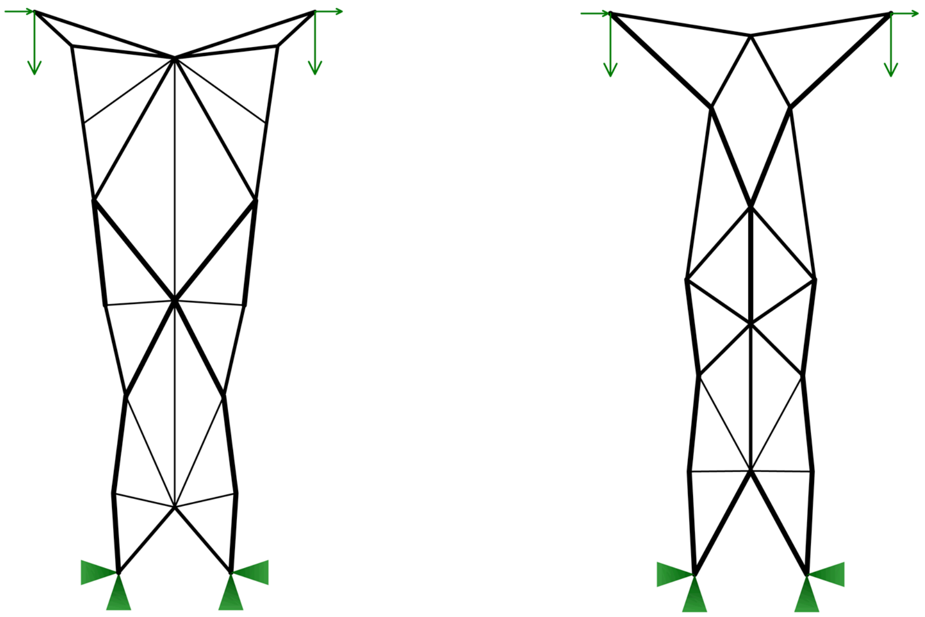
Figure 8.9: Tower designs without a constraint on angles



(a) mass = 895 kg

(b) mass = 938 kg

Figure 8.10: Tower designs with 30° minimum angle



(a) mass = 954 kg

(b) mass = 1077 kg

Figure 8.11: Tower designs using the shape improvement rule; 30° desired minimum angle

Through this study we can see that although its seems to be an intelligent pursuit to incorporate parametric form-function knowledge in the grammar it will not necessarily yield the desired effects. Using functional feedback from the design evaluation concerning the behavioral performance of a form provided sufficient knowledge to the design process in generating good designs. However, between the two examples where angles were restricted, the best designs were found when constraining the design to shapes with angles of 30°. While there may be advantages to using a hard constraint on angles between members for joint construction purposes, in general, it seems a better approach to allow the optimization to determine the appropriate angles for a design problem. The shape improvement rule did not result in better solutions for this problem but it must be considered that the design space is relatively small, fifty members. The shape improvement rule could aid in the generation of more efficient designs when the design space is larger such as the designs shown in Section 6.2 that used a maximum of 150 members.

Syntax could also be added to the grammar for the generation of forms with desired proportional characteristics. A simple example using the golden ratio was shown in Section 7.2 that constrained the structural layout to only golden triangles. Alternatively, parametric syntax could be used in the design generation such that a dynamic grid based on relative proportions exists. While this would restrict the innovation of the design, the patterns generated could be intriguing and the functional implications of imposing one pattern over another could be investigated. One method for applying proportional syntax would be to assign proportion parameters to shapes as described by Stiny (1980) in his discussion of parametric grammars. This would enable a layout to occur on a topological level and then on a spatial level where different geometries of a given topology could meet the required proportions. The difference between using proportional syntax and the current method is that rather than allowing transformations of single points to govern the shape of the topology, the relative proportions of shapes would govern the location of points.

8.3.1.3 Grammar Knowledge Level

The tradeoff between using a grammar that only defines valid topologies and incorporating parametric form-function rules in the grammar involves the time invested in creating the grammar and the number of rule applications to generate a suitable design. The towers shown in Section 8.1.1 were generated in 34,000 iterations from a simple grammar. To accomplish the same task but using special grammar rules for transmission tower design could take significantly less iterations to generate a suitable design. However, the grammar would take longer to formulate and would be limited to the design of transmission towers. In this case the grammar would define a language of transmission towers, which is comparable to shape grammars that have been formulated for the generation of specific design styles such as Palladian villas (Stiny and Mitchell, 1978). The generality of the grammar presented in this work allows for its application in multiple problem domains rather than having to model specific domains. The consequence of this generality is having to apply numerous rules iteratively to generate suitable designs.

Another benefit of formulating a general grammar is that more innovative designs will be generated since the language of designs is less constrained by problem specific knowledge. While constraining the grammar to a limited number of forms could be useful to allow the designer to explore designs within an established domain of forms, this restricts the capability of the method to generate innovative forms.

8.3.1.4 Frame Structures

Up until this point we have limited our discussion to the generation of pin-jointed structures. Other classes of structures frames, plates, shells and solids could also be generated with shape annealing. Since the shape annealing method is a general technique, a different structural element type with a grammar suited to that element can simply be added. We will now look at the effects of changing the element type and analysis model on the generation of simple planar structures. The same grammar for planar trusses will be used but now the analysis will model the lines as beam elements that are capable of supporting bending and shear forces. The joints in the structure are now rigid, that is, in contrast to a pin-jointed structure, such that there is no rotation between elements. The stress violation is now calculated as:

$$\text{stress violation} = \sum_{i=1}^{\text{num members}} \left(1-\frac{F_{x,i}/a_i}{\sigma_a}\right)+\left(1-\frac{M_{z,i}r_i/I_i}{\sigma_b}\right)+\left(1-\frac{F_{y,i}/a_i}{\sigma_s}\right), \qquad \text{Eq. 8.1}$$

where: Fx is the axial force (tension/compression),
Fy is the shear force,
Mz is the bending moment,
I is the moment of inertia,
r is the radius,
a is the area,
σ_a is the allowable axial stress,
σ_b is the allowable bending stress, and
σ_s is the allowable shear stress.

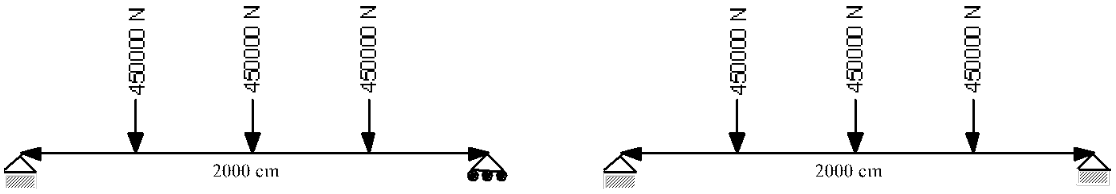
The analysis model was created by dividing each member in the design into three beam elements. The material properties are listed in Table 8.3 and incorporate safety factors of 1.67 for tensile, compressive, and bending stresses and 2.5 for shear stress (Merrit, 1972). The problem specifications for both a simply supported structure and a fixed structure are illustrated in Figure 8.12 and the method parameters are listed in Table 8.4. The loading is a combination of the point loads in Figure 8.12 and self-weight where self-weight for a beam element is distributed load along the length of the beam rather than in a truss model where it is transferred to the nodes as point loads. Beam and truss designs for the simply supported problem are shown in Figure 8.13 with designs for the fixed boundary conditions shown in Figure 8.14.

Table 8.3 Material Properties for Figures 8.13 and 8.14

Material Property	*Steel*
modulus of elasticity, E	6.88 E^6 N/cm^2
allowable tensile stress	14,880 N/cm^2
allowable compressive stress	14,880 N/cm^2
allowable bending stress	14,880 N/cm^2
allowable shear stress	9,920 N/cm^2
mass density, ρ	.00785 kg/cm^3

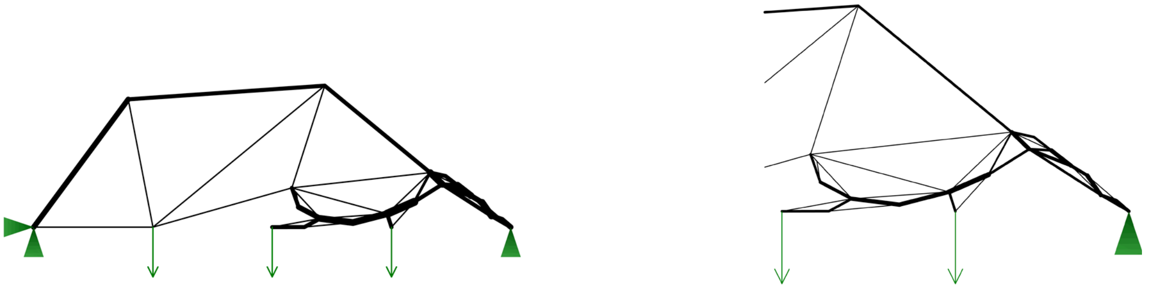
Table 8.3 Method Parameters for Figures 8.13 and 8.14

Method Parameter		*Method Parameter*	
minimum area	.01 cm^2	maximum number of members	50
maximum area	730 cm^2	number of iterations	170
member areas	continuous	number of designs per iteration	200
minimum member length	15 cm	planar truss topology rules	1-5
minimum angle between members	1°	rule selection	Hustin
intersections between members	not allowed	constraint violation normalization	yes
member shape	solid rod		



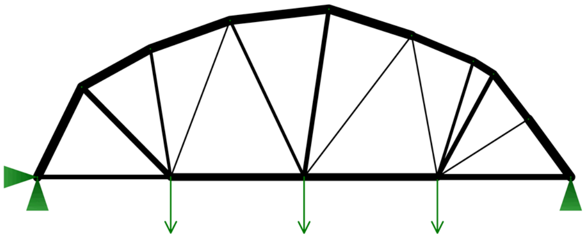
(a) simply supported (b) fixed

Figure 8.12: Problem Specifications for (a) simply supported and (b) fixed boundary conditions



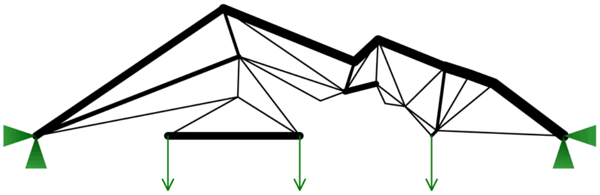
close-up of right hand side

(a) beam design for simply supported boundary conditions; mass = 955 kg

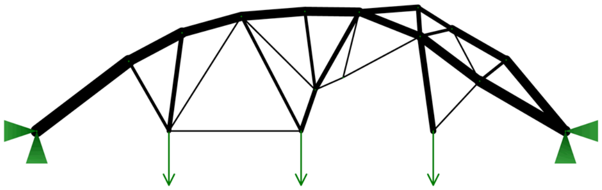


(b) truss design for simply supported boundary conditions; mass = 2,344 kg

Figure 8.13: Beam and truss designs for simply supported boundary conditions



(a) beam design for fixed boundary conditions; mass = 1,783 kg



(b) truss design for fixed boundary conditions; mass = 2023 kg

Figure 8.14: Beam and truss designs for simply supported boundary conditions

The first remark that can be made when comparing the two sets of designs is that a truss design yields much heavier solutions than beam designs for the same problem specification. This difference is accentuated in the simply supported problem and was observed for both problems when comparing six designs generated for each case. The geometric forms of the solutions are also quite different. As a side note, the design in Figure 8.13(b) is a nearly symmetric design for a symmetric load specification illustrating that for simple problems the shape annealing method is capable of generating nearly symmetric topologies without a constraint on symmetry.

Extending the current grammar to the design of frames changes the underlying functional principles of the grammar. The form-function relations in the shape grammar for truss design are an embodiment of Maxwell's rule for pin-jointed structures. For the configuration of frames there is greater topological latitude due to inherent stability from the joint construction.; proposed examples are shown in Figure 8.15. While with truss design it was necessary to have stability knowledge in the grammar such that the structure was always a valid truss structure, a frame grammar could use a very general format as merely a graph of connections. A frame grammar would create a new language of discrete structures.

The design of frames with shape annealing could be implemented two ways, either as a decision made by the designer to select the structural element type (truss or frame) or using the method to switch between element types allowing the optimization to determine the best element type for the design application. A transition grammar could be formulated for moving a truss design to a frame design and back again. Defining a grammar for both truss and frame structures would entail defining three rule sets: the truss rules from Figure 3.1, a transition grammar between trusses and frames (see Figure 8.15(a)) and a frame grammar (see Figure 8.15(b)).

(a) frame-truss transition

(b) frame rules

Figure 8.15: Proposed frame grammar

Another structural language could be defined by extending the planar grammars presented to a three-dimensional truss or frame grammar. Here, the design components would be tetrahedrons in the case of a truss or pyramids in the case of a frame for the generation of double-layer space truss structures. Through the inclusion of new grammars the method would be able to generate designs for different classes of structures. A difficulty in more complex design objects, any structural element other than a truss, is the integrity of the automated finite element model. Much consideration into the syntax of the grammar and the generation of analysis models would be needed to maintain the accuracy of the behavioral evaluation.

8.3.2 Optimization Model for Structural Design

The optimization model presented considered the design goals of efficiency, economy, utility and elegance. Additions to the optimization model could include more elaborate cost functions for economy that trade off the cost of material and the cost of the labor required to construct the design. Additional analyses of the design could also be performed for lighting, acoustics, and airflow such that the effects of the topology on these design attributes could be studied. In order to advance the optimization model, a rigorous investigation into the design considerations of practical structural design would need to be made. Once the model was developed, the current implementation would easily support it as long as the modeled costs are a function of known design attributes. Advancing the optimization model would take the method one step further to being a practical design tool.

8.3.3 Optimization Technique

Configuration optimization of discrete structures is a difficult problem that requires a robust adaptive optimization method. The robustness of simulated annealing requires that a design evolve as a series of small perturbations made to the design. The problem in applying this method to discrete topology design is that adding new members to the design can modify the whole functionality of the structure resulting in a large change of the cost function. This problem has been evident in the application of topology changes that can not be formulated as the small perturbations that simulated annealing desires. Simulated annealing has shown greatest optimization success in the layout of structures for simple, highly constrained problems where the space of design alternatives is relatively small. As the space of possible configurations increases and becomes less constrained, such as with multiobjective design, the convergence of simulated annealing declines. A genetic algorithm, another stochastic technique, could provide for an interesting combination with grammatical structural design since an essay of designs could result from a single run of the method. Although simulated annealing may not provide the optimal solution it is a good technique for navigating a large space to produce feasible, optimally directed designs.

Another modification of the optimization would be to combine a stochastic method with a local optimization method. Here, simulated annealing would be used to move the design in the direction of an optimum while once within the range of the optimum a local method would further optimize the design to reach the true optimum. While this would not ensure the generation of a global optimum, it would allow for a true optimum to be reached.

8.3.4 Analysis

The shape annealing method currently uses finite element analysis (FEA) to provide a behavioral evaluation of a candidate design. FEA was originally chosen for its capability in analyzing many different element types. However, this choice was made when the development of the method was heading in the direction of using shape annealing as a tool purely for structural optimization. The drawback to using FEA is that it accounts for, on average, over 80% of the computation time required to generate one design. The computation time also increases with the size of the problem. The smallest problem presented in this work used a maximum of 25 members and took about 45 minutes to run while the largest problem with a maximum of 150 members averaged five hours of computation time.

Since a large percentage of the design modifications occur as shape or sizing modifications, the computation time could be significantly reduced by the addition of approximation techniques such as that found in (Kirsch, 1995). Using an approximation technique along with FEA to perform a full analysis when topology modification rules were applied could reduce the computation time by about 80%. This would reduce the computation time to generate a design with a maximum of 150 members from five hours to one hour. Computation time significantly increases with the use of frame elements; the designs in Figures 8.13 and 8.14 took about eight hours of computation time each. This run time could be reduced with the addition of an element specific analysis technique such as jointed member analysis, JMA, developed by Degentesh et al. (1996). JMA is capable of analyzing jointed structures, both trusses and frames, and compared to FEA reduces computation time for frames through the reduction of variables. Using JMA, the computation time for the beam designs shown in Figures 8.13 and 8.14 with 50 maximum members could be reduced from eight hours to about one hour.

8.4 Network Flow Application: Land Use and Transportation

The shape annealing method has been presented as a design technique capable of producing optimally directed designs for a network of structural elements. Shape annealing could also be applied to a different network flow problem, the land use and transportation problem, which is to layout a cost-effective road system that connects a specified set of locations over a defined configuration of land (Cagan and Mitchell, 1994); see Figure 8.16. The primary difference between truss layout and a network of roads is that the functional evaluation is different. The function of a network of roads is defined by the connection of all locations and acceptable levels of traffic flow where there are no restrictions on the topology used to achieve this. The cost of a segment of road corresponds to the level of traffic flow and the terrain on which it lies. Defining the land as a grid of costs reflects the relative cost of placing a road on that terrain. For, example it may be much more costly to construct a road over or through a mountain than it would to go around the mountain. Just as in the design of structures, geometric obstacles can be placed to represent impossible spaces to lay a road. Road networks could also require a minimization of the number of intersections similar to reducing the number of joints in a structure. In both problems the layout is a weighted network of line elements.

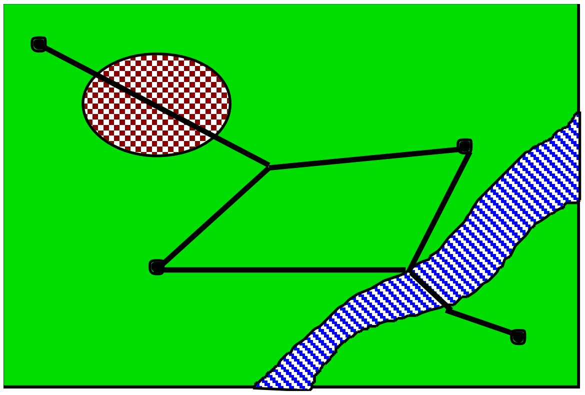


Figure 8.16 Land use and transportation problem

8.5 Design Innovation

The structures generated by shape annealing have often been classified as “innovative” designs. A brief discussion of design innovation can be used to illustrate the possibility that the structural designs presented are spatially innovative. Considering design artifacts, as opposed to the design process, designs can be broken down into creative, innovative or routine. The definitions that will be used to differentiate between these design types were taken from Cagan and Agogino (1989). Routine designs, which are most common, are parametric variations of an existing design. For structural topology design, routine design is shape or sizing optimization of a given topology. Innovative design is then the introduction of new design components, and thus design variables, in the existing design, such that the new design is a variation of the old. By this definition, structural topology optimization could be considered innovative when the initial set of design variables is expanded upon to produce new design variations that use the same structural principle.

Creative design, the hardest but most rewarding class of design, are designs that introduce something completely new. It seems that for computational structural design, creativity could arise from the recognition and manipulation of emergent design features that could result in new structural functions. For example, in the case of truss topology design, by applying a shape rule on an emergent shape a design transformation could result in the formation of a polygon rather than a triangle. This new shape could then be recognized by the algorithm as requiring members that take bending and thus the truss design would become a frame design. It is the recognition of emergent features, or that a design is more than the atomic composition of its elements, that may lead to creative structural design.

While the approach to structural design that has been presented is not capable of generating creative designs, it is, however, capable of innovative design. Often, computer generated design is discounted as not being able to produce innovative designs either due to not simulating an innovative process or the biased computational advantages of a computer over a human designer. Although the process used to generate the structural essays is not innovative, that is it does not simulate an innovative structural designer, the generation of innovative forms cannot be discounted. Thus, in general, the designs generated by shape annealing can be classified as spatially innovative as they are geometric forms that would not often be conceived by humans due to the number of design alternatives that the method can consider and the benefits of computational analysis. Nevertheless, the determination of innovation is left to the designer, the ultimate critic for any computer generated design, who must determine whether, for their purposes, a design is innovative.

8.6 Conclusion

The work presented has explored a grammatical approach to the innovative design of purposeful discrete structures. The method developed here for structural design could provide the foundation for a computational design tool that is capable of generating essays of structural design alternatives that reflect practical design goals and requirements. Although structural optimization was not the primary goal in this work, the method developed also provides an approach to structural topology, shape and sizing optimization that is capable of expanding the topology design space and optimizing discrete design variables. Additionally, the capability of the shape annealing method as an effective approach to design configuration problems has been shown through the application to structural design.

Until now, the current available methods for accomplishing the task of topological and geometric design of structures are structural optimization, which are limited in their topologies and extension to spatial design goals, and languages of structural form, which had a large part in forming this work, but are limited to spatial exploration of designs. It is the combination of structural languages and directed stochastic optimization that provides for the ability to define an infinite set of design alternatives that can be searched for functionally and spatially meaningful structures. The simplicity of the merger between design languages and optimization is where the benefits of the shape annealing method lie and the applications are numerous.

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