Exam Duration: 135 minutes + 15 minutes reading time
Number of Problems: 45
Number of Points: 53

Important:
Questions must be answered on the provided answer sheet; answers given in the booklet will not be considered.

There exist multiple versions of the exam, where the order of the answers has been permuted randomly.

There are two types of questions:

1. **One-best-answer type questions**: One unique correct answer has to be marked. The question is worth one point for a correct answer and zero points otherwise. Giving multiple answers to a question will invalidate the answer. These questions are marked "Choose the correct answer. (1 Point)"

2. **True / false type questions**: All true statements have to be marked and multiple statements can be true. If all statements are selected correctly, the full number of points is allocated; for one incorrect answer half the number of points; and otherwise zero points. These questions are marked "Mark all correct statements. (2 Points)".

No negative points will be given for incorrect answers.

Partial points (Teilpunkte) will not be awarded.

You do not need to justify your answers; your calculations will not be considered or graded.

Use only the provided paper for your calculations; additional paper is available from the supervisors.

Good luck!
1 Control Architectures and Systems

Question 1  Mark all correct statements. (2 Points)

Feedforward control systems...

☐ ...rely on precise knowledge of the plant.  ☐ ...reject external disturbances.
☐ ...can change the dynamics of the plant.

Question 2  Mark all correct statements. (2 Points)

Feedback control systems...

☐ ...can stabilize an unstable plant.  ☐ ...can de-stabilize a stable plant.
☐ ...can feed sensor noise into the system.

Question 3  Mark all correct statements. (2 Points)

All signals are scalars. The system \( \frac{dy(t)}{dt} = (u(t + 1))^2 \) is:

☐ Causal  ☐ Time-Invariant
☐ Memoryless / Static  ☐ Linear

Question 4  Mark all correct statements. (2 Points)

All signals are scalars. The system \( \Sigma(s) = \frac{e^{s}}{s + 1} \) is:

☐ Causal  ☐ Memoryless / Static
☐ Time-Invariant  ☐ Linear

Question 5  Mark all correct statements. (2 Points)

All signals are scalars. The system \( y(t) = t^2 \cdot u(t - 1) \) is:

☐ Linear  ☐ Causal
☐ Time-Invariant  ☐ Memoryless / Static
2 System Modeling

Box 1: Questions 6, 7

Description: Consider an electric motor which you would like to operate at a constant rotational speed $\omega_0$. Applying a voltage $U(t)$ results in a change in the circuit current $I(t)$, which is governed by the differential equation

$$L \cdot \frac{d}{dt} I(t) = -R \cdot I(t) - \kappa \cdot \omega(t) + U(t) \ ,$$  

whereby $L$ is the circuit inductance, $R$ its resistance and $\kappa$ a constant relating the motor speed $\omega(t)$ to an electro motor-force (EMF). The dynamics of the motor speed are given by

$$\Theta \cdot \frac{d}{dt} \omega(t) = -d \cdot \omega(t) + T(t) \ ,$$  

where $\Theta$ represents its mechanical inertia, $d$ a friction constant and $T(t) = \kappa \cdot I(t)$ the current-dependent motor torque.

Box 2: Question 6

Question 6  Choose the correct answer. (1 Point)

Relate the variables in the block diagram above to the correct signals.

- A $u(t) = U(t), x(t) = \begin{bmatrix} \omega(t) \\ I(t) \end{bmatrix}, y(t) = \omega(t), r(t) = \omega_0$
- B $u(t) = U(t), x(t) = \begin{bmatrix} U(t) \\ T(t) \end{bmatrix}, y(t) = T(t), r(t) = \omega_0$
- C $u(t) = U(t), x(t) = \begin{bmatrix} \omega(t) \\ I(t) \end{bmatrix}, y(t) = I(t), r(t) = \omega_0$
- D $u(t) = I(t), x(t) = \begin{bmatrix} \omega(t) \\ U(t) \end{bmatrix}, y(t) = I(t), r(t) = \omega_0$
- E $u(t) = I(t), x(t) = \begin{bmatrix} \omega(t) \\ T(t) \end{bmatrix}, y(t) = \omega(t), r(t) = \omega_0$
**Question 7**  *Choose the correct answer. (1 Point)*
A colleague tells you that the circuit inductance is very small and can actually be neglected. The arising motor model can now be represented as...

(A) ...a second-order system.  
(B) ...a first order system.  
(C) ...an integrator.  
(D) ...a static system.

**Question 8**  *Choose the correct answer. (1 Point)*

Consider the nonlinear system
\[
\frac{dx(t)}{dt} = (x(t) - u(t))^4  
y(t) = x(t) - 20 .
\]

You are measuring a constant output \( y(t) = 10 \).

What is the value of \( x(t) \) and \( u(t) \)?

(A) \( x(t) = 20 \cdot t \) and \( u(t) = 20 \).  
(B) \( x(t) = 10 \) and \( u(t) = 30 \).  
(C) \( x(t) = 20 \) and \( u(t) = 20 \).  
(D) \( x(t) = 30 \) and \( u(t) = 30 \).

**Question 9**  *Choose the correct answer. (1 Point)*

Given the system
\[
\frac{dx(t)}{dt} = x(t)^2 + 5 \cdot u(t) - 10  
y(t) = \frac{4 \cdot x(t) - 12}{u(t)} ,
\]

you have to linearize it around the equilibrium \( x_e = 0, u_e = 2 \).

Which are the state-space matrices \( A, b, c \) and \( d \)?

(A) \( A = 0, b = 5, c = 4, d = -3 \).  
(B) \( A = 5, b = 5, c = 2, d = -3 \).  
(C) \( A = -10, b = 5, c = 2, d = 3 \).  
(D) \( A = 0, b = 5, c = 2, d = 3 \).
3 Linear Systems Analysis

Box 3: Question 10

The figure below shows the step response of a second-order system.

![Step Response Diagram]

**Question 10** *Choose the correct answer. (1 Point)*

Assess the bounded-input bounded-output (BIBO) stability.

- [ ] BIBO unstable
- [B] BIBO stable
- [C] No conclusion possible
Corrected

Box 4: Questions 11, 12

The figure below shows the response of an internal state \( x \) of a system as an impulse is applied to the system’s input.

![Response of internal state](image)

**Question 11**  
*Choose the correct answer. (1 Point)*

- A. No conclusion about the stability is possible.
- B. The system is unstable.
- C. The system is asymptotically stable.
- D. The system is Lyapunov stable.

**Question 12**  
*Choose the correct answer. (1 Point)*

- A. The system is BIBO stable.
- B. No conclusion about the BIBO stability is possible.
- C. The system is BIBO unstable.

**Question 13**  
*Choose the correct answer. (1 Point)*

Given a system with state space representation

\[
x(t) = \begin{bmatrix} -1 & 10 \cdot \alpha^2 \\ 0 & \alpha - 1 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 1 \\ \alpha \end{bmatrix} \cdot u(t) \\
y(t) = \alpha \cdot x(t) + u(t),
\]

where \( \alpha \in \mathbb{R} \). For which values of \( \alpha \) is the system asymptotically stable?

- A. \( \alpha < 1 \).
- B. \( \alpha < 0 \).
- C. \( \alpha \leq 1 \).
- D. \( \alpha \geq 1 \).
Box 5: Question 14

Description: The figure below shows the step response of a first-order system of the form

\[
\dot{x}(t) = a \cdot x(t) + b \cdot u(t) \quad y(t) = x(t) .
\]

Question 14  Choose the correct answer. (1 Point)

Which are the values of \( a \) and \( b \)?

- A  \( a = -2 \) and \( b = 20 \).
- B  \( a = -0.5 \) and \( b = -10 \).
- C  \( a = -0.5 \) and \( b = 20 \).
- D  \( a = -1 \) and \( b = 10 \).

Box 6: Question 15

Description: The figure below shows the impulse responses \( y_A(t) \) and \( y_B(t) \) of two systems \( A \) and \( B \). Both systems are linear time-invariant second order systems. In addition to the responses, an unknown function \( f(t) \) and its tangential line at time \( t = 0 \) is drawn in the plot to the right.

Question 15  Choose the correct answer. (1 Point)

Which are the eigenvalues \( \lambda_A \) and \( \lambda_B \) of both systems?

- A  \( \lambda_A = 2\pi \) and \( \lambda_B = -\frac{1}{2} \).
- B  \( \lambda_A = 0 \pm j \) and \( \lambda_B = -\frac{1}{2} \pm j \).
- C  \( \lambda_A = 0 \pm j \cdot 2\pi \) and \( \lambda_B = -2 \pm j \cdot 2\pi \).
- D  \( \lambda_A = 0 \pm j \cdot 2\pi \) and \( \lambda_B = -\frac{1}{2} \pm j \cdot 2\pi \).
Box 7: Questions 16, 17

Description: A LTI system has the form

\[ A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad d = 0. \]

Question 16  Choose the correct answer. (1 Point)

The system in the above equation is . . .

- [ ] only observable.
- [ ] only controllable.
- [ ] observable and controllable.
- [ ] neither observable nor controllable.

Question 17  Choose the correct answer. (1 Point)

For \( x_1(t) = 0, x_2(t) = 0 \) the initial condition response of the system is . . .

- [ ] \( y(t) = 0 \).
- [ ] \( y(t) = h(t) \).
- [ ] \( y(t) = h(t) \cdot t. \)
- [ ] \( y(t) = h(t) \cdot e^t. \)
4 Frequency Response

Box 8: Questions 18, 19

Description: Given a linear time-invariant system in state-space description.

\[
\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 0 & -8 \\ 1 & 0 & -4 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \cdot u(t) \\
y(t) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \cdot x(t).
\]

Question 18 Choose the correct answer. (1 Point)
The transfer function \( g(s) \) is …

A \( g(s) = \frac{2s+10}{s(s+4)(s+17)} \).

B \( g(s) = \frac{2s+10}{s^2+5s+4} \).

C \( g(s) = \frac{s+5}{s^2+5s+4} \).

D \( g(s) = \frac{2(s+5)}{s^2+5s+4} \).

Question 19 Choose the correct answer. (1 Point)
The system is …

A Asymptotically stable.

B Lyapunov stable.

C Unstable.

D Lyapunov stable.

Question 20 Choose the correct answer. (1 Point)
Given the transfer function \( g(s) \) of a system.

\[
g(s) = \frac{3s + 7}{s(s + 1)}
\]

A unit step is applied to the system’s input:

\[
u(t) = h(t).
\]

The time response \( y(t) \) is …

A \( y(t) = h(t) \cdot (-4 + 7t + 4e^{-t}) \).

B \( y(t) = h(t) \cdot (2 + 7t + 7e^{-t}) \).

C \( y(t) = h(t) \cdot (7 - 4t + 4e^{-t}) \).

D \( y(t) = h(t) \cdot (7 - 4t + 4e^{-t}) \).
Given the transfer function $g(s)$ of a system:

$$g(s) = \frac{s + 3}{s^2 + 0.7s + 1}$$

In addition the figure below shows the four time responses A, B, C and D.

**Question 21** Choose the correct answer. (1 Point)
Which of the four diagrams shows the correct step response of the system?

- A Diagram 3.
- B Diagram 4.
- C Diagram 1.
- D Diagram 2.
Question 22  Choose the correct answer. (1 Point)

Select the transfer function $L(s)$ which matches the Bode Plot given above:

A \[ L(s) = 0.01 \cdot \frac{(s+10)^2}{s+0.1} \]

B \[ L(s) = \frac{10}{s+0.1} \]

C \[ L(s) = \frac{10}{0.1-s} \]

D \[ L(s) = \frac{s+100}{(s+0.1)(s-100)} \]

Question 23  Choose the correct answer. (1 Point)

What is the steady state output of the system

\[ \dot{x}(t) = -x(t) + u(t), \quad y(t) = -x(t), \quad (4) \]

when $u(t) = \sin(\omega t)$?

A \[ y_{ss}(t) = \frac{\omega \cos(\omega t) - \sin(\omega t)}{1+\omega^2} \]

B \[ y_{ss}(t) = \frac{1}{\sqrt{1+\omega^2}} \sin(\omega t - \arctan(\omega)) \]

C \[ y_{ss}(t) = \frac{\sin(\omega t) - \omega^2 \cos(\omega t)}{1+\omega^2} \]
Question 24  Choose the correct answer. (1 Point)

How many poles and zeros does $L(s)$ have?

- **A** $L(s)$ has 2 poles and 0 zeros.
- **B** $L(s)$ has 4 poles and 2 zeros.
- **C** $L(s)$ has 3 poles and 2 zeros.
- **D** $L(s)$ has 3 poles and 0 zeros.
Box 12: Question 25

Consider the following Bode plot of a transfer function \( g(s) \):

![Bode plot for \( g(s) \)]

**Question 25**  *Choose the correct answer. (1 Point)*

Select the transfer function which matches the Bode plot.

- A \( g(s) = \frac{-20}{s+1} \)
- B \( g(s) = \frac{10}{s} \)
- C \( g(s) = \frac{20}{s+1} \)
- D \( g(s) = \frac{10}{s+1} \)
5 Polar Plot and Closed-Loop Stability

Box 13: Question 26

The Nyquist plot for a loop transfer function $L(s)$ with two unstable poles is shown below on the left. On the right is a feedback configuration for the closed loop system.

Question 26  *Mark all correct statements. (2 Points)*

Which of the following statements are true about the closed loop system $T(s) = \frac{kL(s)}{1+kL(s)}$?

A  The closed loop system $T(s)$ is stable when $k = 1$.
B  The closed loop system $T(s)$ is unstable when $k > 2$.
■ The closed loop system $T(s)$ is unstable when $k = 1$.
■ The closed loop system $T(s)$ is stable when $k > 2$. 
**Box 14: Question 27**

The Bode plot for a plant transfer function $P(s)$ is given below.

**Question 27**  Choose the correct answer. (1 Point)

Which of the controller designs will produce a stable closed loop system?

A. $C(s) = \frac{0.01s+1}{0.001s+1}$

B. $C(s) = \frac{1}{0.01s+1}$

C. $C(s) = 0.1$

D. $C(s) = 10.0$
Box 15: Question 28

The Nyquist plot for a loop transfer function of the form

\[ L(s) = \frac{k}{(s + p_1)(s + p_2)(s + p_3)} \]  \hspace{1cm} (5)

is given below.

**Question 28**  Choose the correct answer. (1 Point)

How many unstable poles does the closed loop system \( T(s) = \frac{L(s)}{1 + L(s)} \) have?

- **A** \( T(s) \) has 2 unstable poles.
- **B** \( T(s) \) has 0 unstable poles.
- **C** \( T(s) \) has 1 unstable pole.
- **D** Not enough information is provided.
The bode plot for a system with transfer function $g(s)$ is given below.

**Question 29**  
Mark all correct statements. (2 Points)

Select all answers that correctly classify the stability of the system.

- A $g(s)$ is unstable.
- B $g(s)$ is asymptotically stable.
- Mark $g(s)$ is Lyapunov stable.
- D $g(s)$ is bounded input bounded output (BIBO) stable.
Box 17: Question 30

Consider the following Nyquist plot of an open loop gain $L(s)$:

**Question 30**  Choose the correct answer. (1 Point)

Select the transfer functions which match the Nyquist plot.

- **A** $L(s) = \frac{10}{s - 1}$
- **B** $L(s) = \frac{10}{(s+1)^2}$
- **C** $L(s) = \frac{-10}{(s+1)^2}$
- **D** $L(s) = \frac{10}{(1-s)^2}$
Box 18: Question 31

Consider the system described by the following block diagram

\[ y = C(s) \left( \frac{1}{10s + 1} \right) P(s) e + r \]

where

\[ P(s) = \frac{1}{10s + 1}, \quad C(s) = k. \]

Question 31  Choose the correct answer. (1 Point)

Determine the smallest positive gain \( k \) such that the feedback system is stable and, when \( r(t) \) is a unit step, \( \lim_{t \to \infty} |e(t)| \leq 0.1 \).

The smallest positive gain \( k \) is:

- A 10
- B 2
- C 1
- D 5
- E 9
Question 32  

Choose the correct answer. (1 Point)

We are given the transfer function:

\[ g(s) = \frac{(s + 4)}{(s^4 - 9s^2)}. \]

Which of the following is the root locus plot of \( g(s) \)?
6 Feedback Control and Specifications

Question 33 Choose the correct answer. (1 Point)

For the following system, you have the choice between implementing two different sensors: 
\( y_1(t) = x_1(t) \) or \( y_2(t) = x_2(t) \).

\[
\dot{x}(t) = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)
\]

Which sensor should you choose?

- [ ] \( y_2(t) \)
- [x] \( y_1(t) \)
Box 19: Questions 34, 35, 36

Consider the block diagram given below to answer the following three questions:

Question 34  Choose the correct answer. (1 Point)

Suppose $P_1(s) = \frac{2}{s+1}$, $P_2 = \frac{1}{s+2}$, and $C(s) = 2$. Which of the following choices is the correct transfer function, $G(s) = Y(s)/R(s)$, from $r$ to $y$?

- A $G(s) = \frac{4}{s+1}$
- B $G(s) = \frac{4(s-2)}{(s+1)^2}$
- C $G(s) = \frac{4(s+1)}{(s-1)^2}$
- D $G(s) = \frac{4(s-1)}{(s+1)^2}$
- E $G(s) = \frac{4}{s+1}$

Question 35  Choose the correct answer. (1 Point)

Is the system defined in the above problem internally stable?

- A No.
- B Yes.

Question 36  Choose the correct answer. (1 Point)

Now assume instead that $P_1(s) = P_2(s) = P(s)$ for some $P(s)$ and that $C(s)$ is stable. Which of the following statements is true?

- A The system is internally stable if and only if $P(s)$ has no zeros with positive real part.
- B The system is NOT internally stable for any choice of $P(s)$.
- C The system is internally stable for any choice of $P(s)$.
- D The system is internally stable if and only if $P(s)$ is stable.
Consider the state-space system below, where \( a \) is a real parameter:

\[
\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 1 & -4 \end{bmatrix} x(t) + \begin{bmatrix} a \\ 1 \end{bmatrix} u(t) \\
y(t) = \begin{bmatrix} 2 & a \end{bmatrix} x(t)
\]

**Question 37**  
Choose the correct answer. (1 Point)

Is the system stabilizable for \( a = 0 \)?

- [ ] No.
- [X] Yes.

**Question 38**  
Mark all correct statements. (2 Points)

Mark all of the following statements about control limitations that are true.

- [ ] The dominant right-half-plane pole puts a lower limit on the system’s bandwidth.
- [X] The sensitivity function, \( S(s) \), plus the complementary sensitivity function, \( T(s) \), must equal one for all \( s \).
- [ ] A plant’s right-half-plane zero limits the bandwidth of the closed-loop system.
- [X] When a plant has a right-half-plane zero \( z \), the sensitivity function at \( z \), \( S(z) \), must equal 1.
7 Control Design

Box 21: Question 39

The figure below shows the block diagram of a cascaded controller:

```
\[ r \quad + \quad c_1 \quad + \quad c_2 \quad A(s) \quad B(s) \quad y \]
```

Question 39  Choose the correct answer. (1 Point)

Derive the transfer function \( T(s) \) from \( r \to y \) for the system depicted in the block diagram above.

\[
\begin{align*}
A & : \frac{B(s)}{1 + A(s) + A(s)B(s)} \\
B & : \frac{A(s)B(s)}{1 + A(s) + B(s) + A(s)B(s)} \\
\text{Corrected} & : \frac{A(s)B(s)}{1 + A(s) + A(s)B(s)} \\
D & : \frac{A(s)B(s)}{1 - A(s) + A(s)B(s)} \\
E & : \frac{A(s)B(s)}{1 + A(s)B(s)}
\end{align*}
\]

Question 40  Choose the correct answer. (1 Point)

Let \( L(s) = \frac{(s+5)(s+3)}{(s+2)^2} \) be an open loop transfer function in a standard feedback architecture. Let the input to the system be a unit step input.

What is the static error \( e_\infty \) of the system?

\[
\begin{align*}
A & : \frac{15}{17} \\
\text{Corrected} & : \frac{4}{17} \\
C & : 1 \\
D & : \frac{1}{2} \\
E & : \frac{13}{17}
\end{align*}
\]

Question 41  Choose the correct answer. (1 Point)

To which of the following ideal systems can Ziegler/Nichols-tuning by using critical oscillation not be applied?

\[
\begin{align*}
A & : \frac{7}{(s+1)^7} \\
\text{Corrected} & : \frac{3}{s+1} \\
C & : \frac{4}{s+1}e^{-0.1s} \\
D & : \frac{3}{(s+1)^7}
\end{align*}
\]
Question 42  \textit{Choose the correct answer. (1 Point)}

Your system engineering counterpart gives you the system characteristics below:

- Noise above $\omega_n = 500 \text{ rad/s}$ needs to be suppressed.
- Disturbances below $\omega_d = 5 \text{ rad/s}$ need to be rejected.
- Modeling uncertainty reaches 100 percent at $\omega_2 = 40 \text{ rad/s}$.
- An unstable pole at $s = 0.5$ is present.
- The plant has a nonminimum phase zero at $s = 30$.

You want to design a control system where the crossover frequency satisfies

$$\max\{10 \cdot \omega_d, 2 \cdot \omega_n\} < \omega_c < \min\{0.5 \cdot \omega_2, 0.5 \cdot \omega_{\text{delay}}, 0.5 \cdot \omega_c, 0.1 \cdot \omega_n\}$$ \hspace{1cm} (6)

Can you find a feasible crossover frequency $\omega_c$ in presence of the above specifications?

\begin{itemize}
  \item [A] Yes.
  \item [B] No.
\end{itemize}

Box 22: Question 43

The plot below shows step responses of different variations of PID controllers (P, PI, PD, PID)

\begin{center}
\includegraphics[width=\textwidth]{step_responses.png}
\end{center}

Question 43  \textit{Choose the correct answer. (1 Point)}

Which step response in the figure above belongs to the PD-controller?

\begin{itemize}
  \item [A] 2
  \item [B] 3
  \item [C] 1
  \item [D] 4
\end{itemize}
Box 23: Question 44

Consider the following 4 implementations of an anti-windup scheme:

1. 

2. 

3. 

4.
**Question 44**  Choose the correct answer. (1 Point)
Which of the above is a correct anti-windup scheme?

A 1  
B 3  
C 4  
D 2
Box 24: Question 45

Consider the following Bode plot of a plant:

![Bode Plot](image)

Question 45  Choose the correct answer. (1 Point)

Use the bode plot shown above to determine the approximate crossover frequency $\omega_c$, phase margin $\phi$ and gain margin $\gamma$.

- **A**  $\omega_c \approx 2.5 \text{rad/s}$,  $\phi \approx 30^\circ$,  $\gamma \approx 1$
- **B**  $\omega_c \approx 4.4 \text{rad/s}$,  $\phi \approx 15^\circ$,  $\gamma \approx 3$
- **C**  $\omega_c \approx 2.5 \text{rad/s}$,  $\phi \approx 30^\circ$,  $\gamma \approx 3$
- **D**  $\omega_c \approx 4.4 \text{rad/s}$,  $\phi \approx 30^\circ$,  $\gamma \approx 3$