Steady-State Error of
Discrete-Time Control Systems

Addendum to the Lecture
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Final-Value Theorem
The final-value theorem (FVT) for z-transforms states that if \( \lim_{k \to \infty} x(k) \) exists (i.e. remains finite) it can be calculated by its z-transform as
\[
\lim_{k \to \infty} x(k) = \lim_{z \to 1} (z - 1)X(z).
\]

Steady-State Error
Consider the discrete-time control system in Figure 1. When applying a step (i.e. Heaviside function) as reference
\[
r(k) = h(k)
\]
the system’s output is
\[
y(k) = T(z) \cdot \frac{z}{z - 1}.
\]
Applying the FVT yields
\[
\lim_{k \to \infty} y(k) = \lim_{z \to 1} (z - 1) \cdot T(z) \cdot \frac{z}{z - 1}
= T(1) \cdot 1 = T(1).
\]
Thus, in order to have no steady-state error (“stationärer Nachlaufehler”), i.e. \( y(\infty) = 1 \), the condition \( T(1) = 1 \) has to be satisfied.