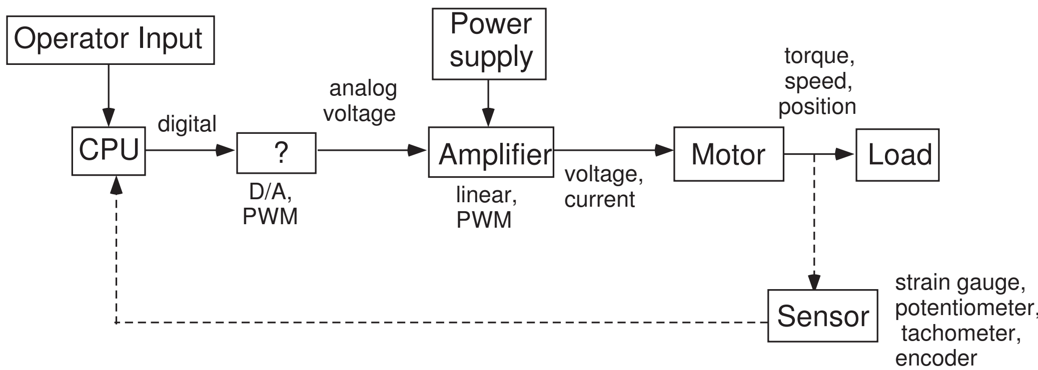
Motor Control

- Suppose we wish to use a microprocessor to control a motor
  - (or to control the load attached to the motor!)



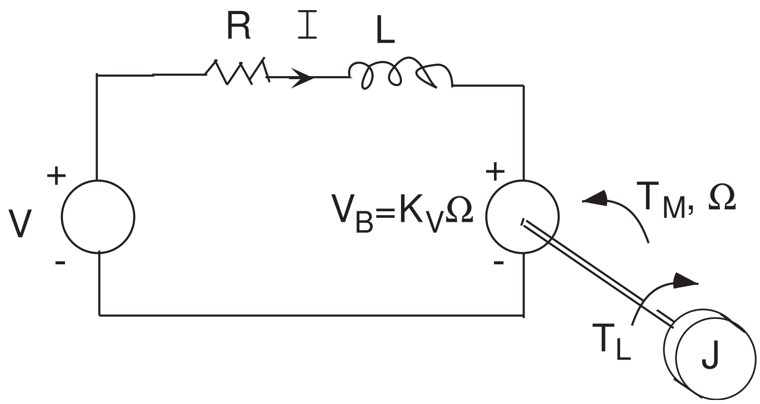
- Convert discrete signal to analog voltage
  - D/A converter
  - pulse width modulation (PWM)
- Amplify the analog signal
  - power supply
  - amplifier
- Types of power amplifiers
  - linear vs. PWM
  - voltage-voltage vs. transconductance (voltage-current)
- DC Motor
  - How does it work?
- What to control?
  - electrical signals: voltage, current
  - mechanical signals: torque, speed, position
- Sensors: Can we measure the signal we wish to control (feedback control)?

Outline

- Review of Motor Principles
  - torque vs. speed
  - voltage vs current control
  - with and without load
- D/A conversion vs. PWM generation
  - harmonics
  - advantages and disadvantages
  - creating PWM signals
- power amplifiers
  - linear vs PWM
  - voltage vs transconductance
- Control
  - choice of signal to control
  - open loop
  - feedback
- References are [5], [3], [1], [4], [8], [7], [6], [9]

Motor Review

- Recall circuit model of motor:

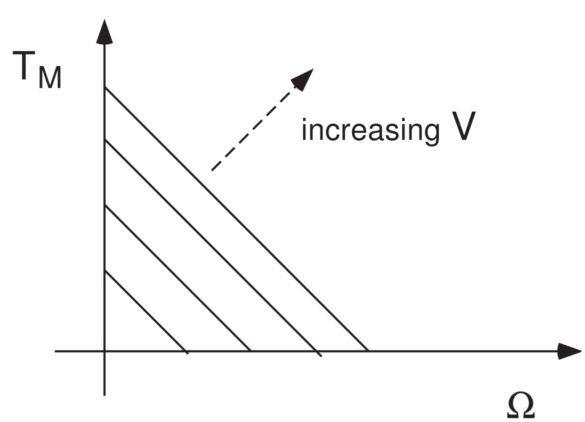


- Suppose motor is driven by a constant voltage source. Then steady state speed and torque satisfy

$$\Omega = \frac{K_M V - R T_L}{K_M K_V + RB}$$

$$T_M = \frac{K_M (VB + K_V T_L)}{K_M K_V + RB}$$

- Torque-speed curve



Voltage Control

- Suppose we attempt to control speed by driving motor with a constant voltage.
- With no load and no friction ($T_L = 0$, $B = 0$)

$$\Omega = \frac{V}{K_V}$$

$$T_M = 0$$

- Recall that torque is proportional to current: $T_M = K_M I$. Hence, with no load and no friction, $I = 0$, and motor draws no current in steady state.
- Current satisfies

$$I = \frac{V - V_B}{R}$$

- In steady state, back EMF balances applied voltage, and thus current and motor torque are zero.
- With a load or friction, ($T_L \neq 0$ and/or $B \neq 0$)

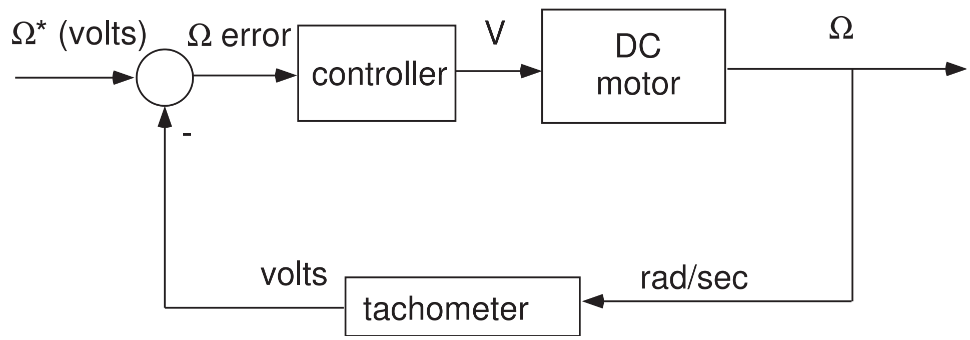
$$\Omega < \frac{V}{K_V}$$

$$T_M > 0$$

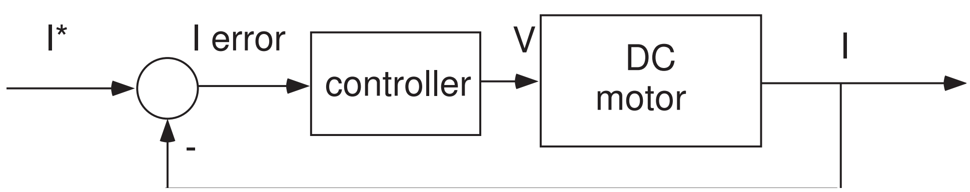
- Speed and torque depend on load and friction
  - friction always present (given in part by motor spec, but there will be additional unknown friction)
  - load torque may also be unknown, or imprecisely known

Issue: Open Loop vs Feedback Control

- Using constant voltage control we cannot specify desired torque or speed precisely due to friction and load
  - an *open loop* control strategy
  - can be resolved by adding a sensor and applying *closed loop*, or *feedback* control
- add a tachometer for speed control



- add a current sensor for torque $(T_M = K_M I)$ control



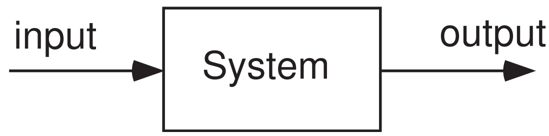
- Will study feedback control in Lecture 7.

Issue: Steady State vs. Transient Response

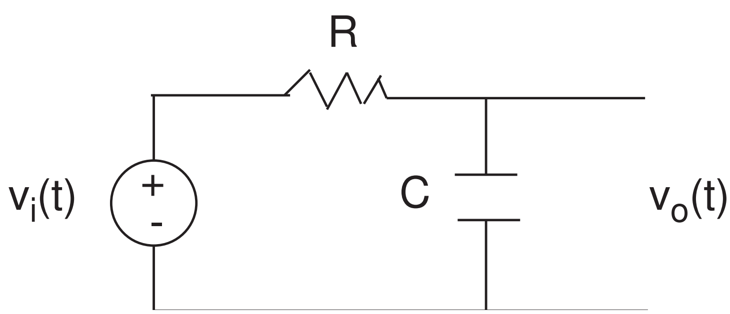
- Steady state response: the response of the motor to a constant voltage input eventually settles to a constant value
  - the torque-speed curves give steady-state information
- Transient response: the preliminary response before steady state is achieved.
- The transient response is important because
  - transient values of current, voltage, speed, . . . may become too large
  - transient response also important when studying response to nonconstant inputs (sine waves, PWM signals)
- The appropriate tool for studying transient response of the DC motor (or any system) is the *transfer function* of the system

System

- A *system* is any object that has one or more inputs and outputs

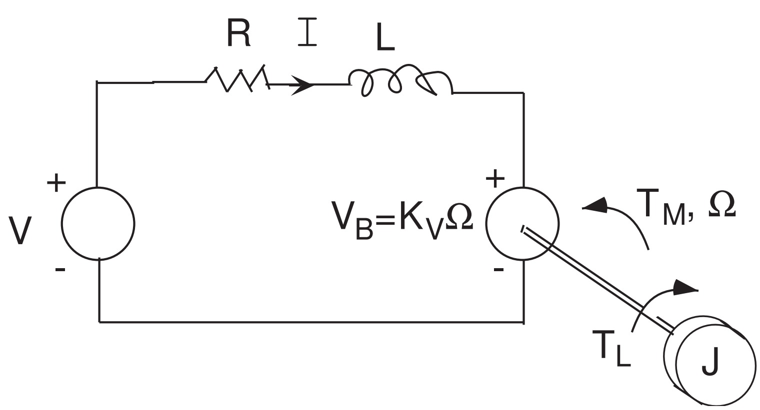


- Input: applied voltage, current, foot on gas pedal, . . .
- Output: other variable that responds to the input, e.g., voltage, current, speed, torque, . . .
- Examples:
  - RC circuit



Input: applied voltage, Output: voltage across capacitor

  - DC motor



Input: applied voltage, Output: current, torque, speed

Stability

- We say that a system is *stable* if a bounded input yields a bounded output
- If not, the system is *unstable*
- Consider DC Motor with no retarding torque or friction
  - With constant voltage input, the steady state shaft speed Ω is constant $\Rightarrow$ the system from V to Ω is stable
  - Suppose that we could hold current constant, so that the steady state torque is constant. Since

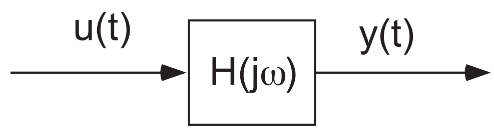
$$\frac{d\Omega}{dt} = \frac{T_M}{J},$$

    the shaft velocity $\Omega \to \infty$ and velocity increases without bound $\Rightarrow$ the system from I to Ω is unstable

- Tests for stability
  - mathematics beyond scope of class
  - we will point out in examples how stability depends on system parameters

Frequency Response

- A linear system has a *frequency response* function that governs its response to inputs:



- If the system is *stable*, then the steady state response to a sinusoidal input, $u(t) = \sin(\omega t)$, is given by $H(j\omega)$:

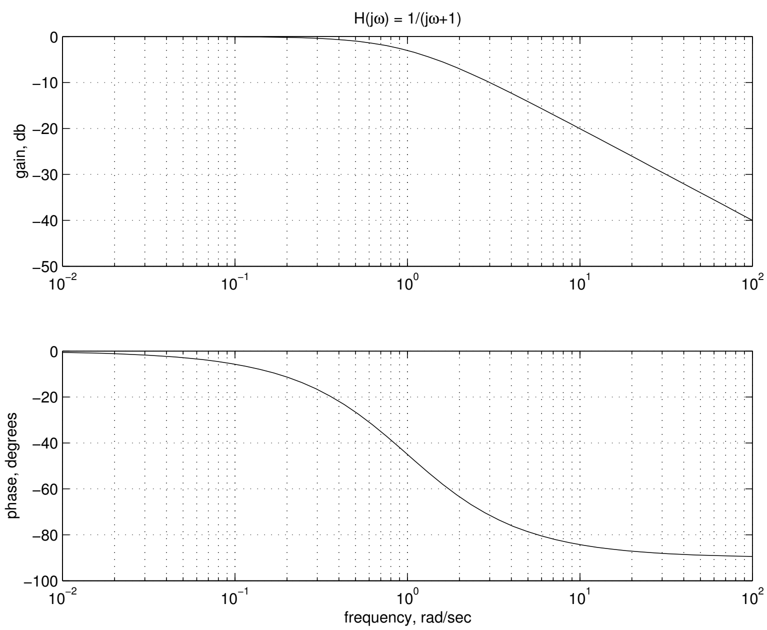
$$y(t) \to |H(j\omega)| \sin(\omega t + \angle H(j\omega))$$

- We have seen this idea in Lecture 2 when we discussed anti-aliasing filters and RC circuits
- The response to a constant, or step, input, $u(t) = u_0, t \geq 0$, is given by the DC value of the frequency response:

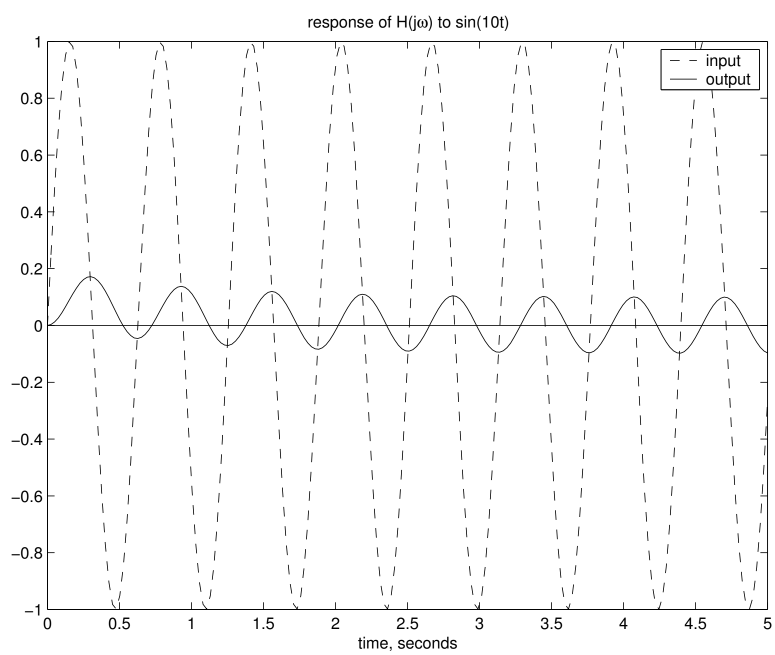
$$y(t) \to H(0) u_0$$

Bode Plot Example

Lowpass filter[1], $H(j\omega) = 1/(j\omega + 1)$



Steady state response to input $\sin(10t)$ satisfies $y_{ss}(t) = 0.1\sin(10t - 85^\circ)$.



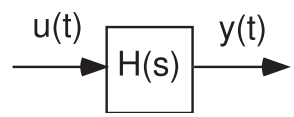
[1]MATLAB file bode_plot.m

Frequency Response and the Transfer Function

- To compute the frequency response of a system in MATLAB, we must use the *transfer function* of the system.
- (under appropriate conditions) a time signal $v(t)$ has a Laplace transform

$$V(s) = \int_0^\infty v(t)e^{-st}dt$$

- Suppose we have a system with input $u(t)$ and output $y(t)$



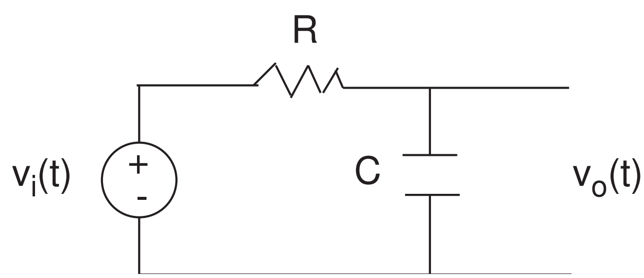
- The transfer function relates the Laplace transform of the system output to that of its input:

$$Y(s) = H(s)U(s)$$

- for simple systems $H(s)$ may be computed from the differential equation describing the system
- for more complicated systems, $H(s)$ may be computed from rules for combining transfer functions
- To find the frequency response of the system, set $s = j\omega$, and obtain $H(j\omega)$

Transfer Function of an RC Circuit

- RC circuit
  - Input: applied voltage, $v_i(t)$.
  - Output: voltage across capacitor, $v_o(t)$



- differential equation for circuit
  - Kirchoff's Laws: $v_i(t) - I(t)R = v_o(t)$
  - current/voltage relation for capacitor: $I(t) = C\frac{dv_O(t)}{dt}$
  - combining yields

$$RC\frac{dv_o(t)}{dt} + v_o(t) = v_i(t)$$

- To obtain transfer function, replace
  - each time signal by its Laplace transform: $v(t) \rightarrow V(s)$
  - each derivative by "s" times its transform: $\frac{dv(t)}{dt} \rightarrow sV(s)$
  - solve for $V_o(s)$ in terms of $V_i(s)$:

$$V_o(s) = H(s)V_i(s), \qquad H(s) = \frac{1}{RCs + 1}$$

- To obtain frequency response, replace $j\omega \rightarrow s$

$$H(j\omega) = \frac{1}{RCj\omega + 1}$$

Transfer Functions and Differential Equations

- Suppose that the input and output of a system are related by a differential equation:

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + a_2 \frac{d^{n-2} y}{dt^{n-2}} + \dots a_{n-1} \frac{dy}{dt} + a_n y =$$
$$b_1 \frac{d^{n-1} u}{dt^{n-1}} + b_2 \frac{d^{n-2} u}{dt^{n-2}} + \dots b_{n-1} \frac{du}{dt} + b_n u$$

- Replace $d^m y/dt^m$ with $s^m Y(s)$:

$$\left(s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n\right) Y(s) =$$
$$\left(b_1 s^{n-1} + b_2 s^{n-2} + \dots b_{n-1} s + b_n\right) U(s)$$

- Solving for $Y(s)$ in terms of $U(s)$ yields the transfer function as a ratio of polynomials:

$$Y(s) = H(s)U(s), \qquad H(s) = \frac{N(s)}{D(s)}$$

$$N(s) = b_1 s^{n-1} + b_2 s^{n-2} + \dots b_{n-1} s + b_n$$
$$D(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n$$

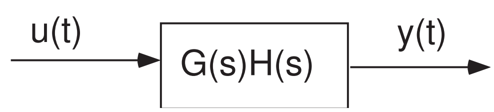
- The transfer function governs the response of the output to the input with all initial conditions set to zero.

Combining Transfer Functions

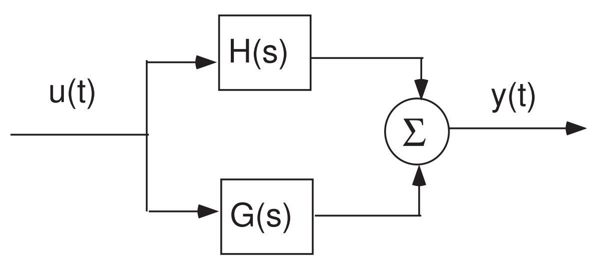
- There are (easily derivable) rules for combining transfer functions

  - Series: a series combination of transfer functions

reduces to



  - Parallel: a parallel combination of transfer functions



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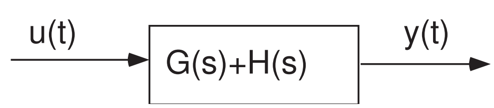
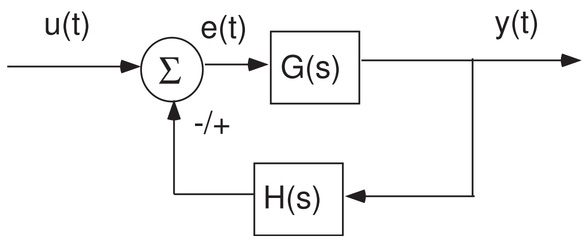
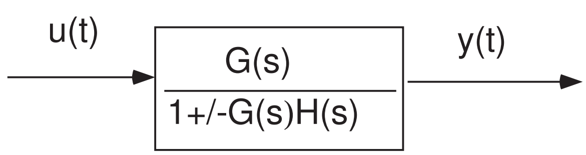


Feedback Connection

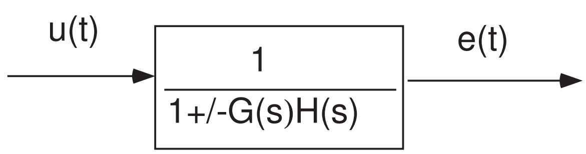
- Consider the feedback system



- *Feedback equations*: the output depends on the error, which in turn depends upon the output!
  (a) $y = Ge$
  (b) $e = u \mp Hy$
- If we use "negative feedback", and $H = 1$, then $e = y - u$
  - the input signal u is a "command" to the output signal y
  - e is the error between the command and the output
- Substituting (b) into (a) and solving for y yields



- The *error signal* satisfies



Motor Transfer Functions, I

- Four different equations that govern motor response, and their transfer functions
  - Current: Kirchoff's Laws imply

$$L\frac{dI}{dt} + RI = V - V_B$$

$$I(s) = \left(\frac{1}{sL + R}\right)(V(s) - V_B(s)) \qquad (1)$$

  - Speed: Newton's Laws imply

$$J\frac{d\Omega}{dt} = T_M - B\Omega - T_L$$

$$\Omega(s) = \left(\frac{1}{sJ + B}\right)(T_M(s) - T_L(s)) \qquad (2)$$

  - Torque:

$$T_M(s) = K_M I(s) \qquad (3)$$

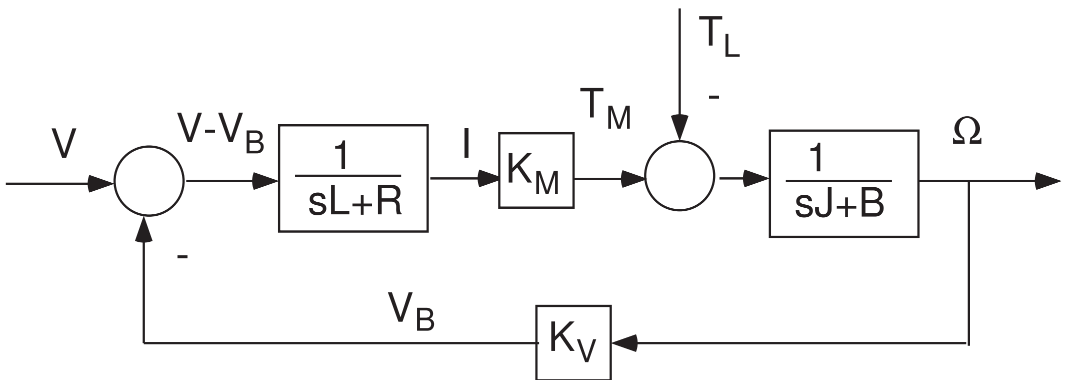
  - Back EMF:

$$V_B(s) = K_V \Omega(s) \qquad (4)$$

$\Rightarrow$ We can solve for the outputs $T_M(s)$ and $\Omega(s)$ in terms of the inputs $V(s)$ and $T_L(s)$

Motor Transfer Functions, II

- Combine (1)-(4):



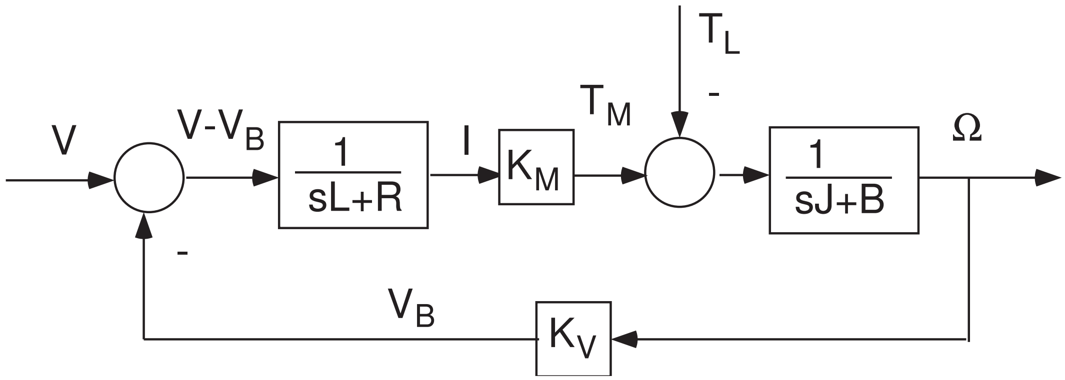
- Linear systems theory ⇒ the superposition principle holds ⇒ the response of Ω to V and T_L is equal to the *sum* of the response to V and the response to T_L.
- Transfer function from Voltage to Speed (set $T_L = 0$):
  - First combine (1)-(3)

$$\Omega(s) = \frac{K_M}{(sJ+B)}\frac{1}{(sL+R)}(V(s) - V_B(s))$$

  - Then substitute (4) and solve for $\Omega(s)$ in terms of $V(s)$:

$$\Omega(s) = \left(\frac{\frac{K_M}{(sL+R)}}{1 + \frac{K_M K_V}{(sJ+B)}\frac{1}{(sL+R)}}\right)\frac{1}{(sJ+B)}V(s) \qquad (*)$$

Motor Transfer Functions, III



- Transfer function from Voltage to Motor Torque (set $T_L = 0$):
  - First combine (1) and (3)

$$T_M(s) = \frac{K_M}{(sL+R)}(V(s) - V_B(s))$$

  - Then substitute (4) and (2) and solve for $T_M(s)$ in terms of $V(s)$:

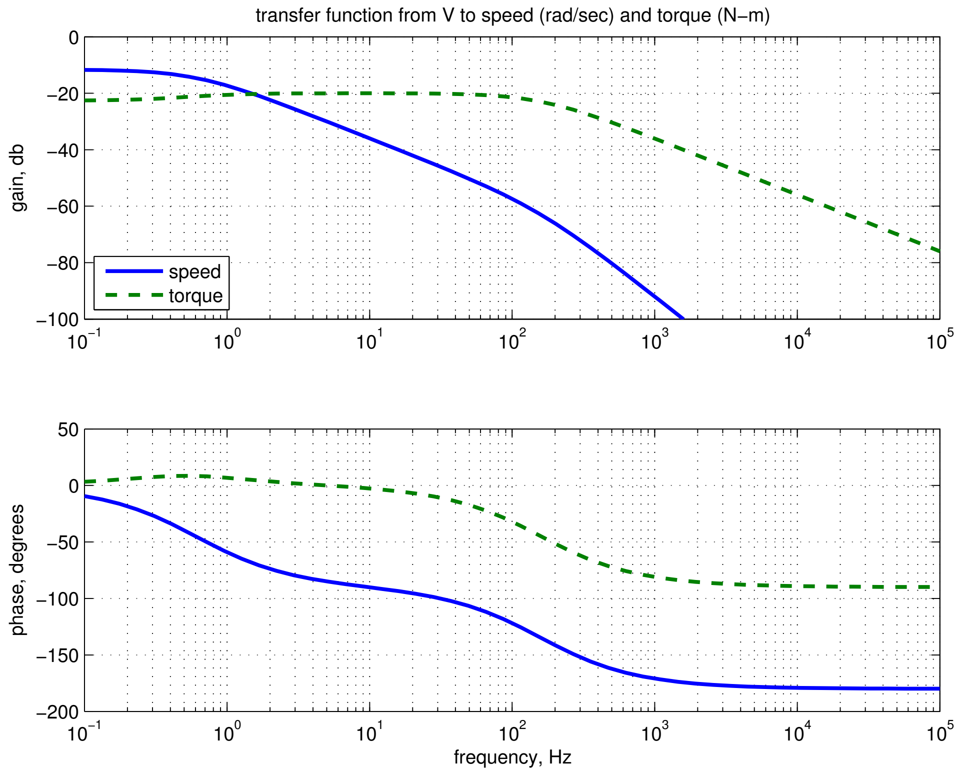
$$T_M(s) = \left(\frac{\frac{K_M}{(sL+R)}}{1 + \frac{K_M K_V}{(sJ+B)} \frac{1}{(sL+R)}} \right) V(s) \qquad (**)$$

- Comparing (*) and (**), we see that the speed response is equal to the torque response passed through a first order filter representing the mechanical motor dynamics.
- The steady state response of speed and torque to a constant voltage input V is obtained by setting $s = 0$ (cf. Lecture 5):

$$\Omega_{ss} = \frac{K_M V}{RB + K_M K_V}, \qquad T_{Mss} = \frac{K_M B V}{RB + K_M K_V}$$

Motor Frequency Response

- DC Motor is a lowpass filter[2]. Speed is filtered more than torque:

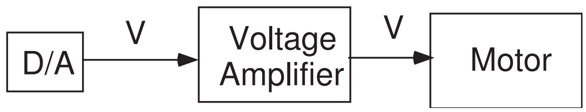


- Parameter Values
  - $K_M = 1$ N-m/A
  - $K_V = 1$ V/(rad/sec)
  - $R = 10$ ohm
  - $L = 0.01$ H
  - $J = 0.1$ N-m/(rad/sec)^2
  - $B = 0.28$ N-m/(rad/sec)
- Why is frequency response important?
  - Linear vs. PWM amplifiers . . .

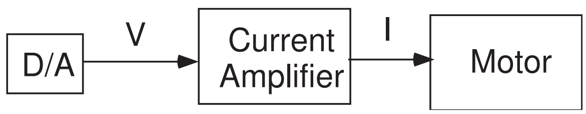
[2]Matlab m-file DC_motor_freq_response.m

Linear Power Amplifier

- Voltage amplifiers:



  - output voltage is a scaled version of the input voltage, gain measured in V/V.
  - Draws whatever current is necessary to maintain desired voltage
  - Motor speed will depend on load: $\Omega = \frac{K_M V - RT_L}{K_M K_V + RB}$
- Current (transconductance) amplifiers:



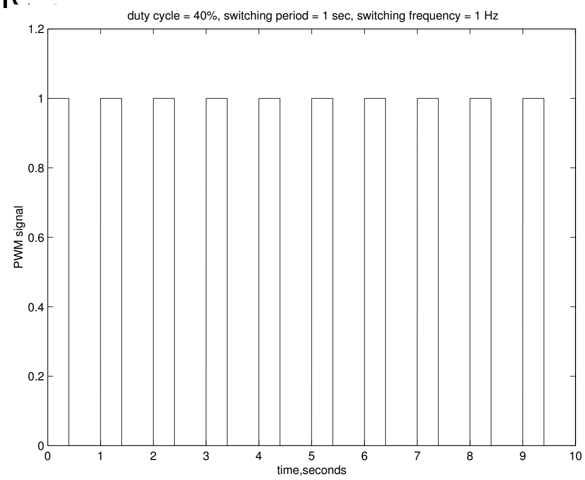
  - output current is a scaled version of the input voltage, gain measured in A/V.
  - Will produce whatever output voltage is necessary to maintain desired current
  - Motor torque will not depend on load: $T_M = K_M I$
- Advantage of linearity: Ideally, the output signal is a constant gain times the input signal, with no distortion
  - In reality, bandwidth is limited
  - Voltage and/or current saturation
- Disadvantage:
  - inefficient unless operating "full on", hence tend to consume power and generate heat.

Pulse Width Modulation

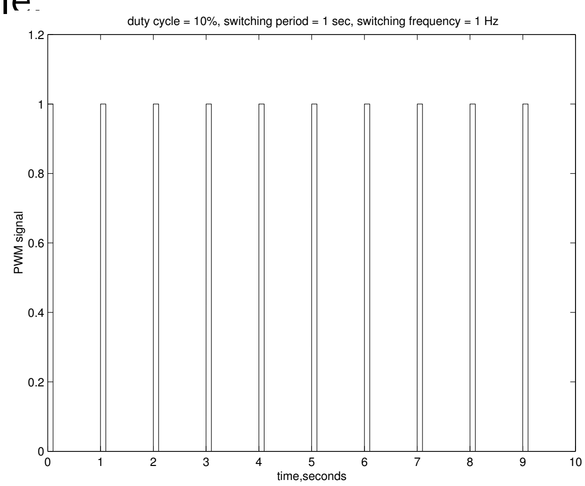
- Recall:
  - with no load, steady state motor speed is proportional to applied voltage
  - steady state motor torque is proportional to current (even with a load)
- With a D/A converter and linear amplifier, we regulate the level of applied voltage (or current) and thus regulate the speed (or torque) of the motor.
- PWM idea: Apply full scale voltage, but turn it on and off periodically
  - Speed (or torque) is (approximately) proportional to the average time that the voltage or current is on.
- PWM parameters:
  - switching period, seconds
  - switching frequency, Hz
  - duty cycle, %
- see the references plus the web page [2]

PWM Examples

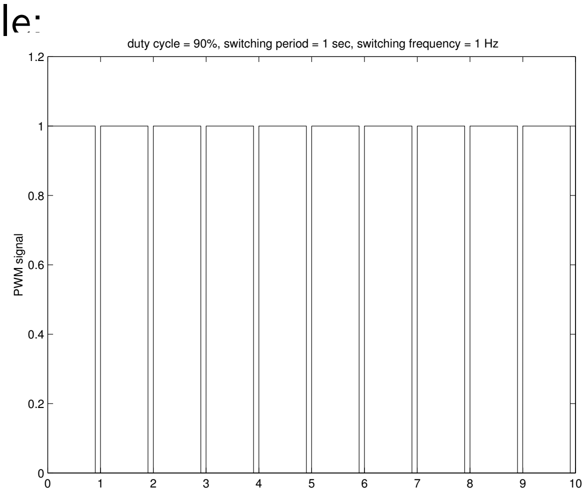
- 40% duty cycle[3]:



- 10% duty cycle:



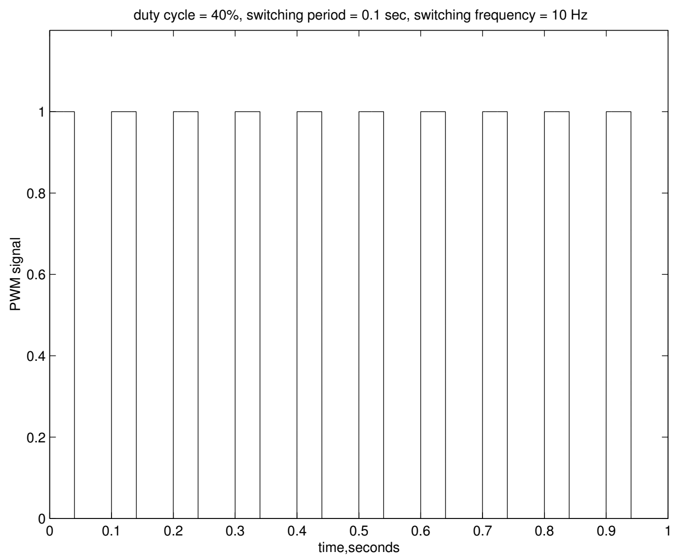
- 90% duty cycle:



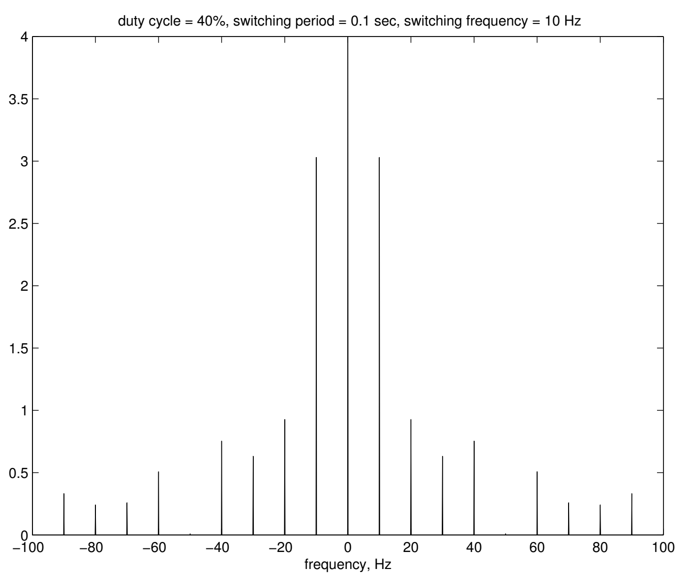
[3] Matlab files PWM_plots.m and PWM.mdl

PWM Frequency Response, I

- Frequency spectrum of a PWM signal will contain components at frequencies k/T Hz, where T is the switching period
- PWM input: switching frequency 10 Hz, duty cycle 40%[4]:



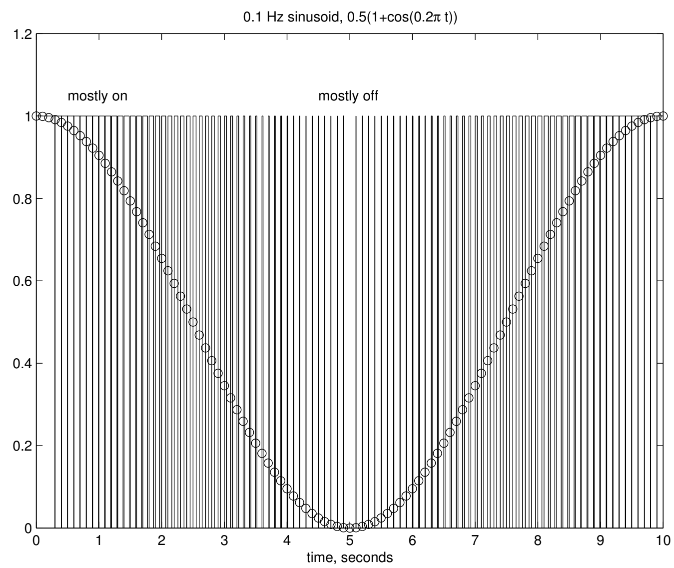
- Frequency spectrum will contain
  - a nonzero DC component (because the average is nonzero)
  - components at multiples of 10 Hz



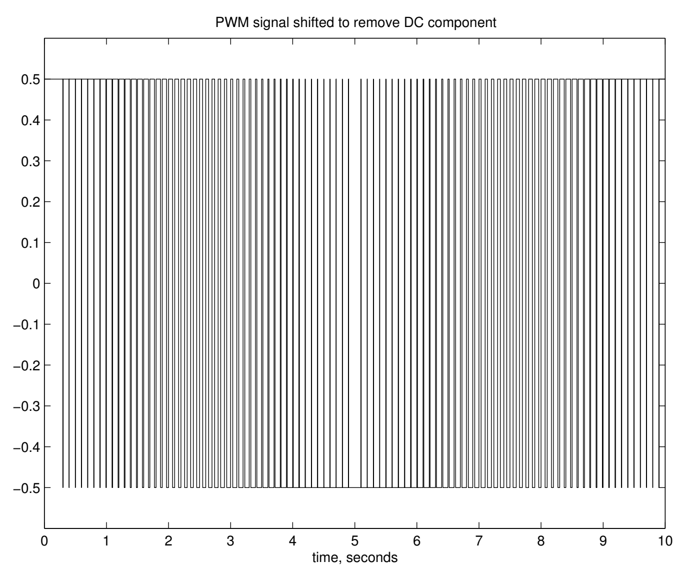
[4]Matlab files PWM_spectrum.m and PWM.mdl

PWM Frequency Response, II

- PWM signal with switching frequency 10 Hz, and duty cycle for the k'th period equal to $0.5(1+\cos(.2\pi kT))$ (a 0.1 Hz cosine shifted to lie between 0 and 1, and evaluated at the switching times $T = 0.1$ sec)[5]



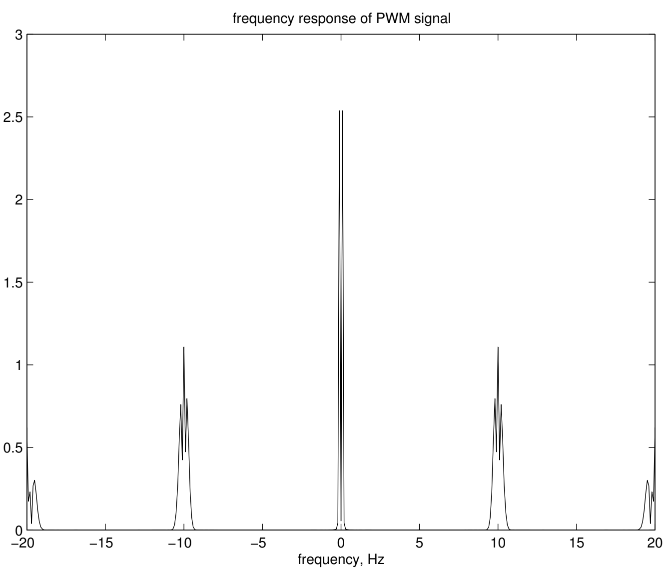
- Remove the DC term by subtracting 0.5 from the PWM signal



[5] Matlab files PWM_sinusoid.m and PWM.mdl

PWM Frequency Response, III

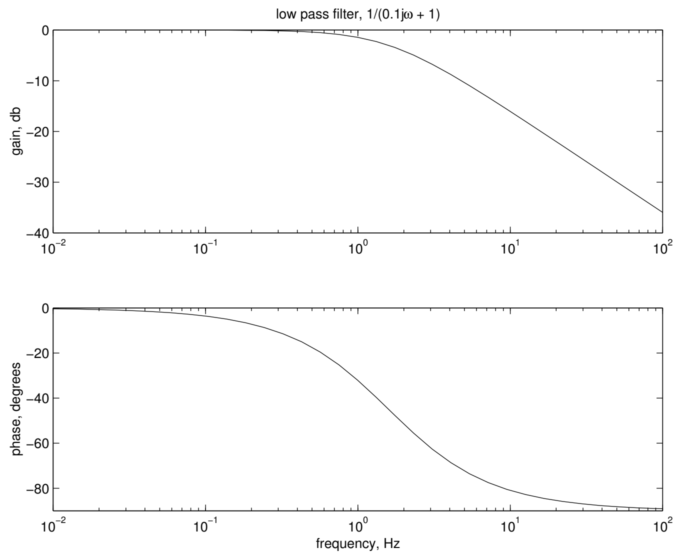
- Frequency spectrum of PWM signal has
  - zero DC component
  - components at ± 0.1 Hz
  - components at multiples of the switching frequency, 10 Hz



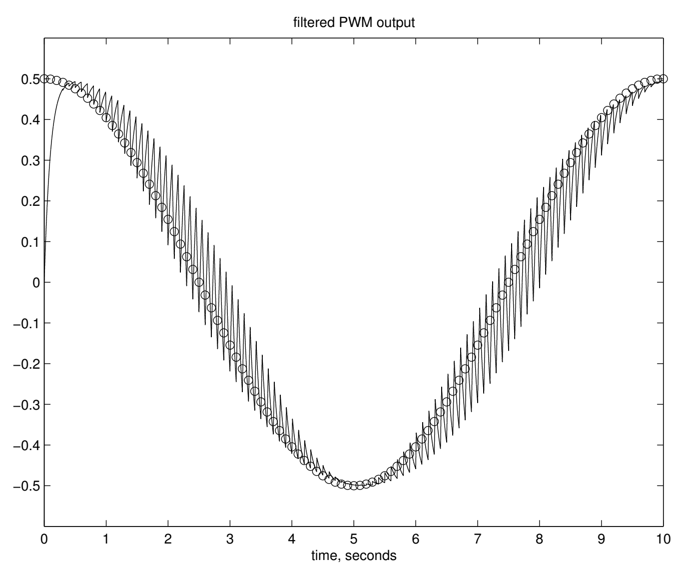
- Potential problem with PWM control:
  - High frequencies in PWM signal may produce undesirable oscillations in the motor (or whatever device is driven by the amplified PWM signal)
  - switching frequency usually set ≈ 25 kHz so that switching is not audible

PWM Frequency Response, IV

- Suppose we apply the PWM output to a lowpass filter that has unity gain at 0.1 Hz, and small gain at 10 Hz

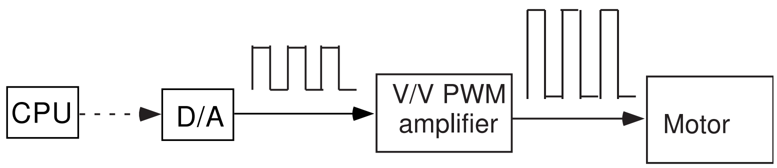


- Then, after an initial transient, the filter output has a 0.1 Hz oscillation.

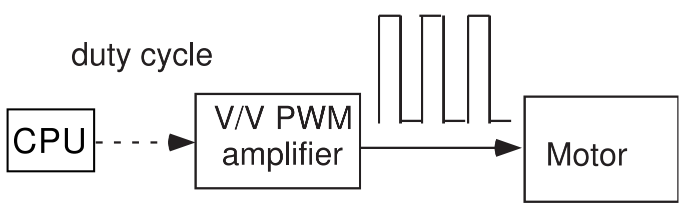


PWM Generation

- Generate PWM using D/A and pass it through a PWM amplifier



- techniques for generating analog PWM output ([6]):
  - software
  - timers
  - special modules
- Feed the digital information directly to PWM amplifier, and thus bypass the D/A stage



- PWM voltage or current amplifiers
- must determine direction
  - normalize so that
    * 50% duty cycle represents 0
    * 100% duty cycle represents full scale
    * 0% duty cycle represents negative full scale
    * what we do in lab, plus we limit duty cycle to $35\% - 65\%$
  - use full scale, but keep track of sign separately

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