Modeling and Analysis of Dynamic Systems

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Outline

Definitions: Modeling and Analysis of Dynamic Systems

Dynamic Systems

systems that are not static, i.e., their state evolves w.r.t. time, due to:

- input signals,
- external perturbations,
- or naturally.

For example, a dynamic system is a system which changes:

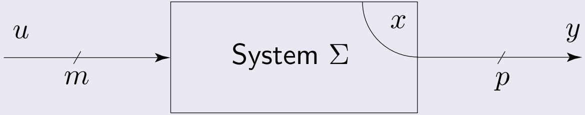
- its trajectory $\rightarrow$ changes in acceleration, orientation, velocity, position.
- its temperature, pressure, volume, mass, etc.
- its current, voltage, frequency, etc.

Examples of “Dynamic Systems”

Definition: "Modeling and Analysis"

the field of science which formulates a mathematical representation of a system:

1. for analysis/understanding (unstable, stable, observable, controllable, etc.)
2. simulation
3. control purposes.

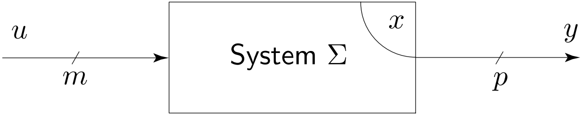


Usually, we have to deal with nonlinear time-varying system.

Nonlinear System

A system for which the output is not directly proportional to the input. Example of nonlinearities?

Definition: “Modeling and Analysis”



$$\dot{x}(t) = f(x(t), u(t), t), \qquad x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$$
$$y(t) = g(x(t), u(t), t), \qquad y(t) \in \mathbb{R}^p$$

or as a transfer function (linear time-invariant system)

$$Y(s) = \left[D + C(sI - A)^{-1}B\right]U(s), \qquad y(t) \in \mathbb{C}^p,\ u(t) \in \mathbb{C}^m$$

Model Synthesis

Types of Model

- “black-box models”: derived from experiments only
- “grey-box models” : model-based, experiments need for parameter identification, model validation
- “white-box models”: no experiments at all

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Model-based System Description

- Based on physical first principles.
- This approach has 2 major benefits (comp. to exp. methods), the models obtained:
  1. are able to *extrapolate* the system behavior (valid beyond the operating conditions used in model validation).
  2. useful, if the real system is not available (still in planning phase or too dangerous for real experiments).

Why Use Models?

1. System analysis and synthesis
2. Feedforward control systems
3. Feedback control systems

Why Use Models?

Imagine you are to design a system. Good practice in engineering is to consider:

1. System analysis

- What are the optimal system parameters (performance, safety, economy, etc.)?
- Can the system be stabilized and, if yes, what are the "best" (cost, performance, etc.) control and sensor configurations?
- What happens if a sensor or an actuator fails and how can the system's robustness be increased?

If the real system is not available for experimentation $\rightarrow$ a mathematical model must be used to answer these questions.

Why Use Models?

1. System analysis

Example: Geostationary Satellite

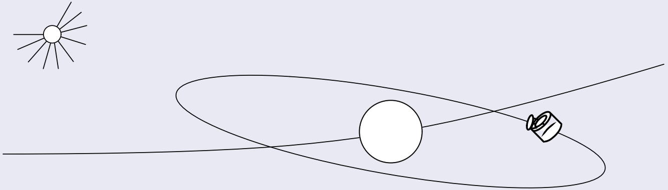


Figure: Geostationary Satellite

Constant altitude, circular orbit, constant angular velocity, despite external disturbances

→ Need for a stabilizing controller

Why Use Models?

1. System analysis

Example: Geostationary Satellite

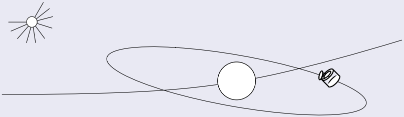


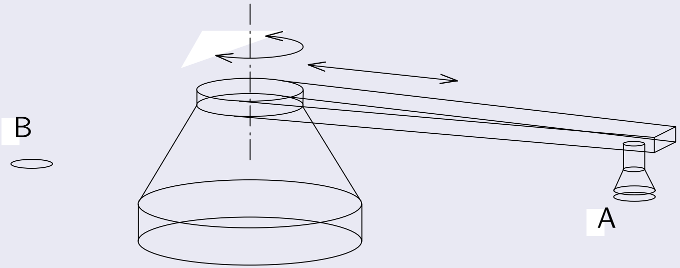
Figure: Geostationary Satellite

1. What is an optimal geometric thruster configuration?
2. Minimum thruster size? amount of fuel?
3. What kind of sensors are necessary for stabilization?
4. What happens if an actuator fails?

Why Use Models?

2. Feedforward control systems

- What are the control signals that yield optimal system behavior (shortest cycle time, lowest fuel consumption, etc.)?
- How can the system response be improved: speed, precision..?
- How much is lost when trading optimality for safety, reliability..?



Why Use Models?

3. Feedback control systems

- How can system stability be maintained for a given set of expected modeling errors?
- How can a specified disturbance rejection (robustness) be guaranteed for disturbances acting in specific frequency bands?
- What are the minimum and maximum bandwidths that a controller must attain for a specific system in order for stability and performance requirements to be guaranteed?

Why Use Models?

3. Feedback control systems

Example: Magnetic Bearing

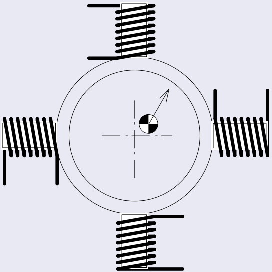


Figure: Cross section of a magnetic bearing

Scope of the lecture

Questions addressed in this lecture:

1. How are these mathematical models derived?
2. What properties of the system can be inferred from these models?

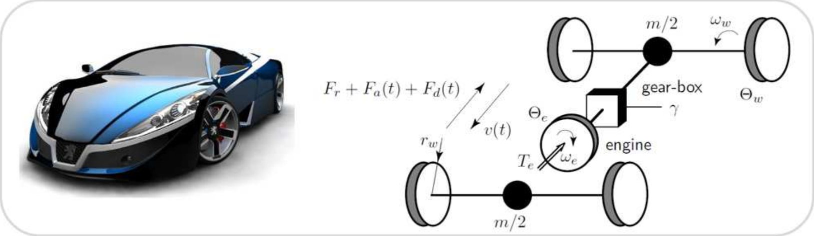
Objectives:

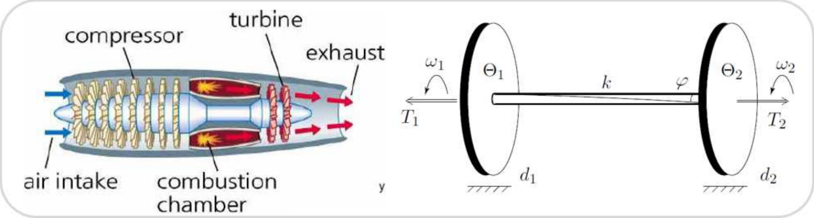
1. assemble some methods for model design in a unified way
2. suggest a methodology to formulate mathematical models (on any arbitrary system).

Keep in mind: however hard we try to model a system, it will always contain:

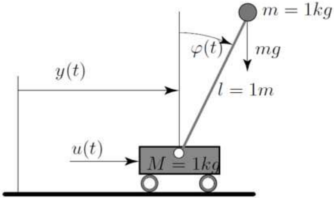
1. approximations
2. uncertainties
3. modeling or parameter errors ...

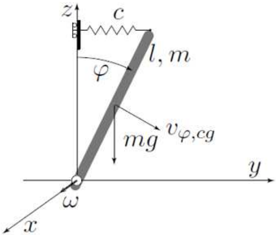
Case Studies

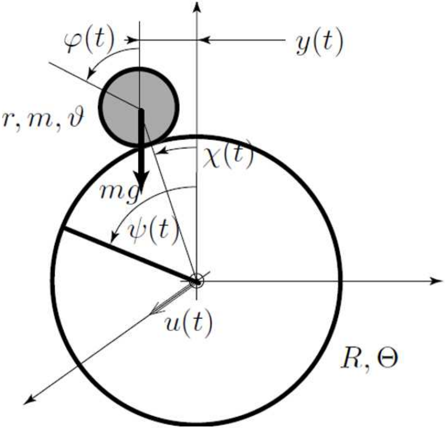




Case Studies







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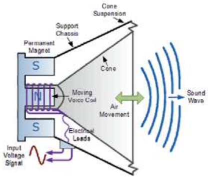

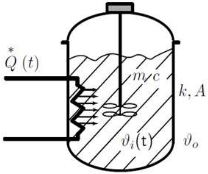

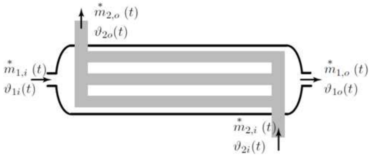

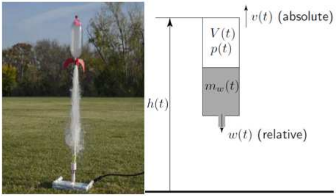

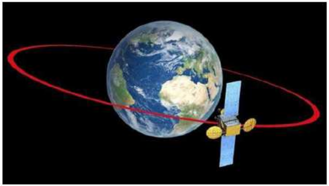

2. System Modeling for Control

Types of Modeling: Definitions

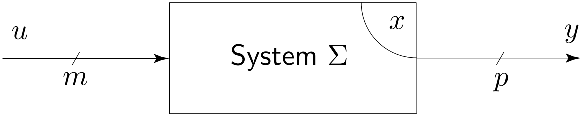

Figure: General definition of a system, input $u(t) \in \mathbb{R}^m$, output $y(t) \in \mathbb{R}^p$, internal state variable $x(t) \in \mathbb{R}^n$.

Mathematical models of dynamic systems can be subdivided into two broad classes

1. parametric models (PM)
2. non-parametric models (NPM) .

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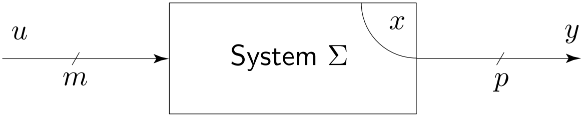

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Question:

What are the differences between these 2 classes of modeling?

Types of Modeling: Example

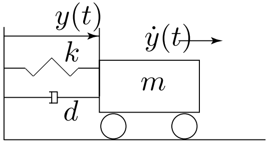


Figure: Spring mass system with viscous damping

Parametric Model

Differential equation

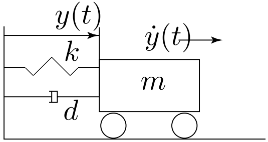
$$m\ddot{y}(t) + d\dot{y}(t) + ky(t) = F(t)$$

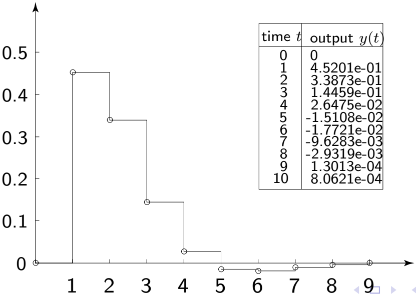
Parameters: mass: m, viscous damping: d, spring constant: k

Types of Modeling: Example

Non Parametric Model

Impulse response of a damped mechanical oscillator





time t	output $y(t)$
0	0
1	4.5201e-01
2	3.3873e-01
3	1.4459e-01
4	2.6475e-02
5	-1.5108e-02
6	-1.7721e-02
7	-9.6283e-03
8	-2.9319e-03
9	1.3013e-04
10	8.0621e-04

Types of Modeling: Discussion

Non parametric models have several drawbacks

1. they require the system to be accessible for experiments
2. they cannot predict the behavior of the system if modified
3. not useful for systematic design optimization

During this lecture, we will only consider parametric modeling.

Parametric Models

2 types:

1. "forward" (regular causality)
2. "backward" (inverted causality)

causality? causes/effects, inputs/outputs

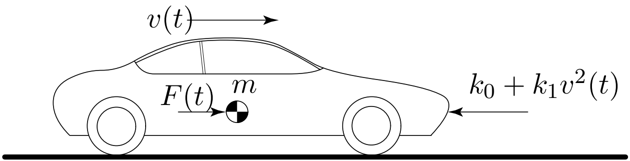


Figure: Car moving in a plane.

Parametric Models

"Forward" models

$$m\frac{d}{dt}v(t) = -\{k_0 + k_1 v(t)^2\} + F(t)$$

System input: Traction force F [N].

System output: actual fuel mass flow $\overset{*}{m}(t)$ (or its integral)

$$\overset{*}{m}(t) = \{\mu + \epsilon F(t)\}v(t) \tag{1}$$

Parametric Models

"Backward" models
Look at the speed history:

$$v(t_i) = v_i, \quad i = 1, \ldots, N, \quad t_i - t_{i-1} = \delta$$

Invert the causality chain to reconstruct the applied forces

$$F(t_i) \approx m \frac{v(t_i) - v(t_{i-1})}{\delta} + k_0 + k_1 \left(\frac{v(t_i) + v(t_{i-1})}{2} \right)^2$$

Insert resulting force $F(t_i)$ and known speed $v(t_i)$ into (1) compute the total consumed fuel:

$$\sum_{i=1}^{N} \overset{*}{m} (t_i)\delta$$

Modeling Fundamentals

- b) signals with “relevant” dynamics;
- a) signals with “fast” dynamics;
- c) signals with “slow” dynamics.

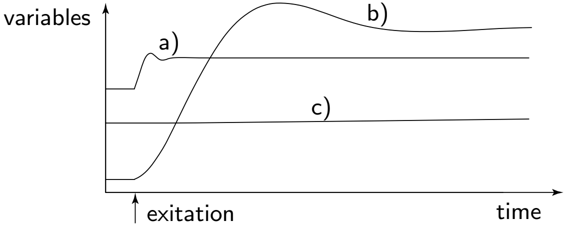


Figure: Classification of variables

When modeling any physical system: 2 main classes of objects to take into account:

1. "reservoirs," accumulative element, for ex: of thermal or kinetic energy, of mass or of information;
2. "flows," for instance of heat, mass, etc. flowing between the reservoirs.

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1. "reservoirs," accumulative element, for ex: of thermal or kinetic energy, of mass or of information;
2. "flows," for instance of heat, mass, etc. flowing between the reservoirs.

Fundamental notions

- The notion of a reservoir is fundamental: only systems including one or more reservoirs exhibit dynamic behavior.
- To each reservoir there is an associated "level" variable that is a function of the reservoir's content (in control literature: "state variable").
- The flows are typically driven by the differences in the reservoir levels. Several examples are given later.

Modeling Methodology: Reservoir-based Approach

1. define the system-boundaries (what inputs, what outputs, . . .);

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Modeling Methodology: Reservoir-based Approach

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3. formulate the differential equations (“conservation laws”) for all relevant reservoirs as shown in eq. (2)

$$\frac{d}{dt}(\text{reservoir content}) = \sum \text{inflows} - \sum \text{outflows};$$

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6. identify the unknown system parameters using some experiments;

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6. identify the unknown system parameters using some experiments;
7. validate the model with experiments that have not been used to identify the system parameters.

Example: Water Tank

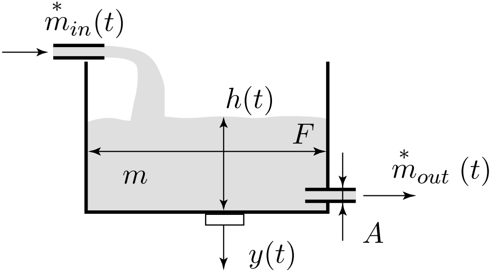

Figure: Water tank system, $m(t)$ mass of water in tank, $h(t)$ corresponding height, F tank-floor area, $A =$ out flow orifice area

Modeling the water tank

Step 1: Inputs/Outputs

- System input is: incoming mass flow $\overset{*}{m}_{in}(t)$.
- System output is: water height in the tank $h(t)$.

Modeling the water tank

Step 1: Inputs/Outputs

- System input is: incoming mass flow $\overset{*}{m}_{in}(t)$.
- System output is: water height in the tank $h(t)$.

Step 2: Reservoirs and associated levels

- One relevant "reservoir": mass of water in tank: $m(t)$.
- Level variable: height of water in tank : $h(t)$.
  Assumptions: Sensor very fast (type a) variable. Water temperature (density) very slow (type c) variable $\rightarrow$ mass and height strictly proportional.

Modeling the water tank

Step 3: Differential equation

$$\frac{d}{dt}m(t) = u(t) - \overset{*}{m}_{out}(t)$$

Modeling the water tank

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$$\frac{d}{dt}m(t) = u(t) - \overset{*}{m}_{out}(t)$$

Step 4: formulate algebraic relations of flows btw reservoirs

Mass flow leaving the tank given by Bernoulli's law

$$\overset{*}{m}_{out}(t) = A\rho v(t), \quad v(t) = \sqrt{2\Delta p(t)/\rho}, \quad \Delta p(t) = \rho g h(t)$$

therefore

$$\frac{d}{dt}m(t) = \rho F \frac{d}{dt}h(t) = u(t) - A\rho\sqrt{2gh(t)}$$

Modeling the water tank

Causality Diagrams

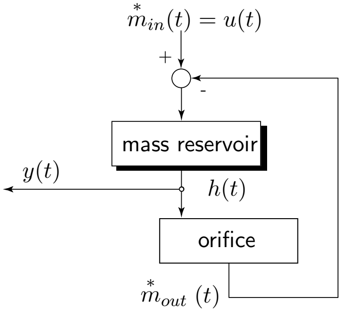

Figure: Causality diagram of the water tank system, shaded blocks represent dynamical subsystems (containing reservoirs), plain blocks represent static subsystems.