Outline

1 Introduction to the Pelton Turbine
   - Invention
   - Modern Pelton Wheels

2 Hydroelectric Power Plant Using Pelton Turbines
   - Most Powerful Dam using Pelton Turbines
   - Key Figures
   - Pelton Turbine: presentation
   - Pelton Turbine: modeling

3 Electromagnetic Systems
   - Recalls
   - Electric Oscillator

4 Electromechanical Systems
   - Permanently Excited DC Motor: introduction
   - Modeling of a DC Motor
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Lester Allan Pelton

Born: 5 Sept. 1829, Ohio, USA
Passed away: in 1908 (78 years old), Oakland, California, USA
Inventor: in field of hydropower and hydroelectric power
Pelton Wheel: Patent in Oct. 1880
Original Pelton Wheel (Efficiency: about 80%)

1st commercial sample, built by Miners Foundry, Nevada City, 1880.
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Modern Pelton Wheel (Efficiency: More than 90 %)

Old Pelton wheel from Walchensee Hydroelectric Power Station, Germany.
Assembly of a Pelton wheel at Walchensee Hydroelectric Power Station, Germany.
Modern Pelton Turbine with 6 water injectors
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Where is located the hydro-electric power plant holding 3 world records?

- highest altitude difference for the water fall: 1880m
- most powerful Pelton turbines (400 MW/turbine)
- highest power per electrical generator pole (37.5 MVA)
Barrage de la Grande-Dixence, Lac des Dix, Bieudron, Valais, CH

http://www.myswitzerland.com/fr-ch/grande-dixence.html
Barrage de la Grande-Dixence, Lac des Dix, Bieudron, Valais, CH, 2364m
Barrage de la Grande-Dixence, Lac des Dix, Bieudron, Valais, CH, 2364m
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Introduction to the Pelton Turbine
Hydroelectric Power Plant Using Pelton Turbines
Electromagnetic Systems
Electromechanical Systems

Most Powerful Dam using Pelton Turbines

Key Figures

- Pelton Turbine: presentation
- Pelton Turbine: modeling

Barrage de la Grande-Dixence, Lac des Dix, Bieudron, Valais, CH, 2364m

Key figures:

- Height : 285 m
- Width (top): 15 m
- Width (bottom): 195 m
- Lake length: 5 km
- Construction start: 3 Aug. 1953
- Operation start: 22 Sept. 1961
- Nb of Pelton turbines: 3
- Total power of the installation : 2000 MW
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Example: Pelton Turbine in a Hydro-electric Power Plant
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Questions: (answered during the class on the board)

1. What is the mean force acting on the Pelton turbine? \( F_T = f(w, \omega, V^*) \)?
2. What is the resulting wheel torque: \( T_T = g(w, \omega, V^*) \)?
3. What is the power being transferred from the fluid to the turbine?
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**Introduction**

Electromagnetic systems often can be formulated as RLC networks:

- resistances (R)
- inductances (L)
- capacitances (C)

**Two classes of reservoir elements are important:**

- **magnetic energy**: stored in magnetic fields \( B \); and
- **electric energy**: stored in electric fields \( E \).
### Mathematical modeling

<table>
<thead>
<tr>
<th>Element</th>
<th>Capacitance</th>
<th>Inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy</strong></td>
<td>$W_E = \frac{1}{2} C \cdot U^2(t)$</td>
<td>$W_M = \frac{1}{2} L \cdot I^2(t)$</td>
</tr>
<tr>
<td><strong>Level variable</strong></td>
<td>$U(t)$ (voltage)</td>
<td>$I(t)$ (current)</td>
</tr>
<tr>
<td><strong>Conservation law</strong></td>
<td>$C \cdot \frac{d}{dt} U(t) = I(t)$</td>
<td>$L \cdot \frac{d}{dt} I(t) = U(t)$</td>
</tr>
</tbody>
</table>

The electrical power is $P(t) = U(t) \cdot I(t)$. 
Working with RLC networks, two rules ("Kirchhoff’s laws") are useful:

1: The algebraic sum of all currents in each network node is zero.

2: The algebraic sum of all voltages following a closed network loop is zero.

These rules are equivalent to the energy balance and are usually more convenient to use.
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Step 1: Inputs and Outputs

1. Input: \( u(t) \), input voltage
2. Output: \( y(t) \), output voltage

Step 2: Energy reservoirs

1. Magnetic energy in \( L \)
2. Electric energy in \( C \)

Step 3: "Equivalent energy balance": Kirchhoff rule

\[
U_L(t) + U_R(t) + U_C(t) = u(t)
\]
Step 4: Differential equations

- “C” and ”L law”:
  \[
  U_L(t) = L \cdot \frac{d}{dt} I(t), \quad I(t) = C \cdot \frac{d}{dt} U_C(t)
  \]

- and Ohm’s law:
  \[
  U_R(t) = R \cdot I(t)
  \]

Step 5: Reformulation and result

- Definitions: \( y(t) = U_C(t), \; I(t) = \frac{d}{dt} Q(t) \)

- Reformulation: \( I(t) = C \cdot \frac{d}{dt} y(t), \; \frac{d}{dt} I(t) = C \cdot \frac{d^2}{dt^2} y(t) \)

- Result: \( L \cdot C \cdot \frac{d^2}{dt^2} y(t) + R \cdot C \cdot \frac{d}{dt} y(t) + y(t) = u(t) \)
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Most electric motors used in control loops are rotational. Classification: according to the commutation mechanisms used:

1. **Classical DC drives** have a mechanical commutation of the current in the rotor coils and constant (permanent magnet) or time-varying stator fields (external excitation).

2. Modern **brushless DC drives** have an electronic commutation of the stator current and permanent magnet on the rotor (i.e., no brushes).

3. **AC drives** have an electronic commutation of the stator current and use self-inductance to build up the rotor fields.
DC motor

Figure: Principle of a DC motor (picture from Wikipedia)
DC Motor Principle

\[ B = \mu_0 \frac{N}{l} I \]

North Magnet

South Magnet
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Step 1: Input & Output

- System input voltage $u(t)$ (control input) and load torque $T_l(t)$ (disturbance).
- System output measurement of rotor speed $\omega(t)$.

Remarks:
- The motor is permanently excited, parameters $\kappa$ in the motor and generator laws are constant.
- The mechanical part has friction losses.
Step 2: relevant reservoirs

- the magnetic energy stored in the rotor coil, level variable $I(t)$;
- the kinetic energy stored in the rotor, level variable $\omega(t)$.

Step 3: two energy conservation laws

$$L \cdot \frac{d}{dt} I(t) = -R \cdot I(t) - U_{\text{ind}}(t) + u(t)$$

$$\Theta \cdot \frac{d}{dt} \omega(t) = T_m(t) - T_i(t) - d \cdot \omega(t)$$
Step 4: generator and the motor laws

\[ P_{\text{elec}} = P_{\text{mech}} \]

\[ U_{\text{ind}}(t) \cdot I(t) = \kappa \cdot \omega(t) \cdot I(t) = T_m(t) \cdot \omega(t) \]  \( (2) \)

\[ \rightarrow T_m(t) = \kappa \cdot I(t) \]

Inserting equation (2) into equation (1),

\[ L \cdot \frac{d}{dt} I(t) = -R \cdot I(t) - \kappa \cdot \omega(t) + u(t) \]  \( (3) \)

\[ \Theta \cdot \frac{d}{dt} \omega(t) = \kappa \cdot I(t) - T_l(t) - d \cdot \omega(t) \]
Upcoming Exercise

Next exercises: **Hydro Power Plant**