Stochastic programming and multi-horizon modeling with applications to hydro power planning

Hubert Abgottspon

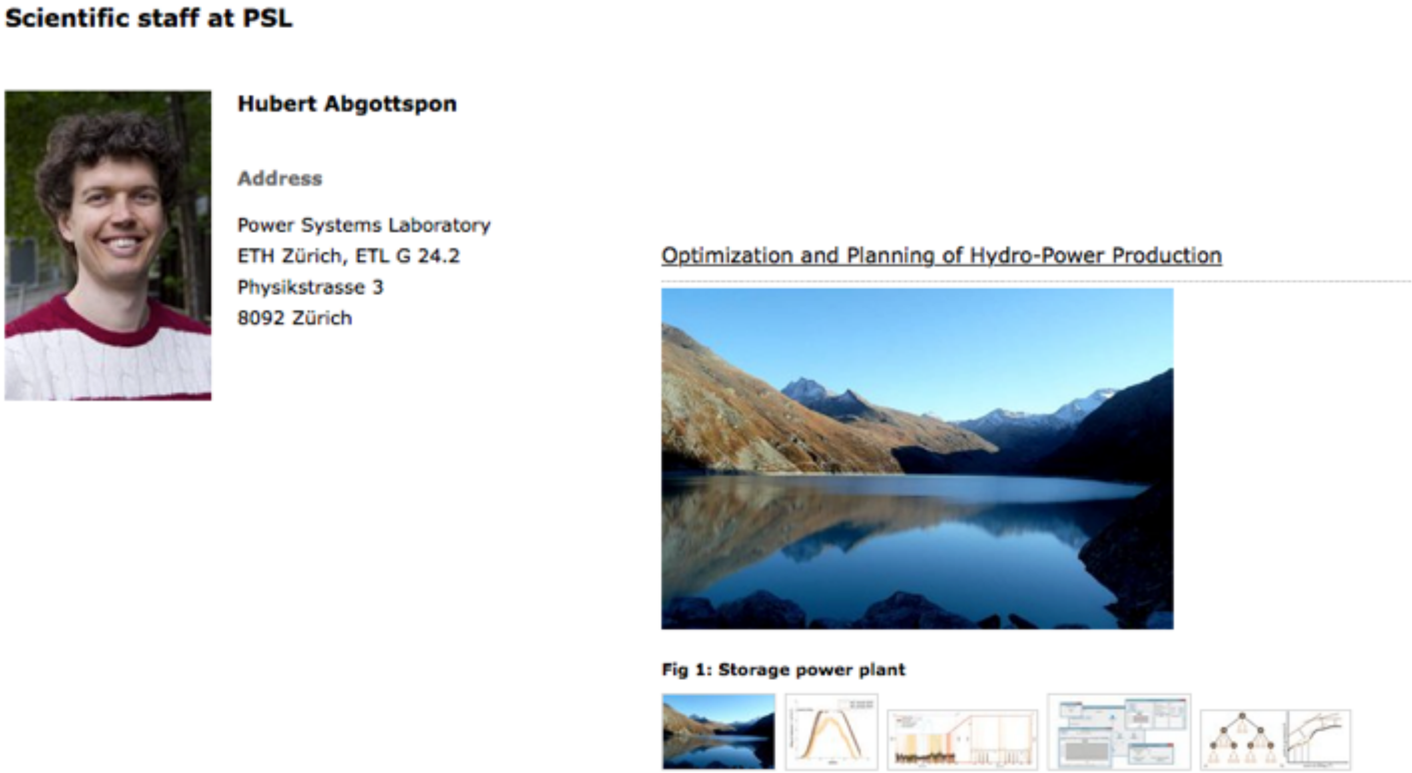
Power Systems Laboratory ETH Zürich

- Staff
  - 1 Professor: Göran Andersson
  - 6 Senior Researchers
  - 12 PhD Students
  - 1 Secretary
  - 4 External Lecturers

- Research Areas
  - Power System Dynamics and Control
  - Future Energy Systems and Networks
  - Energy and Power Markets

- Focus on development of
  - Models
  - Methods
  - Analysis Tools

About me



Scientific staff at PSL

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Optimization and Planning of Hydro-Power Production

Fig 1: Storage power plant

Outline

1. Hydro scheduling in research
2. Hydro power in Switzerland
3. Optimal hydro operation scheduling
4. Stochastic optimization
5. Examples
6. Multi-horizon decision trees

1. Hydro scheduling in research

Hydro scheduling in research

Stochastic programs:
- 50’s and early 60’s: introduction of uncertainty in math. programs

Uncertainty in hydro power:
- Ideas: 1946 (P. Massé), 1955 (J. Little)

Dynamic programming:
- Applied for hydro: 1967 (G. Young)

Stochastic dynamic programming:
- Applied for hydro and extended in 70’s and early 80’s
- review: 1982

Stochastic dual dynamic programming:
- approximate dynamic programming: since 50’s, very active in 90’s
- SDDP (1985 J. Birge, 1989/1991 M. Pereira)
- mathematically: convergence, statistical properties etc.

Hydro scheduling in research

Current research:

- Brazil, Norway, USA, France, New Zealand, Austria/Germany, (China)
- Improve NEWAVE, EMPS/EOPS
- Risk management (NEWAVE)
- Scenario tree generation
- Price-maker, bidding, head dependencies
- Approximate DP for distributed storage

Issues:

- multi-reservoir (> 10) multi-period (> 10) optimization
- consideration of different markets
- modeling: more meaningful, more efficient

2. Hydro power in Switzerland

- some numbers
- how to operate them
- electricity markets

Hydro power in Switzerland

Stromhandel Schweiz - EU

Werbetour für Wasserkraft

4.3.2015, 21:17 Uhr

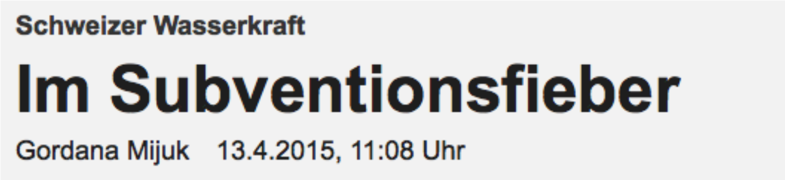
Energiestrategie 2050

Mehr Gewicht für die Wasserkraft

Christof Forster, Bern 3.4.2015, 18:19 Uhr

Die Schweizer Grosswasserkraft ist akut bedroht

David Thiel, IWB 13.3.2015, 05:30 Uhr



Schweizer Wasserkraft

Im Subventionsfieber

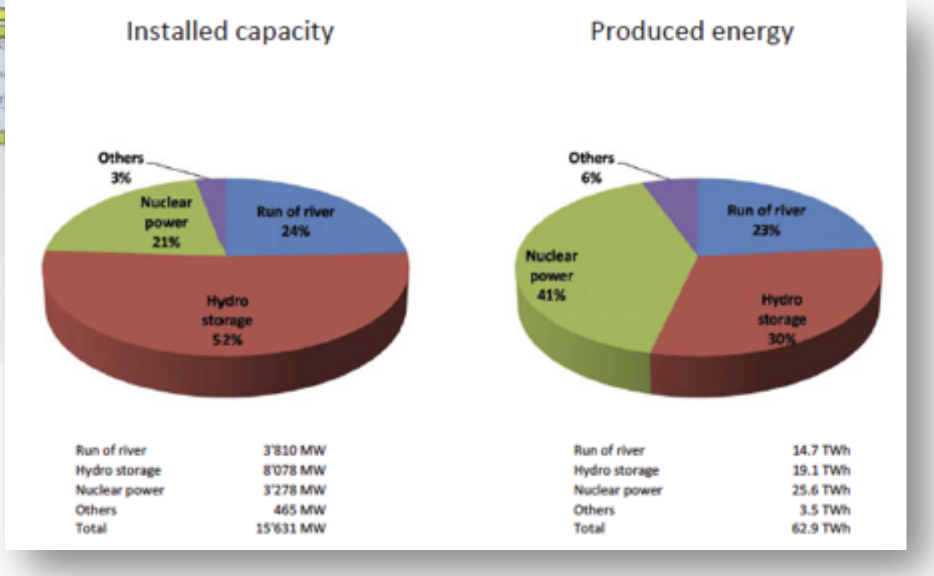
Gordana Mijuk 13.4.2015, 11:08 Uhr

Energiestrategie 2050

Neue Wasserkraft-Abgabe für Stromkunden stösst auf Kritik

Davide Scruzzi 25.3.2015, 05:30 Uhr

Hydro power in Switzerland


Installed capacity

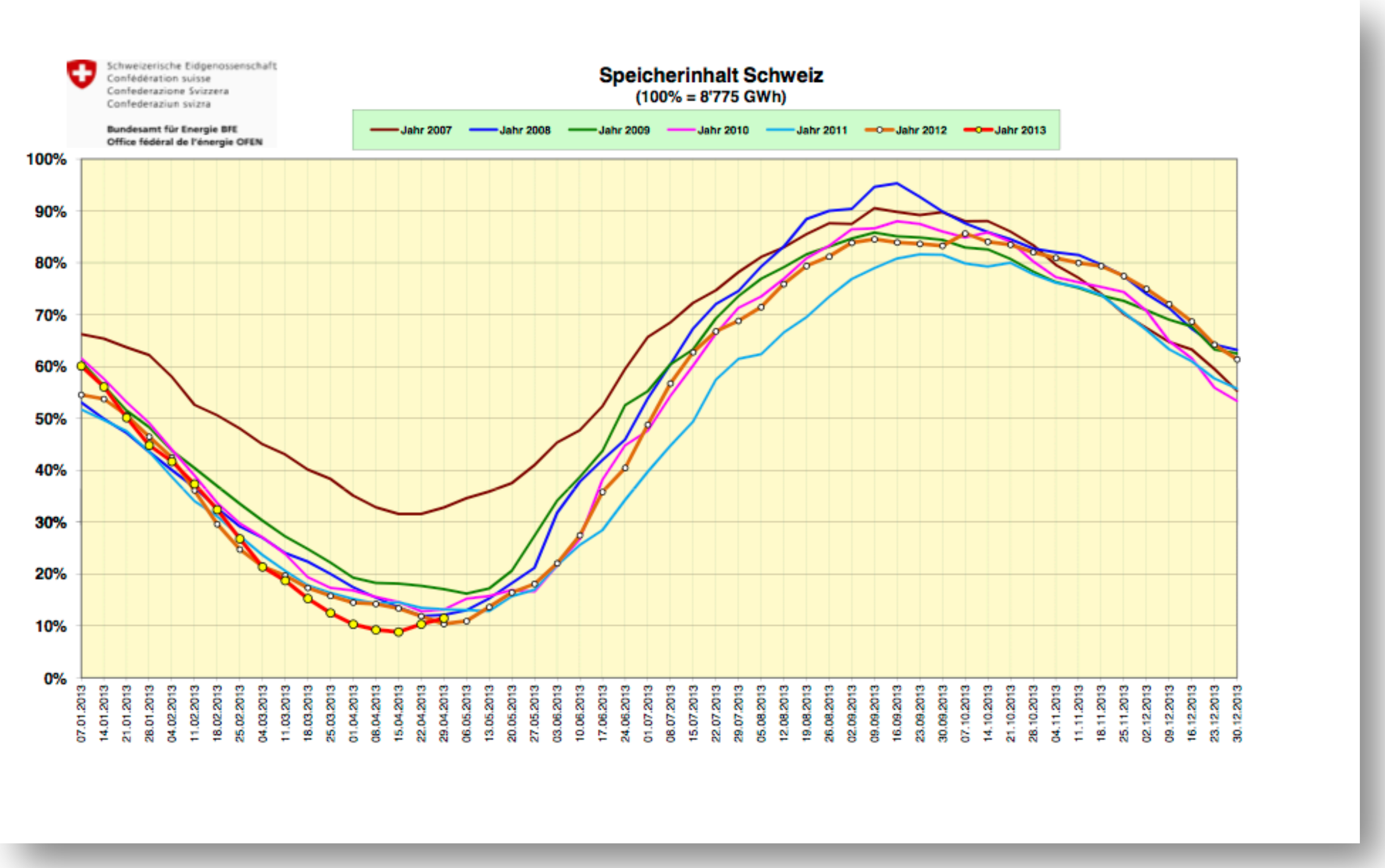
Produced energy

Run of river	3'810 MW
Hydro storage	8'078 MW
Nuclear power	3'278 MW
Others	465 MW
Total	15'631 MW

Run of river	14.7 TWh
Hydro storage	19.1 TWh
Nuclear power	25.6 TWh
Others	3.5 TWh
Total	62.9 TWh

Hydro power in Switzerland: (2/6)

seasonal operation



Hydro power in Switzerland: (3/6)

How to operate them?

- When produce how much?
- Produce now or use the water later?
- Costs? Marginal production costs?

Usual approach:

- Asset management group:
  - Hydro plant valuation
  - Find “most optimal” operation over time
- Trading group
  - “buy” plant from asset management group
  - Don’t deviate “too much” from optimal operation

Hydro power in Switzerland: (4/6)

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Hydro power in Switzerland: (5/6)

Electricity markets

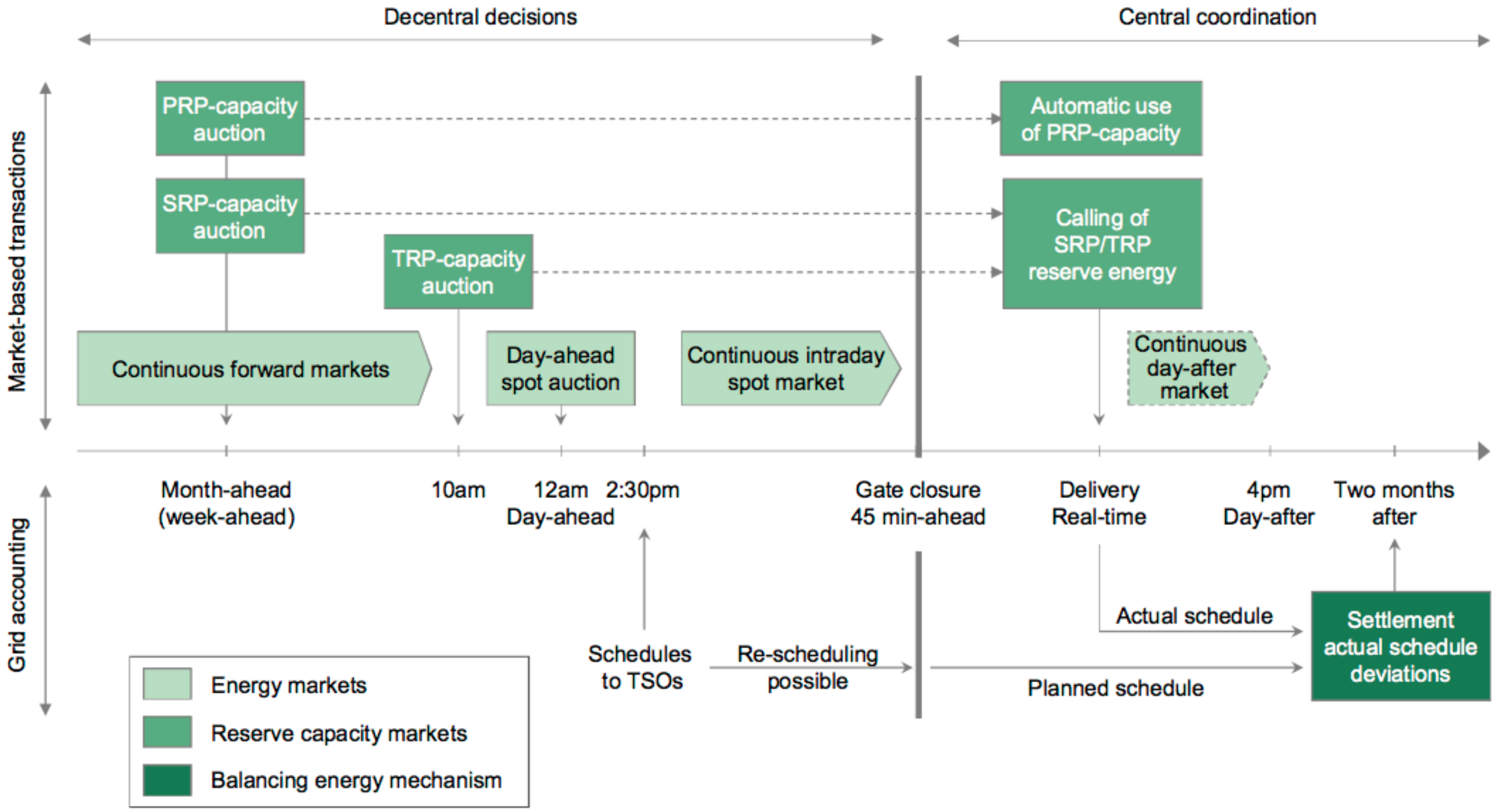


FIGURE 1: SEQUENCE OF ELECTRICITY MARKETS IN GERMANY

Hydro power in Switzerland: (6/6)

Electricity markets

EPEX SPOT
EUROPEAN POWER EXCHANGE

- Spot market:
- intraday and day-ahead
- Single hours, base, peak
- Germany, France, Switzerland, Austria

eex

- Futures & Options
- German and French futures, with physical and financial settlement
- Weekly, monthly, quarterly, yearly, and base, peak, off-peak
- Options on German futures
- CO2 certificates, gas

- However, around 60% of all exchanges in Switzerland are still OTC

3. Optimal hydro power scheduling

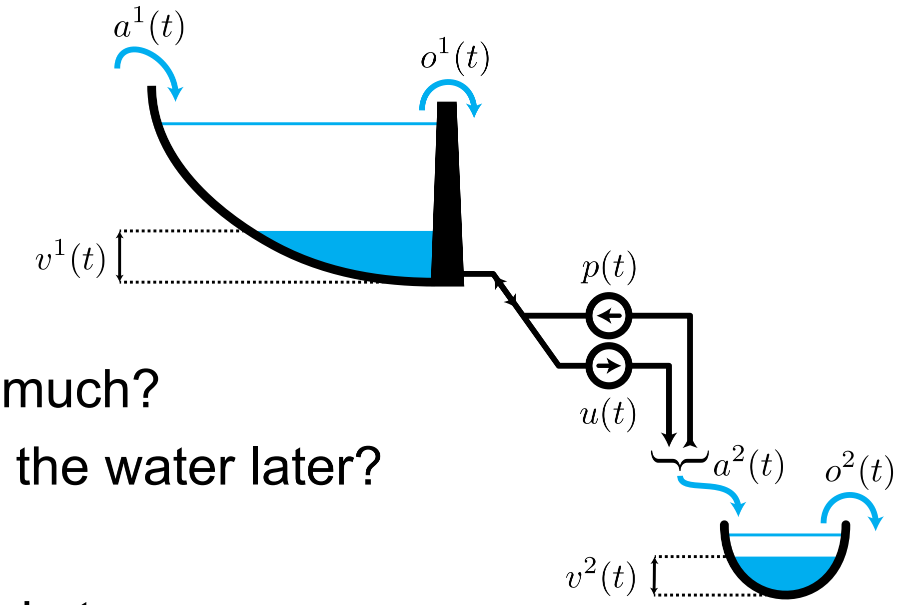
Optimal hydro operation scheduling

- No marginal production costs
- Since energy storage: produce only for “high prices”
  - -> opportunity costs

- Idea: simulate optimal power plant operation
  - -> opportunity costs
  - -> asset valuation

Optimal hydro operation scheduling: (2/3)
Model

- Parameters:
  - reservoirs
  - pump/turbines capacity, efficiencies
- Water inflows (forecast, uncertain)



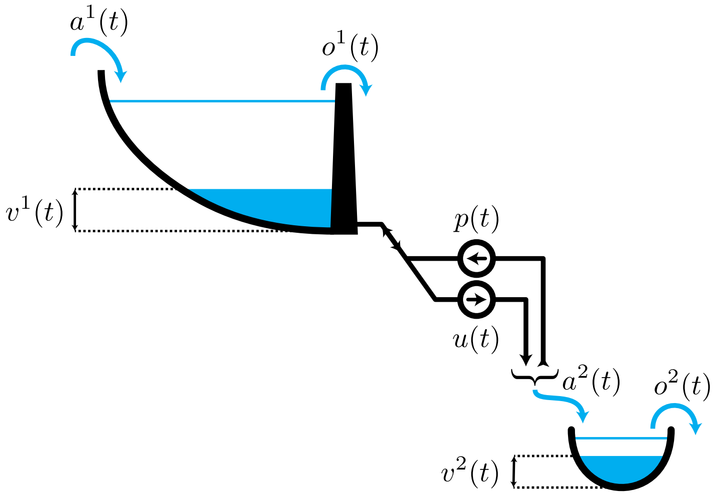
- When produce how much?
- Produce now or use the water later?
- prices?
  - -> Electricity markets

Optimal hydro operation scheduling: (3/3)
Model

- Parameters:
  - reservoirs
  - pump/turbines capacity, efficiencies
- Water inflows (forecast, uncertain)
- Market prices (forecast, uncertain)

Optimal operation:

- Stochastic optimization



Small recap:

- Hydro power in Switzerland:
  - important
  - seasonal operation
  - optimal operation is sought
  - electricity markets -> market prices

- Optimal hydro power scheduling:
  - power plant data
  - forecast of stochastic water inflows
  - forecast of stochastic market prices
  - -> Stochastic optimization

4. Stochastic Optimization

- model
- solver

Stochastic Optimization

“optimization under uncertainty”

Two important aspects:

- model
- solver

Stochastic Optimization: (2/10)

Model

Mathematical programming:

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ \text{and} & \mathbf{x} \geq \mathbf{0} \end{array}$$

No common language in stochastic programming...

Stochastic Optimization: (3/10)

Model

Especially important:

- disclosure and “flow” of information

Two-stage stochastic program:

decision, realization of random data, recourse decision

$$x, \xi, y$$

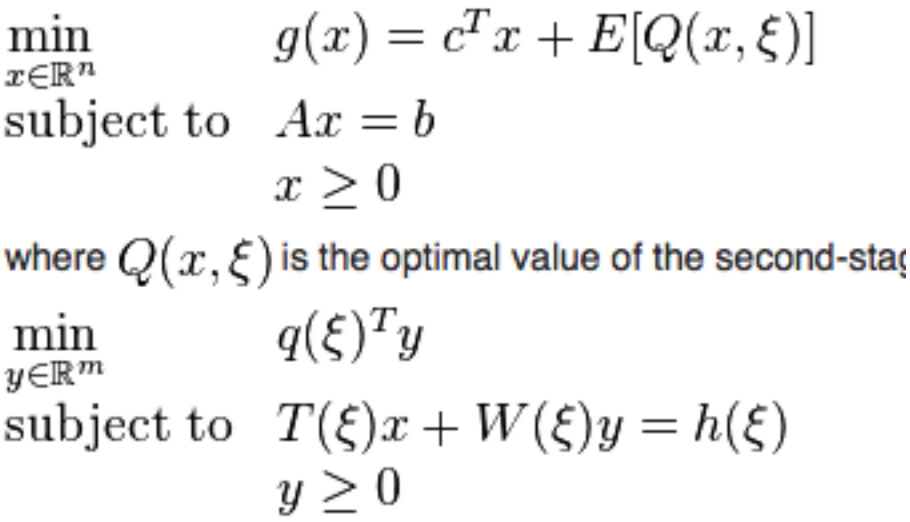
Multi-stage stochastic program:

$$x_1, \xi_1, x_2, \ldots, \xi_{T-1}, x_T$$

Stochastic Optimization: (4/10)

Model

Classical two-stage problem formulation:



$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & g(x) = c^T x + E[Q(x,\xi)] \\ \text{subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

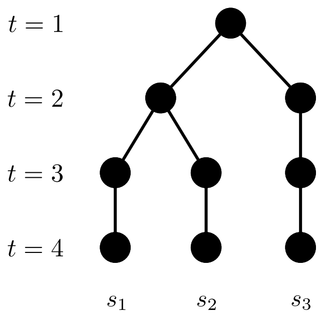
where $Q(x,\xi)$ is the optimal value of the second-stag

$$\begin{aligned} \min_{y \in \mathbb{R}^m} \quad & q(\xi)^T y \\ \text{subject to} \quad & T(\xi)x + W(\xi)y = h(\xi) \\ & y \geq 0 \end{aligned}$$

Stochastic Optimization: (5/10)

Model

Scenario tree:



$\mathcal{A}_1 = \{s_1, s_2, s_3\}$

$\mathcal{A}_2 = \{s_1, s_2\}, \mathcal{B}_2 = \{s_3\}$

$\mathcal{A}_3 = \{s_1\}, \mathcal{B}_3 = \{s_2\}, \mathcal{C}_3 = \{s_3\}$

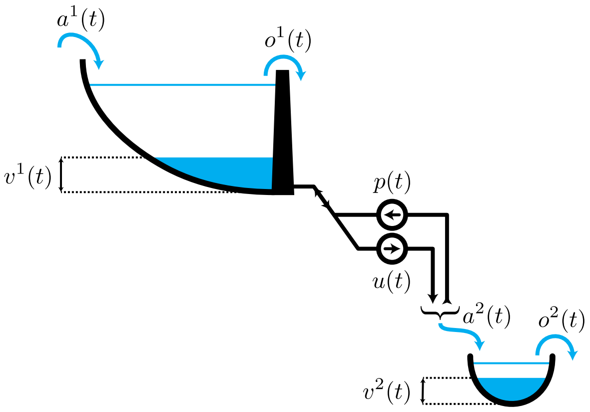
$\mathcal{A}_4 = \{s_1\}, \mathcal{B}_4 = \{s_2\}, \mathcal{C}_4 = \{s_3\}$

$\mathcal{S} = \{s_1, s_2, s_3\}$

Multi-stage stochastic program applied to hydro scheduling as its deterministic equivalent:

$$\max \; c_1(u_1 - p_1) + \sum_{t=2}^{T} \sum_{\mathcal{A}_t \in \Lambda_t} \rho_{\mathcal{A}_t} c_t(u_{t,\mathcal{A}_t} - p_{t,\mathcal{A}_t}) \tag{3.1}$$

$$\text{s.t.:} \begin{cases} v^1_{t,\mathcal{B}_t} = v^1_{t-1,\mathcal{A}_{t-1}} - u_{t-1,\mathcal{A}_{t-1}} + p_{t-1,\mathcal{A}_{t-1}} \\ \qquad - o^1_{t-1,\mathcal{A}_{t-1}} + a^1_{t-1,\mathcal{A}_{t-1}} \forall t \in 2,\ldots,T, \forall \mathcal{A}_{t-1} \in \Lambda_{t-1}, \forall \mathcal{B}_t \in U(\mathcal{A}_{t-1}) \\ v^2_{t,\mathcal{B}_t} = v^2_{t-1,\mathcal{A}_{t-1}} - o^2_{t-1,\mathcal{A}_{t-1}} + a^2_{t-1,\mathcal{A}_{t-1}} \\ \qquad \forall t \in 2,\ldots,T, \forall \mathcal{A}_{t-1} \in \Lambda_{t-1}, \forall \mathcal{B}_t \in U(\mathcal{A}_{t-1}) \\ \mathrm{lb}_t \leq u,v,o,a \leq \mathrm{ub}_t \\ u,v,o,a \in \mathbb{R} \end{cases}$$



Stochastic Optimization: (6/10)

Solver

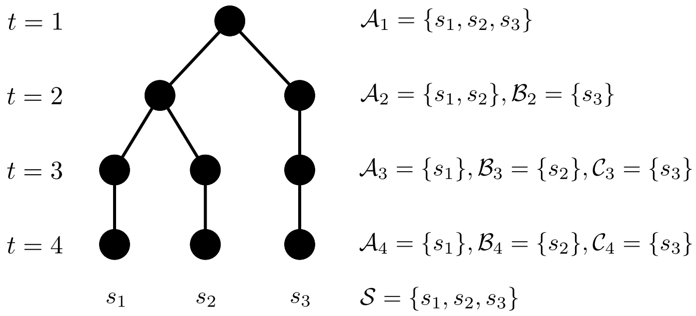
How to get to a solution? -> Solver

For linear (or even convex) problems: “good” solvers available

Problem with such a formulation:

“curse of dimensionality”

-> exponential growth



Hydro scheduling:

- hourly market prices
- time horizon a few years

  -> more than 10000 time steps. (2^10000 = 2*10^3010)

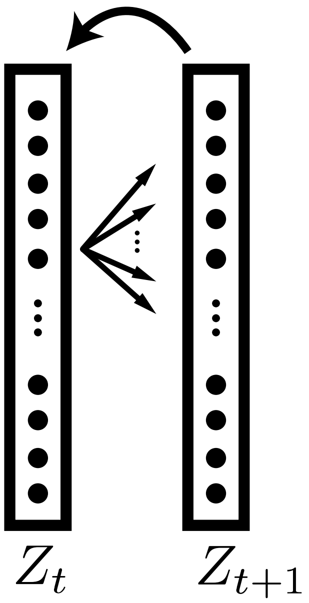
  -> solver?

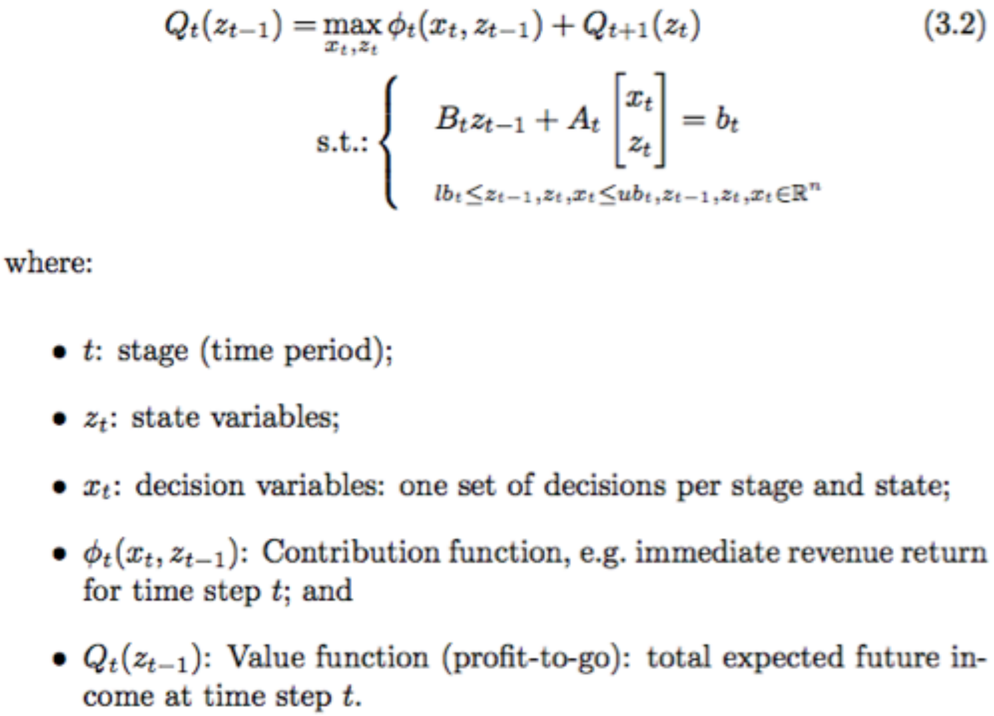
Stochastic Optimization: (7/10)

Solver

Second option: Dynamic programming:

“break down problem into simpler subproblems”





$$Q_t(z_{t-1}) = \max_{x_t, z_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \tag{3.2}$$

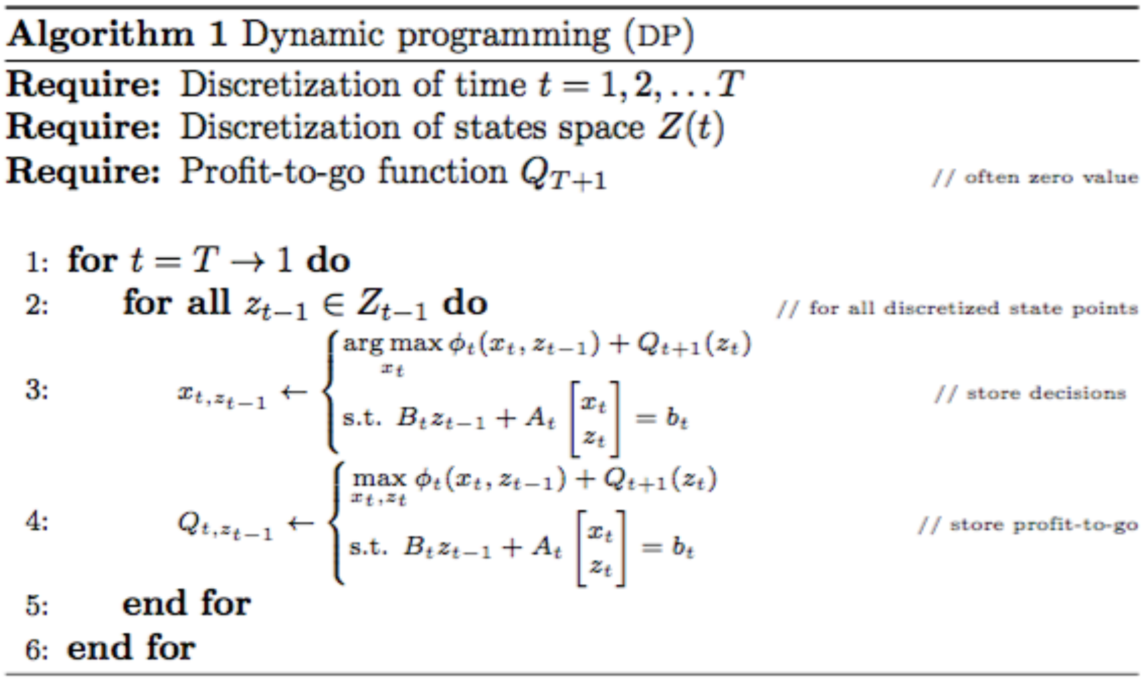
$$\text{s.t.:} \begin{cases} B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \\ lb_t \leq z_{t-1}, z_t, x_t \leq ub_t, z_{t-1}, z_t, x_t \in \mathbb{R}^n \end{cases}$$

where:

- t: stage (time period);
- z_t: state variables;
- x_t: decision variables: one set of decisions per stage and state;
- $\phi_t(x_t, z_{t-1})$: Contribution function, e.g. immediate revenue return for time step t; and
- $Q_t(z_{t-1})$: Value function (profit-to-go): total expected future income at time step t.

Stochastic Optimization: (8/10)
Solver

Dynamic programming solver:



Algorithm 1 Dynamic programming (DP)

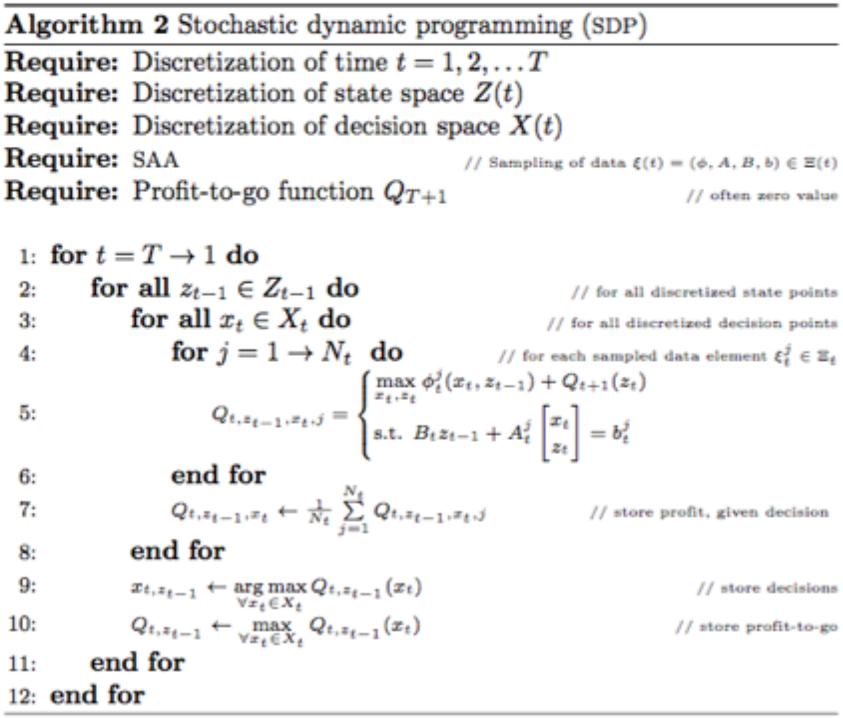
```
Require: Discretization of time $t = 1, 2, \dots T$
Require: Discretization of states space $Z(t)$
Require: Profit-to-go function $Q_{T+1}$                    // often zero value

for $t = T \rightarrow 1$ do
    for all $z_{t-1} \in Z_{t-1}$ do                        // for all discretized state points
        $x_{t,z_{t-1}} \leftarrow \begin{cases} \arg\max_{x_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \end{cases}$        // store decisions
        $Q_{t,z_{t-1}} \leftarrow \begin{cases} \max_{x_t, z_t} \phi_t(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t \end{cases}$        // store profit-to-go
    end for
end for
```

Stochastic Optimization: (9/10)
Solver

What about stochastics?

Stochastic dynamic programming solver:



Algorithm 2 Stochastic dynamic programming (SDP)

```
Require: Discretization of time $t = 1, 2, \dots T$
Require: Discretization of state space $Z(t)$
Require: Discretization of decision space $X(t)$
Require: SAA                                  // Sampling of data $\xi(t) = (\phi, A, B, b) \in \Xi(t)$
Require: Profit-to-go function $Q_{T+1}$       // often zero value

for $t = T \to 1$ do
    for all $z_{t-1} \in Z_{t-1}$ do          // for all discretized state points
        for all $x_t \in X_t$ do              // for all discretized decision points
            for $j = 1 \to N_t$ do            // for each sampled data element $\xi_t^j \in \Xi_t$
                $Q_{t,z_{t-1},x_t,j} = \begin{cases} \max\limits_{x_t, z_t} \phi_t^j(x_t, z_{t-1}) + Q_{t+1}(z_t) \\ \text{s.t. } B_t z_{t-1} + A_t^j \begin{bmatrix} x_t \\ z_t \end{bmatrix} = b_t^j \end{cases}$
            end for
            $Q_{t,z_{t-1},x_t} \leftarrow \frac{1}{N_t} \sum_{j=1}^{N_t} Q_{t,z_{t-1},x_t,j}$      // store profit, given decision
        end for
        $x_{t,z_{t-1}} \leftarrow \arg\max\limits_{\forall x_t \in X_t} Q_{t,z_{t-1}}(x_t)$      // store decisions
        $Q_{t,z_{t-1}} \leftarrow \max\limits_{\forall x_t \in X_t} Q_{t,z_{t-1}}(x_t)$      // store profit-to-go
    end for
end for
```

Hydro power in Switzerland:
How to operate them?

- When produce how much?
- Produce now or use the water later?
- Costs? Marginal production costs?

Usual approach:

- Asset management group:
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- Trading group
  - “buy” plant from asset management group
  - Don’t deviate “too much” from optimal operation

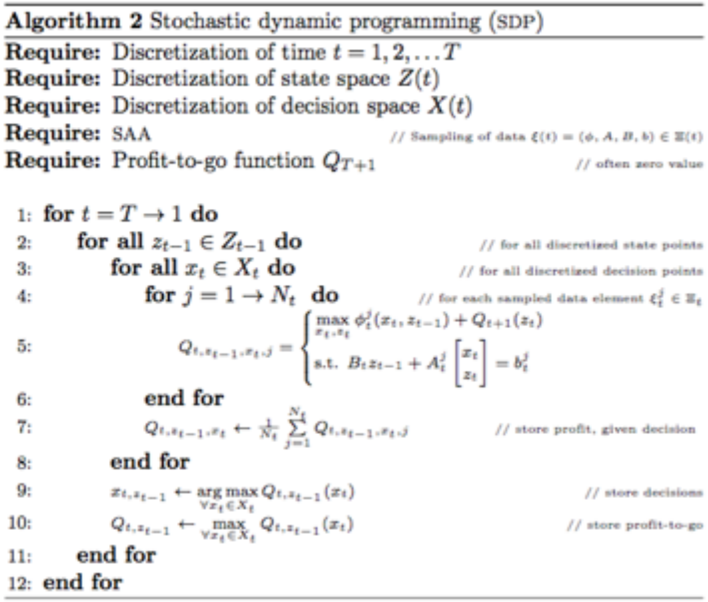
Stochastic Optimization: (10/10)

Solver

Problem of SDP solvers:

“curse of dimensionality”

-> make some approximations



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    end for
end for
```

Small Recap 2:

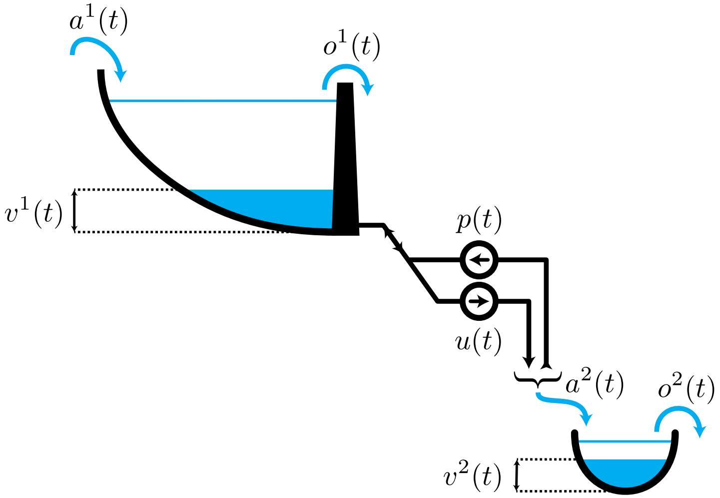
Stochastic optimization:

- model / solvers
- Model:
  - no common language
  - “flow” of information difficult to formulate
  - scenario trees
- Solvers:
  - deterministic equivalent -> LP
  - dynamic programming -> “for-loop”
  - “curse of dimensionality”

5. Examples

Example 1

Input data for time stage t:		
Inflow ι_t^j:	$= \{0, 200\}$	$[1000\,\mathrm{m}^3]$
Price $c_t^j, \forall j$:	$= 50$	[€/MWh]
Turbine $p(t)$:	$\in [0, 100]$	[MW]
Pump $u(t)$:	$= 0$	[MW]



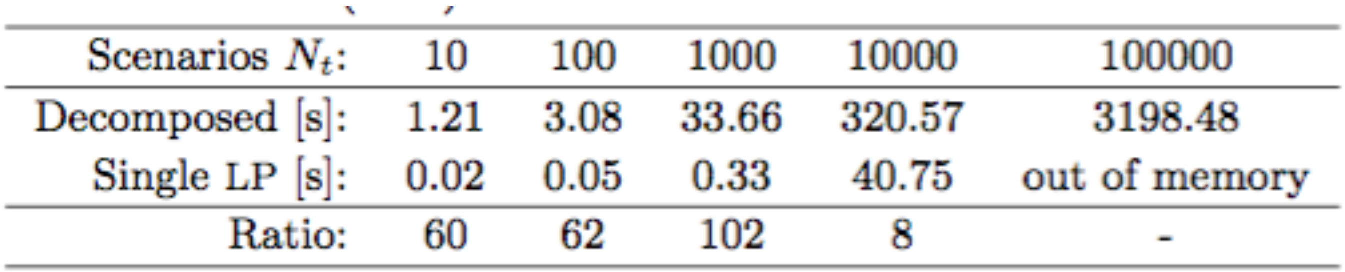
Results:	wait-and-see	here-and-now	deterministic
Obj. value: [€]	2500	0	5000
Turbine p_t^j: [MW]	0/100	0	100
Water values: [€/1000m^3]	25/0	50/0	0/0

Example 2

Computational complexity of stochastic solvers:

Solver 1: formulated as deterministic equivalent, solved with LP-solver

Solver 2: formulated and solved as stochastic dynamic programming



Scenarios N_t:	10	100	1000	10000	100000
Decomposed [s]:	1.21	3.08	33.66	320.57	3198.48
Single LP [s]:	0.02	0.05	0.33	40.75	out of memory
Ratio:	60	62	102	8	-

Small Recap 3:

Examples:

- different models possible
- solvers each have their advantages

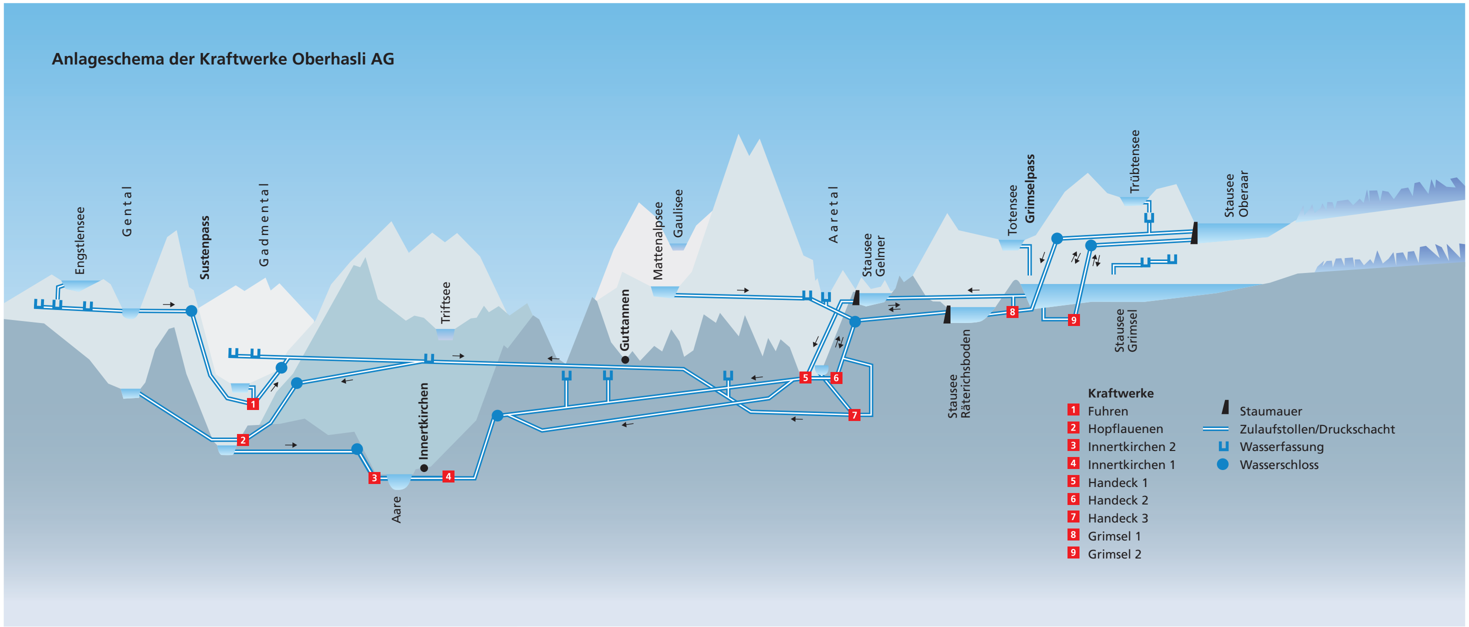
6. Multi-horizon decision trees

Multi-horizon decision trees: (1/5)

Motivation

- 10 reservoirs
- 20 (aggregated) pumps and turbines
- a few operation rules

- 3 years time horizon
- hourly dynamics
- stochastic inflows and prices
- daily rerun



source: kwo.ch

Multi-horizon decision trees: (2/5)

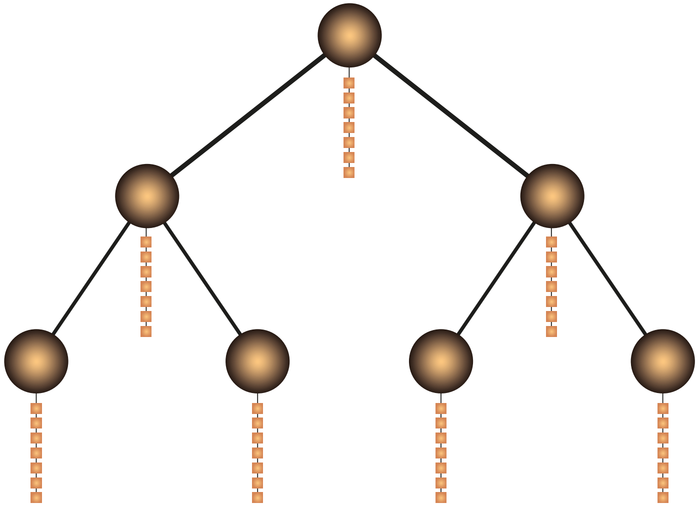
General Idea

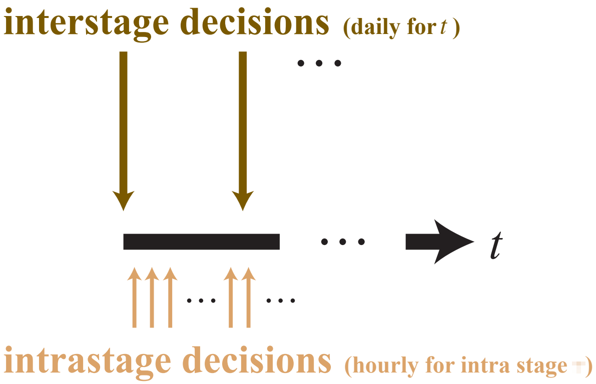
Idea: mimic operators thinking

- only seasonal reservoirs matter (balancing reservoirs not)
- daily operation of power plant

Multi-horizon decision trees: (3/5)

Implementation and results





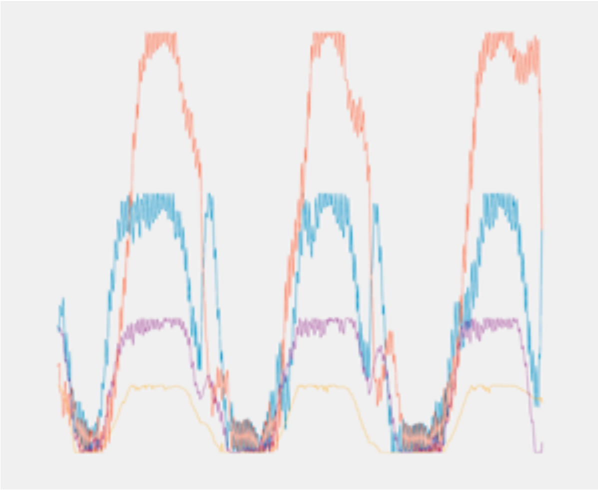
interstage decisions (daily for t)

. . .

. . . t

. . . . . .

intrastage decisions (hourly for intra stage τ)



-> combination of deterministic equivalents and dynamic programs!

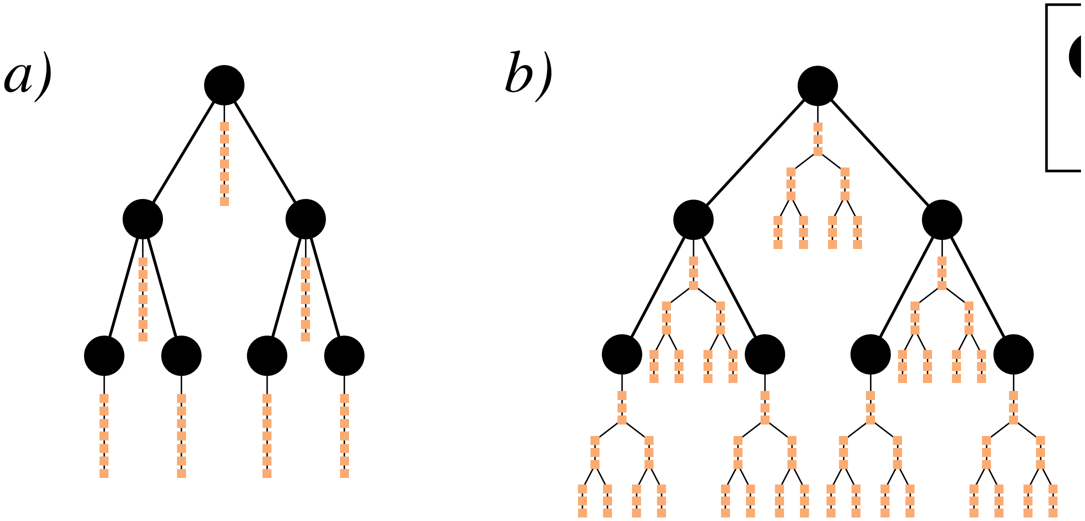
Multi-horizon decision trees: (4/5)

Comparison of alternative methods

	Multi-horizon	hourly time steps
Comp. complexity (SDP, one core):	- 33 Mio. subproblems - 38 days solving time	no balancing reservoirs: - 788 Mio. subproblems - 91 days solving time with balancing reservoirs: - 50 Mia. subproblems - 5800 days solving time
Implementation:	complex	simple(r)
Issues:	deterministic subproblems, no hourly seasonal fillings	discretization error in balancing reservoirs, no storing
Results:	(very) roughly +5% more revenue	

Multi-horizon decision trees: (5/5)

Generalization



Small Recap 4:

Multi-horizon decision trees:

- combination of deterministic equivalents and dynamic programming
- advantages: both computationally and from the modeling point of view

7. Wrap up

Wrap up

1. Hydro scheduling in research
2. Hydro power in Switzerland
3. Optimal hydro power scheduling
4. Stochastic optimization:
    - deterministic equivalents
    - dynamic programming
5. Examples
6. Multi-horizon decision trees

Wrap u

Thank you for your attention!

1. Hydro scheduling in research

2. Hydro power in Switzerland

3. Optimal hydro power scheduling

4. Stochastic optimization:
   - deterministic equivalents
   - dynamic programming

5. Examples

6. Multi-horizon decision trees

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