Designing Incentive-Compatible and Coalition-Proof Payment Mechanisms for Electricity Markets

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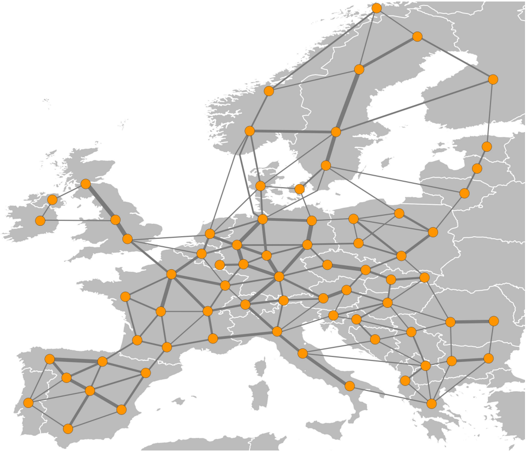
Electricity markets for stability

- Transformation to deregulated competitive markets
- *Stability*: Supply and demand balance at every instance



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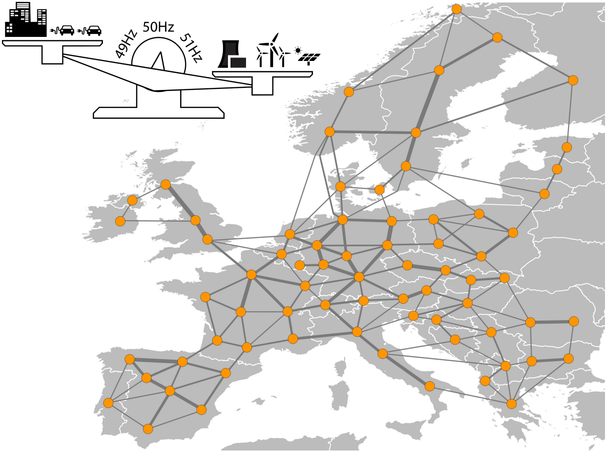

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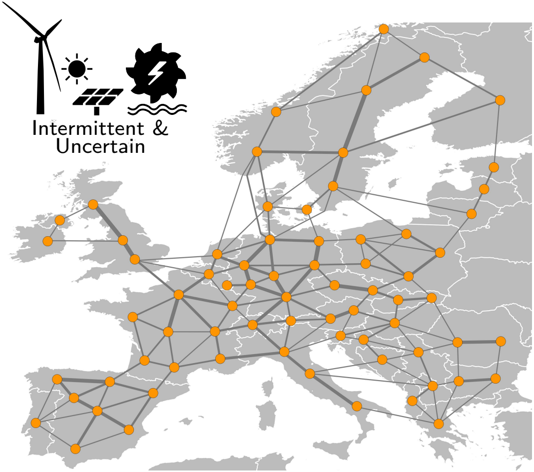
EU 2020 Target
20% renewables



Electricity markets for stability

- Transformation to deregulated competitive markets
- *Stability*: Supply and demand balance at every instance
- Role of electricity markets in ensuring this stability

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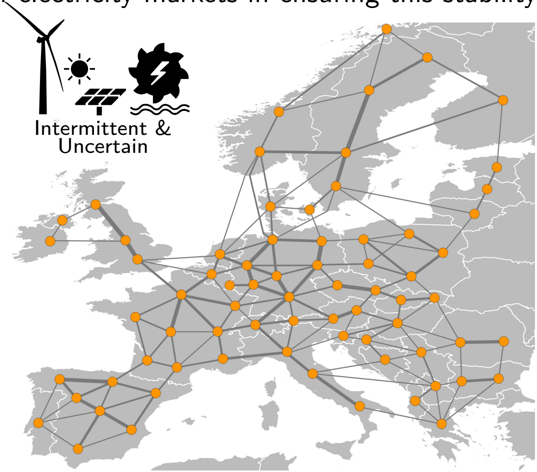
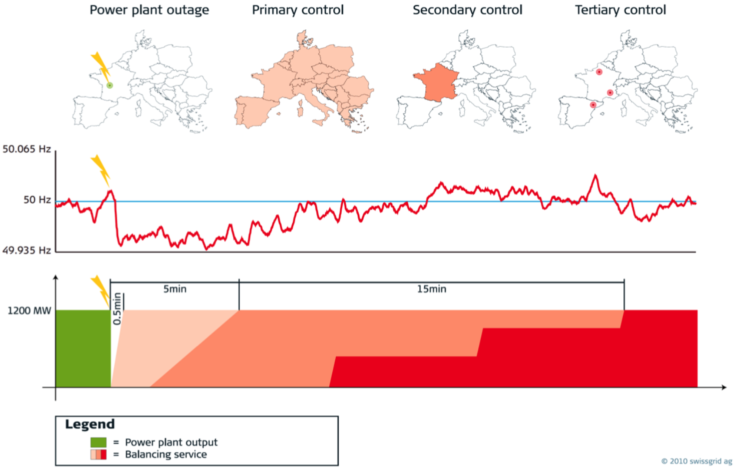


Example 1: Control reserves market



- Different supplies depending on **speed and direction (sign)**
- Involves probabilistic dimensioning criteria

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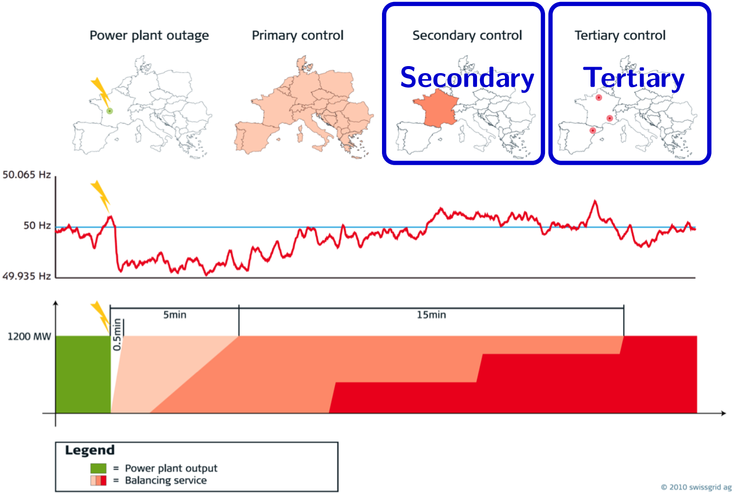

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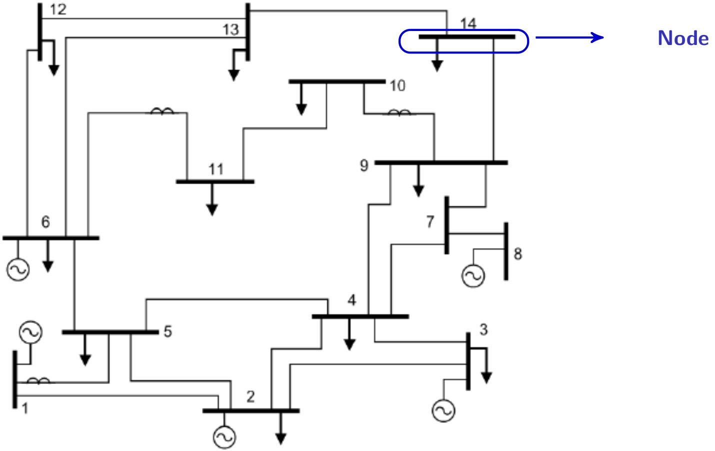

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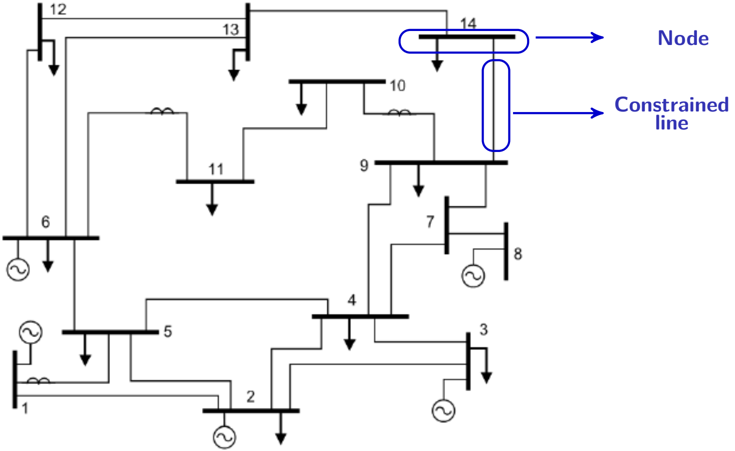

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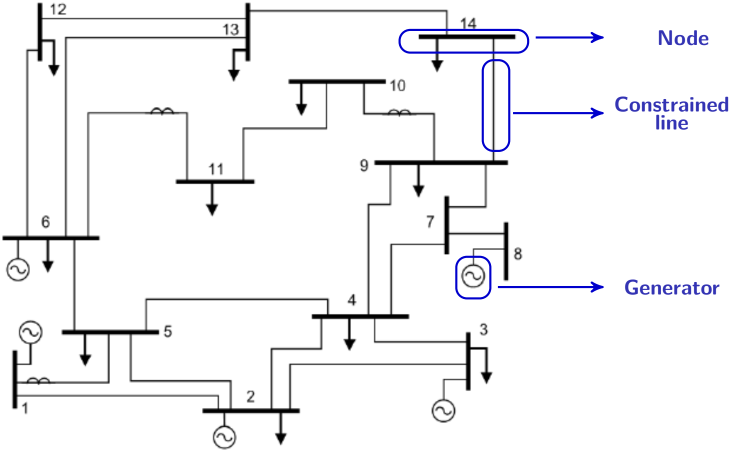

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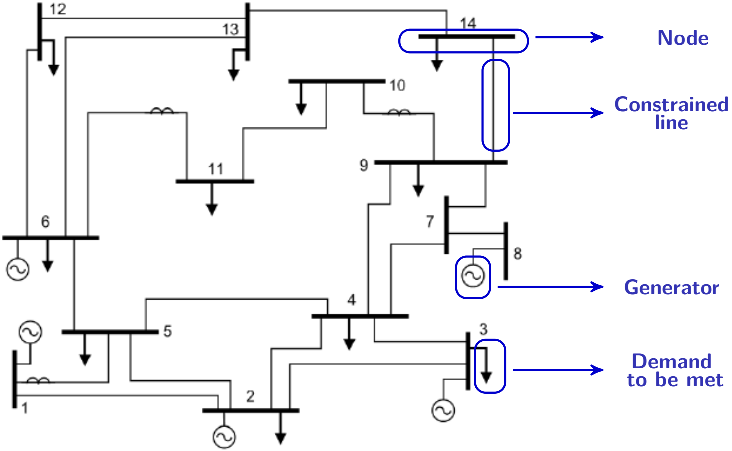

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Market design criteria

Efficiency: Immunity to strategic manipulations

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How can we **eliminate strategic manipulations** to achieve a stable and an efficient grid?

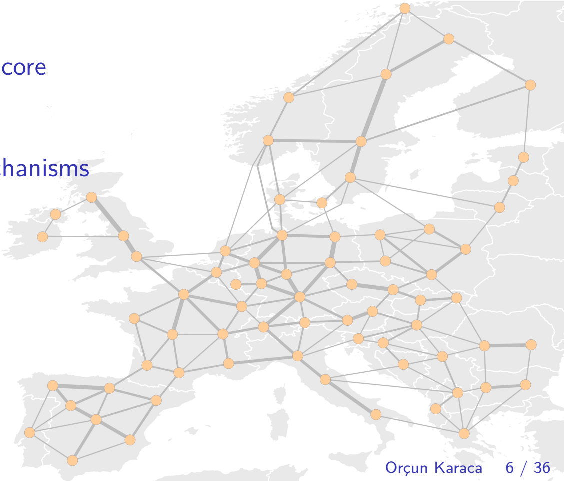
Outline

Market framework and incentive-compatibility

Coalition-proofness using the core

Designing coalition-proof mechanisms

Numerical results



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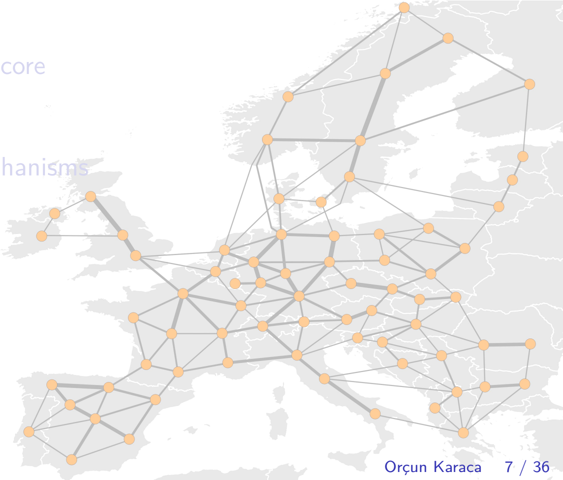
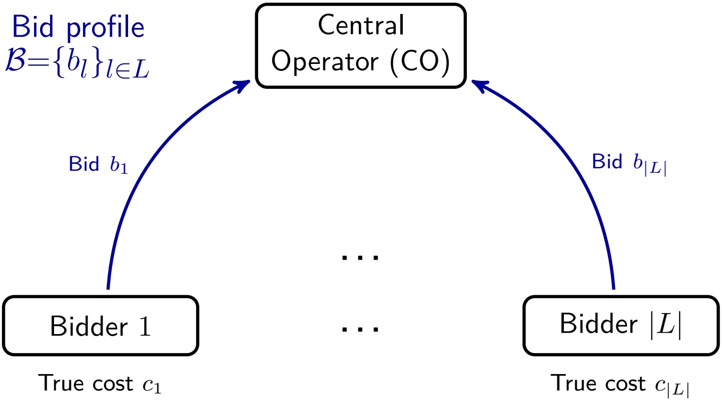


Electricity market framework

- Wholesale electricity markets, control reserve markets, and many others; generalization of **reverse auctions**



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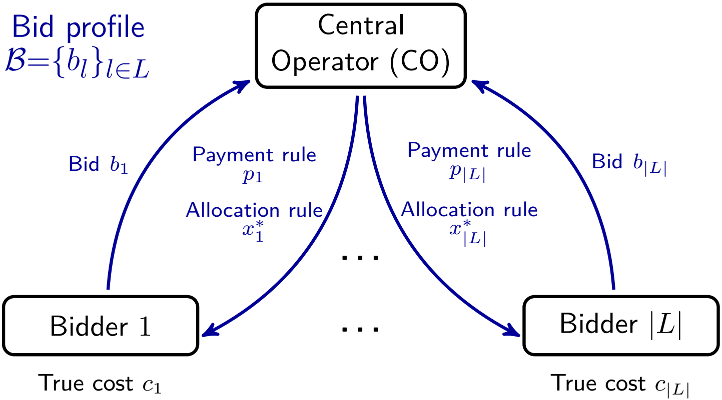

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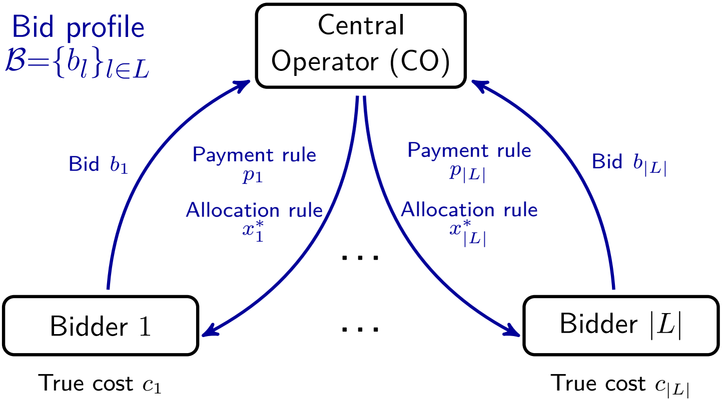
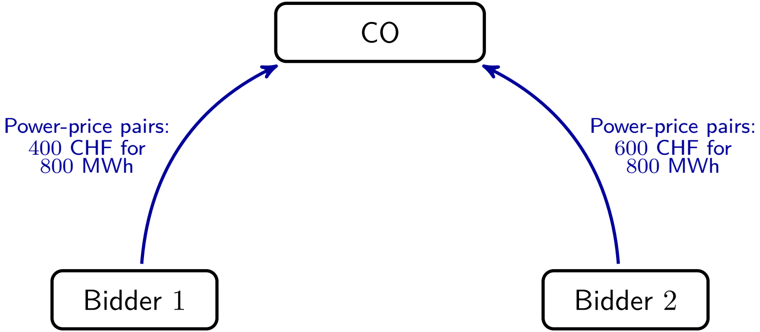


Utility of bidders = Payment − True cost

Utility of CO = − Total payment

Payment design on a simple procurement auction

- Procure 800 MWh from 2 generators by minimizing the cost



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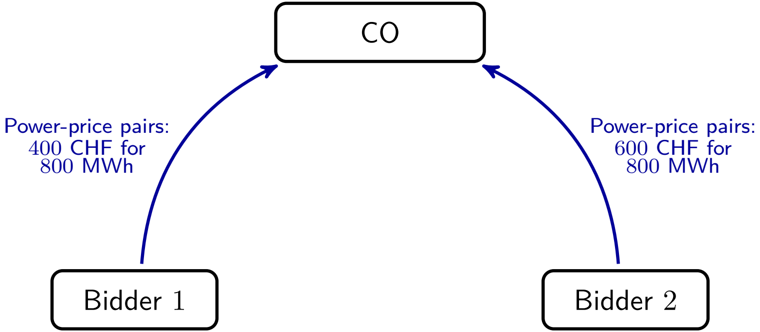

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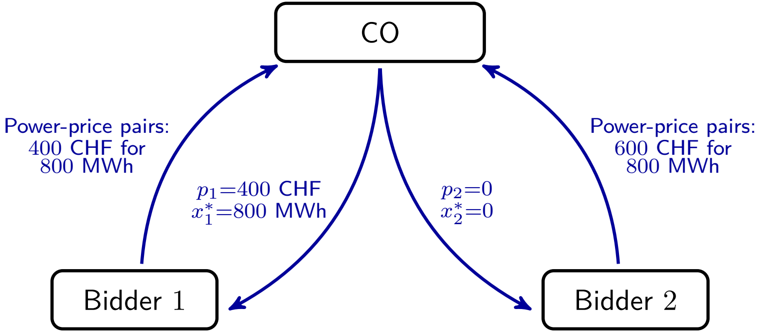

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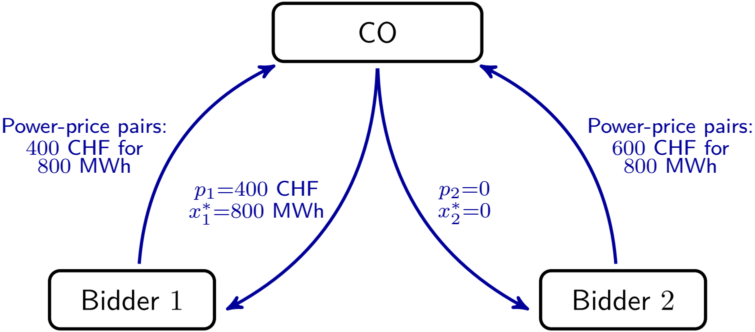
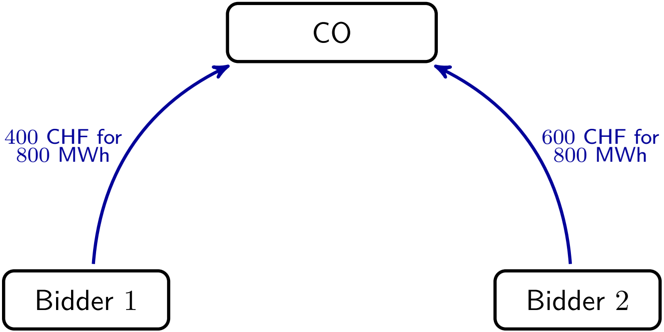


- Payment rule: pay winners their bid
- Bid very large, hard to predict

Vickrey auction and its desirable properties

- Payment rule: pay the 2^{nd} price



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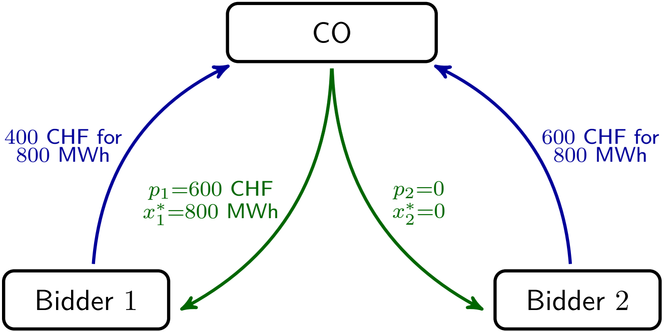

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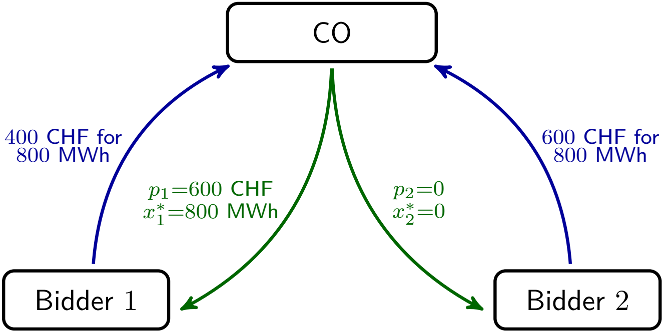

- *Incentive-compatible:* truthfulness is the dominant-strategy
  [Vickrey 1961]

How do we ensure incentive-compatibility for complex electricity markets?

Allocation rule as an optimization problem

- Private true cost of bidder l

$$c_l : \mathbb{X}_l \rightarrow \mathbb{R}_+ \text{ such that } 0 \in \mathbb{X}_\mathrm{l} \subset \mathbb{R}_+ \text{ and } c_l(0) = 0$$

- Reported cost of bidder l

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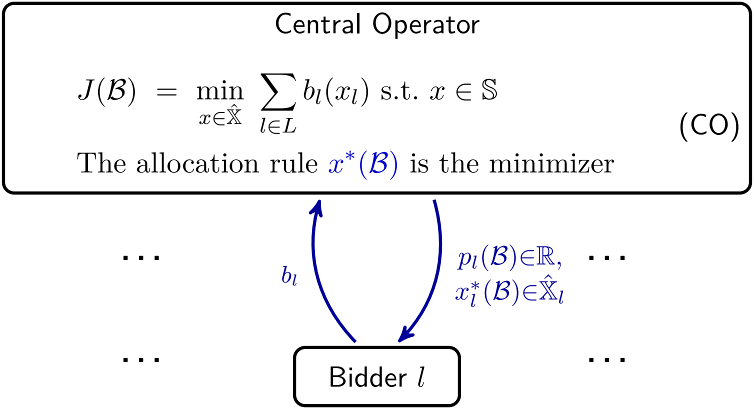
- **The central operator solves for the economic dispatch**

$$J(\mathcal{B}) = \min_{x \in \hat{\mathbb{X}}} \sum_{l \in L} b_l(x_l) \quad \text{s.t.} \quad x \in \mathbb{S}$$

  - Production limits $\hat{\mathbb{X}} = \hat{\mathbb{X}}_1 \times \cdots \times \hat{\mathbb{X}}_{|L|}$
  - Market constraints $\mathbb{S} \subset \mathbb{R}_+^{|L|}$—*e.g.*, **security constraints**

Updating the framework with the allocation rule

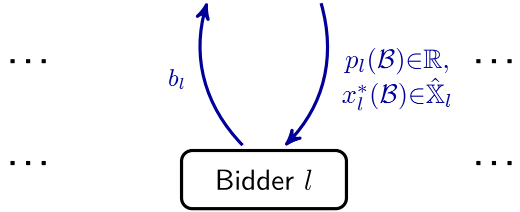


Updating the framework with the allocation rule

Central Operator

$$J(\mathcal{B}) \; = \; \min_{x \in \hat{\mathbb{X}}} \sum_{l \in L} b_l(x_l) \text{ s.t. } x \in \mathbb{S} \qquad \text{(CO)}$$

The allocation rule $x^*(\mathcal{B})$ is the minimizer



- Bidder's utility: $u_l(\mathcal{B}) = p_l(\mathcal{B}) - c_l(x_l^*(\mathcal{B}))$

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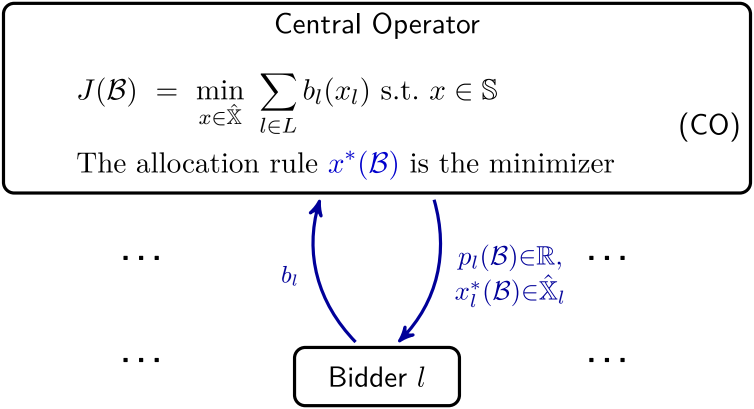

- Bidder's utility: $u_l(\mathcal{B}) = p_l(\mathcal{B}) - c_l(x_l^*(\mathcal{B}))$
- CO's utility: $u_{\text{CO}}(\mathcal{B}) = -\sum_{l \in L} p_l(\mathcal{B})$

Desirable properties for the payment rules

- *Individually rational:* Nonnegative utilities for bidders
- *Efficient:* Sum of all utilities is maximized

$$\underbrace{u_{\mathsf{CO}}(\mathcal{B}) + \sum_{l \in L} u_l(\mathcal{B})}_{\textbf{maximize}} = \underbrace{-\sum_{l \in L} c_l(x_l^*(\mathcal{B}))}_{\textbf{minimize}}$$

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- Widely used mechanisms
  - *Pay-as-bid mechanism*:

$$p_l(\mathcal{B}) = b_l(x_l^*(\mathcal{B}))$$

  - *Locational marginal pricing (LMP) mechanism*:

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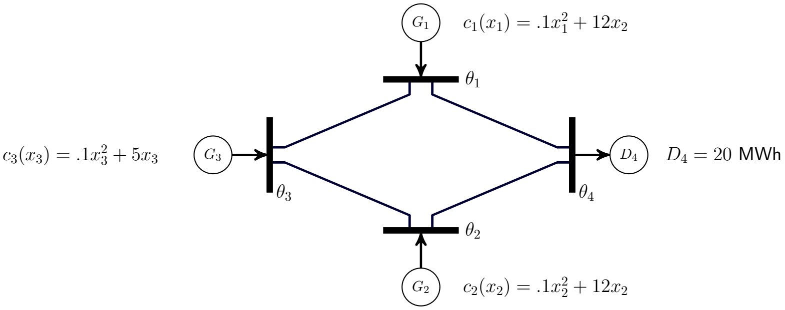
- **Not incentive-compatible, not efficient!**
- Manipulations risk the stability of the grid [Wolfram 1997], [Joskow 2001]

Example: Four-node three-generator network



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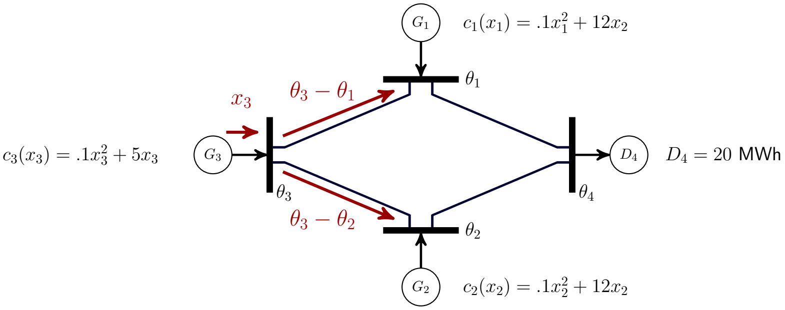

DC power flow model with identical lossless lines:

$\theta_i - \theta_j$: Power flow from Node i to Node j

$\exists \theta \in \mathbb{R}^n$ such that $x_i - D_i = \sum_j \theta_i - \theta_j \; : \; \underbrace{\lambda_i}_{\text{Lagrange M.}}, \; \forall i$ (Nodal Balance)

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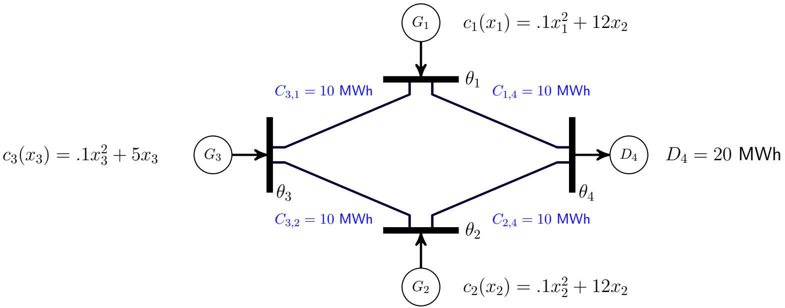

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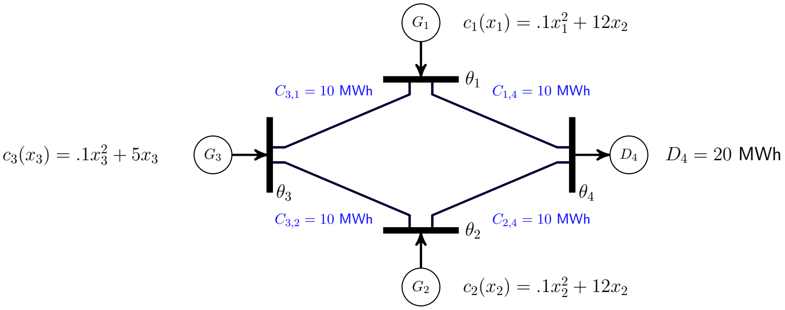

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$$\theta_i - \theta_j \leq C_{ij}, \forall i, j$$ (Line Limits)

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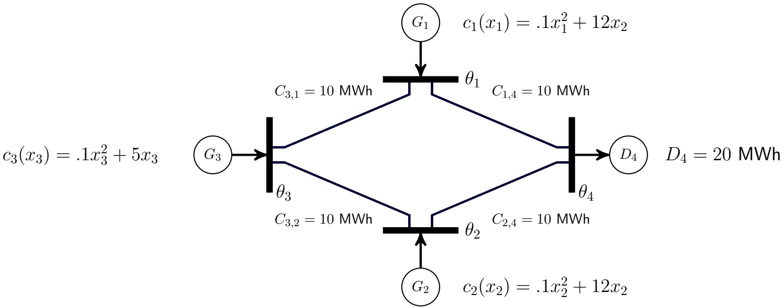

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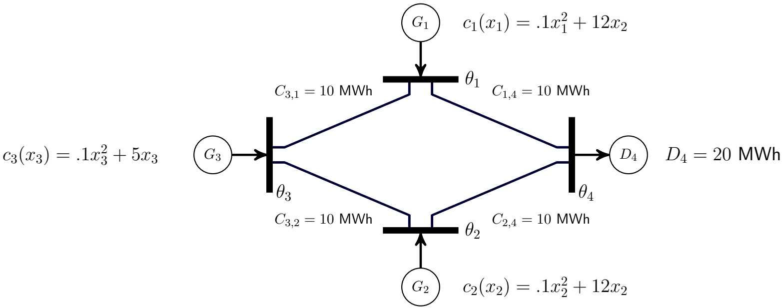

Table: LMP outcomes for the model (CHF) (p: payment, u: utility)

	Truthful Bidding			
	p (u)	x		
Generator 1	0 (0)	0		
Generator 2	0 (0)	0		
Generator 3	180 (40)	20		

$$p_3 = \lambda_3 \times x_3 = 9 \times 20$$

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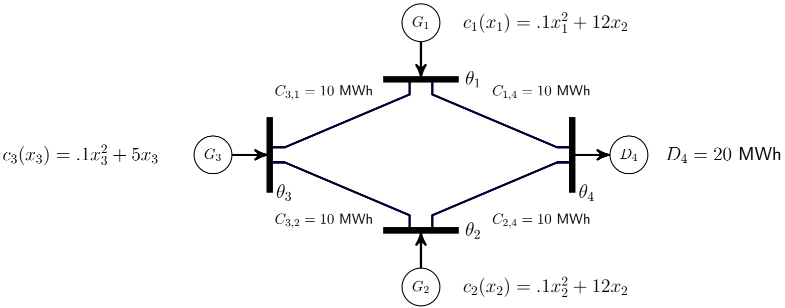

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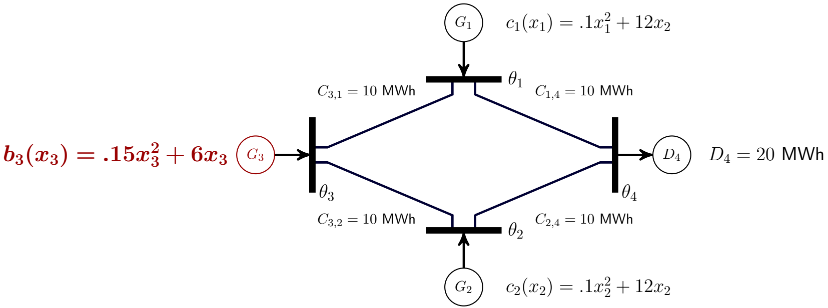

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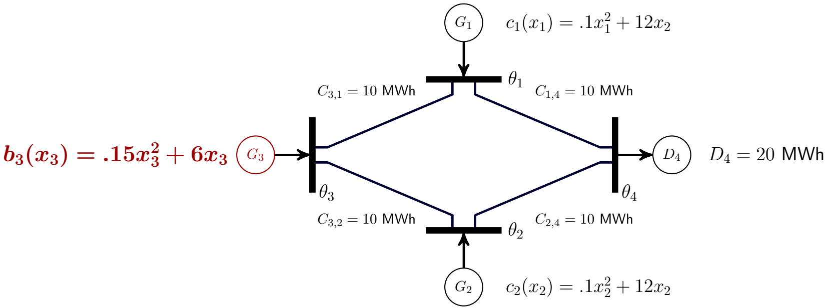

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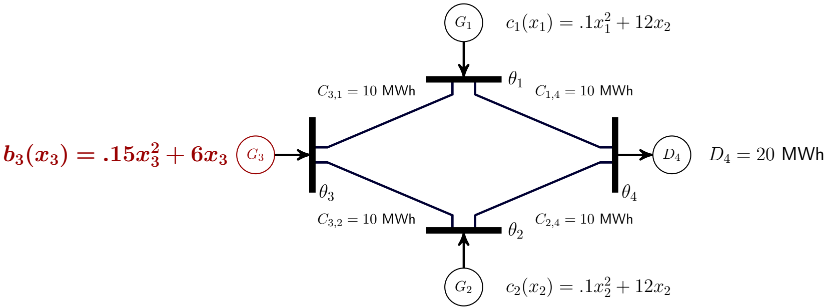

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Under LMP, unilateral deviation is profitable for bidders

The Vickrey-Clarke-Groves (VCG) mechanism

[Vickrey 1961], [Clarke 1971], [Groves 1973]

- Define optimal value of (CO) without bidder l

$$J(\mathcal{B}_{-l}) \geq J(\mathcal{B})$$

where

$$\begin{aligned} J(\mathcal{B}_{-l}) \; = \; \min_{x \in \hat{\mathbb{X}}} \; & \sum_{l \in L} b_l(x_l) \\ \text{s.t.} \;\; & x \in \mathbb{S}, \, x_l = 0 \end{aligned}$$

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$$p_l(\mathcal{B}) = \underbrace{J(\mathcal{B}_{-l})}_{\text{cost of others } \textbf{in the absence of } l} - \underbrace{(J(\mathcal{B}) - b_l(x_l^*(\mathcal{B})))}_{\text{cost of others } \textbf{when } l \textbf{ is present}}$$

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Theorem 1

Given (CO), the VCG mechanism is

a) *Incentive-compatible*
b) *Efficient*
c) *Individually rational*

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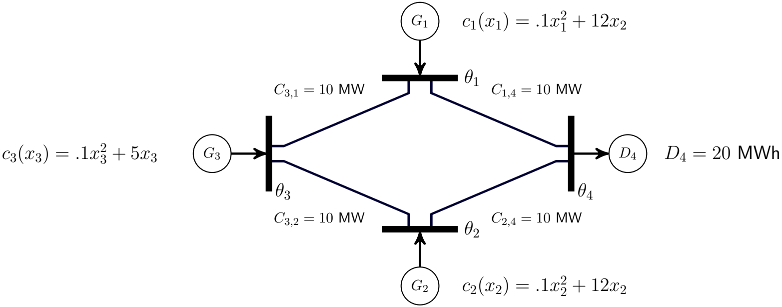
- Generalization of the 2^{nd} price mechanism

The lovely but lonely VCG mechanism [Ausubel and Milgrom 2006]



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$$p_3 = J(\mathcal{B}_{-3}) - (J(\mathcal{B}) - b_3(x_3^*(\mathcal{B})))$$

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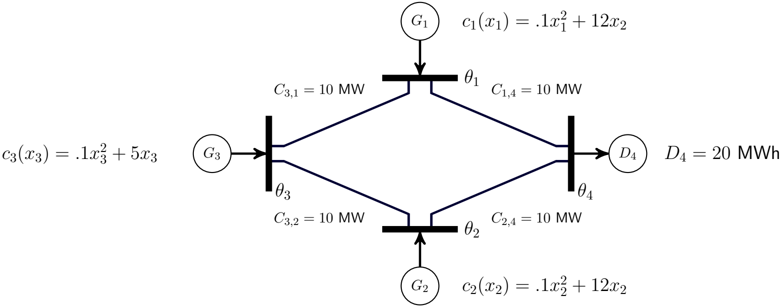

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$$p_3 = 260 - (140 - 140) = 260$$

The lovely but lonely VCG mechanism [Ausubel and Milgrom 2006]

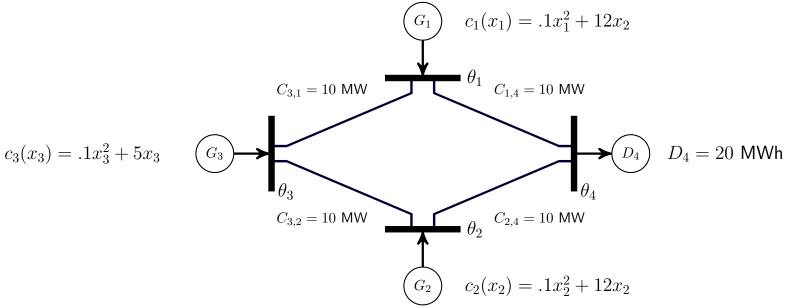

Table: VCG outcomes for the model (CHF) (p: payment, u: utility)

	Truthful Bidding			
	p (u)	x		
Generator 1	0 (0)	0		
Generator 2	0 (0)	0		
Generator 3	260 (120)	20		

Truthful bidding is the dominant strategy

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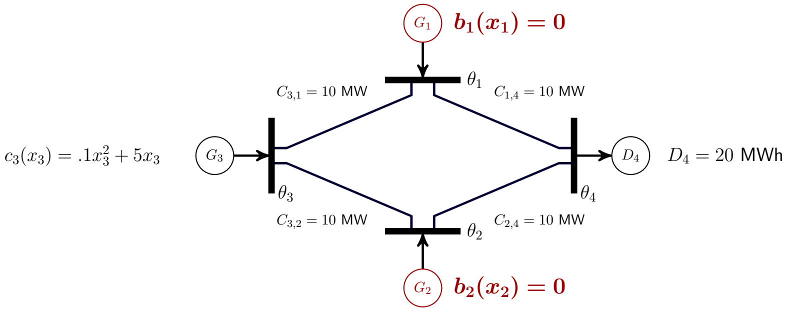

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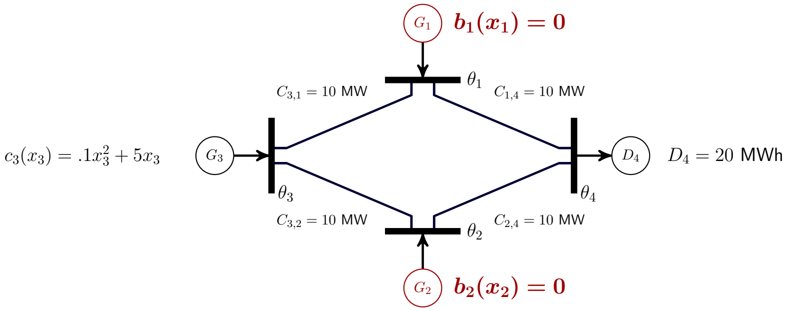

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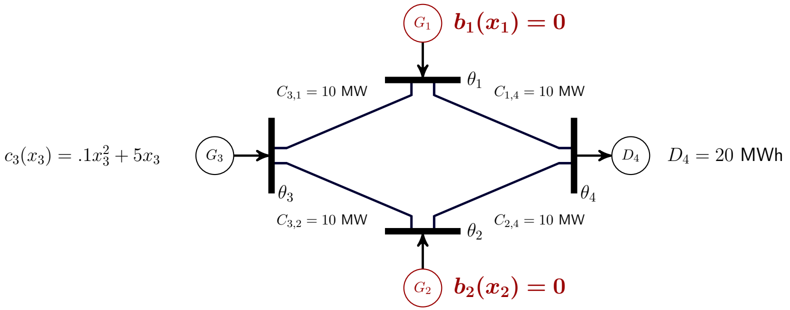

- Another important property:
- ***Coalition-proofness***
  - Joint deviation is not profitable for losing bidders
  - Bidding with multiple identities is not profitable for any bidder

Which mechanisms attain the **coalition-proofness** property?

Outline

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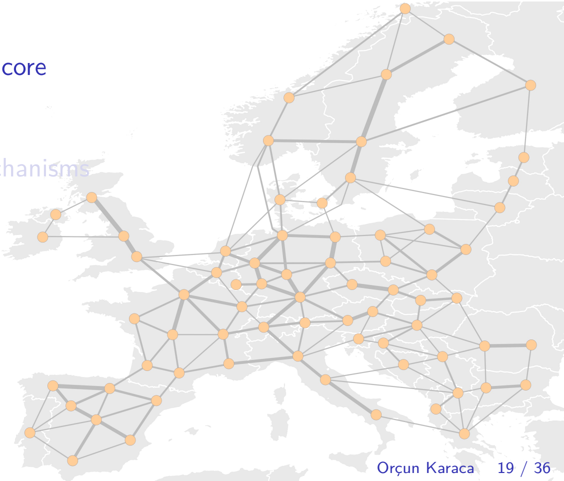

Bringing in the core from coalitional game theory

- Bidder's utility: $u_l(\mathcal{B}) = p_l(\mathcal{B}) - c_l(x_l^*(\mathcal{B}))$
- Central operator's utility:

$$u_{\mathsf{CO}}(\mathcal{B}) = -\sum_{l \in L} p_l(\mathcal{B})$$

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- Bidder's *revealed* utility: $\bar{u}_l(\mathcal{B}) = p_l(\mathcal{B}) - b_l(x_l^*(\mathcal{B}))$
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- Objective value under the profile $\mathcal{B}_S = \{b_l\}_{l \in S}$, $S \subseteq L$

$$J(\mathcal{B}_S) = \min_{x \in \hat{\mathbb{X}}} \sum_{l \in S} b_l(x_l) \quad \text{s.t. } x \in \mathbb{S},\ x_{-S} = 0$$

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Characterization of coalition-proof mechanisms

- Core-selecting payment rule

$$p_l(\mathcal{B}) = b_l(x_l^*(\mathcal{B})) + \bar{u}_l(\mathcal{B}),\ \forall l,\ \text{where } \bar{u} \in Core(\mathcal{B})$$

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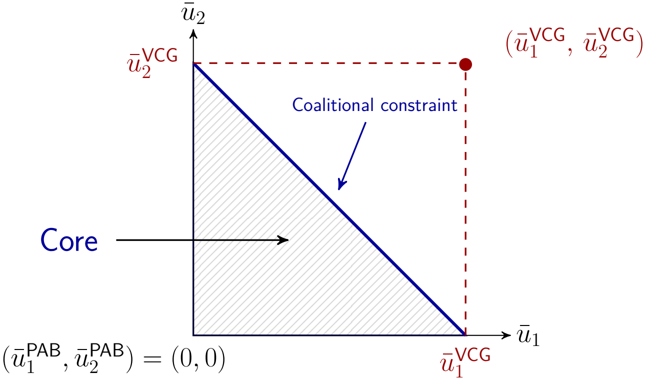
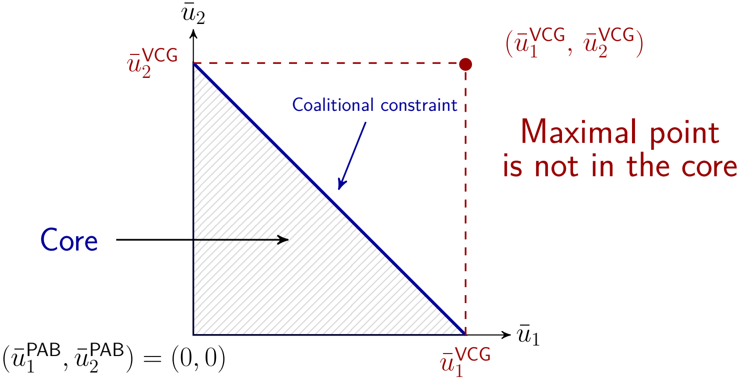


$\bar{u}_2$
$(\bar{u}_1^{\mathsf{VCG}},\ \bar{u}_2^{\mathsf{VCG}})$
$\bar{u}_2^{\mathsf{VCG}}$
Coalitional constraint
Core
$\bar{u}_1$
$(\bar{u}_1^{\mathsf{PAB}}, \bar{u}_2^{\mathsf{PAB}}) = (0, 0)$
$\bar{u}_1^{\mathsf{VCG}}$



$\bar{u}_2$
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Coalitional constraint
Maximal point
is not in the core
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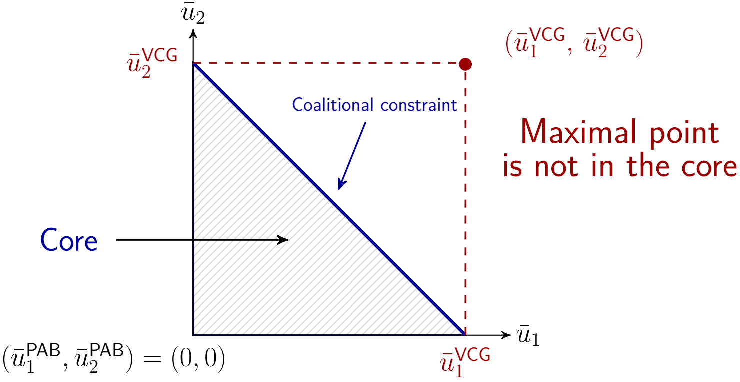

The VCG mechanism is in general not core-selecting!

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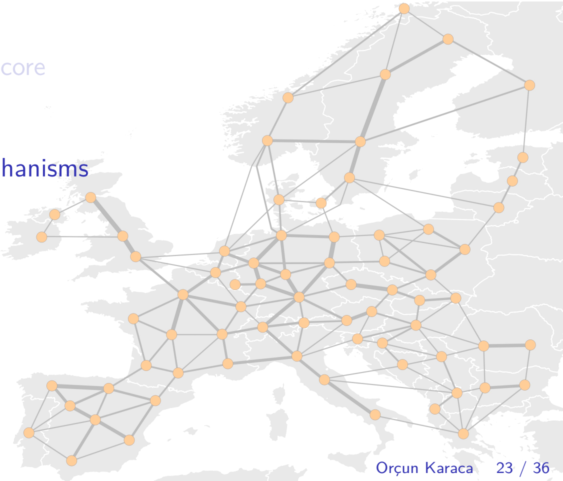

Core-selecting is in general **not incentive-compatible** and there are **many points** to choose from the core...

Can core-selecting mechanisms **approximate incentive-compatibility** while ensuring coalition-proofness?

Approximating incentive-compatibility using core-selecting

- We **quantify the violation of incentive-compatibility** under any core-selecting mechanism

Lemma 1

The maximum gain of bidder l by a unilateral deviation from its true cost is tightly upperbounded by

$$\bar{u}_l^{\mathsf{VCG}}(\mathcal{C}_l, \mathcal{B}_{-l}) - \bar{u}_l(\mathcal{C}_l, \mathcal{B}_{-l})$$

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- Idea: **The closer** you get to the VCG payments, **the better** you approximate incentive-compatibility

Maximum payment core-selecting mechanism

- *Maximum payment core-selecting* (MPCS) mechanism:

$$\bar{u}^{\mathsf{MPCS}}(\mathcal{B}) = \underset{\bar{u} \in \mathsf{Core}(\mathcal{B})}{\arg\min} \sum_{l \in L} \left(\bar{u}_l - \bar{u}_l^{\mathsf{VCG}}(\mathcal{B}) \right)^2$$

Theorem 3

The MPCS mechanism minimizes the sum of maximum gains from unilateral deviations

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- Problem size is **exponential in the number of bidders**!
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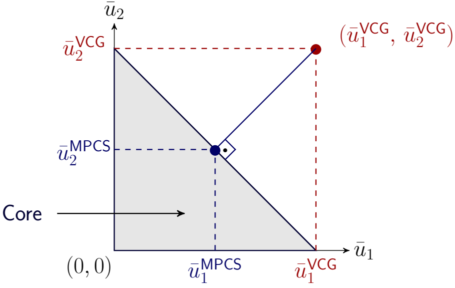
  - Can be tackled via **iterative constraint generation** [Dantzig *et al.* 1954], [Hallefjord *et al.* 1995]

Comparison of revealed utilities under different mechanisms



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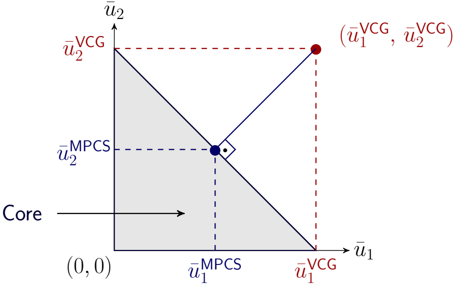

The MPCS mechanism:

+ Approximate incentive-compatibility
+ Exact coalition-proofness and individual-rationality
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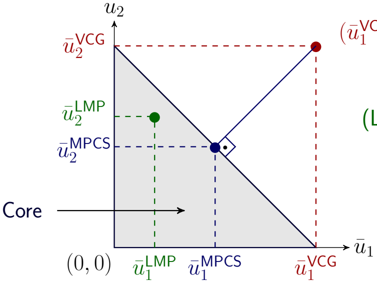

Bring in LMP
(Lagrange-based payments)

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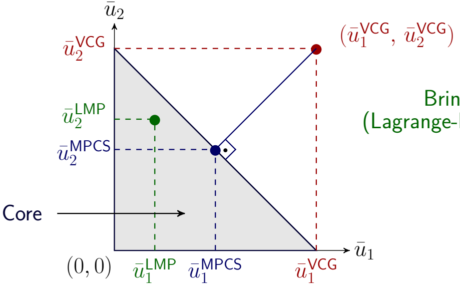

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+ Approximate incentive-compatibility
+ Exact coalition-proofness and individual-rationality
+ Equivalent to the VCG if VCG is core-selecting
+ (Compared to LMP) Applicable to the general setting
− (Compared to LMP) Payments are nonlinear

We extend our model to **exchanges** (and two-sided markets)

Can we quantify the **budget-balance** of the MPCS mechanism?

Budget-balance in exchanges

- Exchange extends the domains of the functions to $\mathbb{R}$

$$c_l : \mathbb{X}_l \to \mathbb{R} \text{ such that } 0 \in \mathbb{X}_l \subset \mathbb{R} \text{ and } c_l(0) = 0$$
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- The VCG mechanism is **not** always budget-balanced
  [Myerson and Satterthwhite 1983], [Krishna and Perry 1998]

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Theorem 4

Any core-selecting mechanism is budget-balanced

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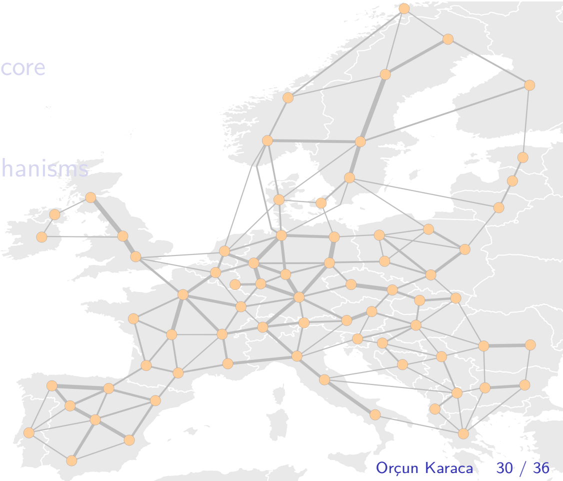


Swiss reserve procurement auctions

- Two-stage stochastic weekly market for secondary and tertiary reserves [Abbaspourtobati and Zima 2016]
- *Mutually exclusive* bids are submitted

$$J(\mathcal{B}) = \min_{x \in \hat{\mathbb{X}}, y} \sum_{l \in L} b_l(x_l) + d(y)$$
$$\text{s.t. } g(x, y) \leq 0$$

- $x \in \hat{\mathbb{X}}$: Power to be purchased in the weekly market
- $y \in \mathbb{R}^p_+$: Power to be purchased in the daily market
- $d : \mathbb{R}^p_+ \to \mathbb{R}$: Expected daily market cost
- Reserves ensure a deficit probability of less than 0.2%

Swiss reserve procurement auctions

- Based on 2014 data—67 bidders

Table: Total payments of the two-stage auction

Total Pay-as-bid payment	2.293 million CHF
Total MPCS payment	2.437 million CHF
Total VCG payment	2.529 million CHF

- Computation times for different mechanisms
  - VCG: 580.6 seconds
  - MPCS: 659.2 seconds

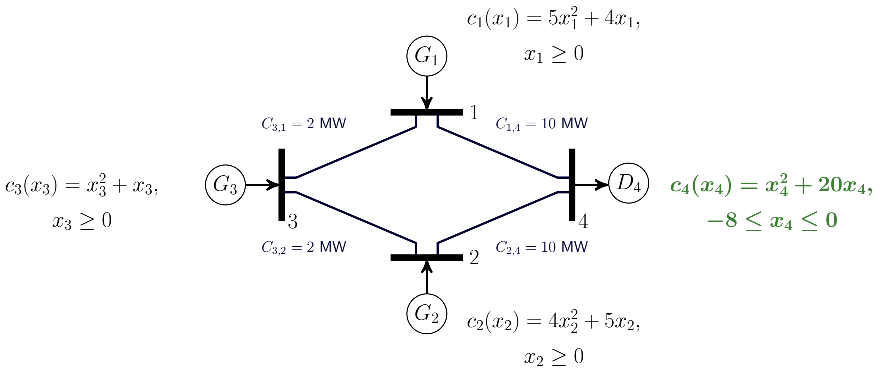
IEEE test systems with power flow constraints

Table: Total payment in the IEEE test systems

Mechanism	14-bus, line limits	118-bus, no line limits
Pay-as-bid	\$9715.2	\$125947.8
Loc. marg. pricing	\$10361.0	\$167055.8
MPCS	\$11220.1	\$169300.4
VCG	\$11432.1	\$169300.4

- **VCG is core-selecting when there are no line limits!**
- Similar results are obtained for other IEEE test systems

Two-sided markets with power flow constraints



Two-sided markets with power flow constraints

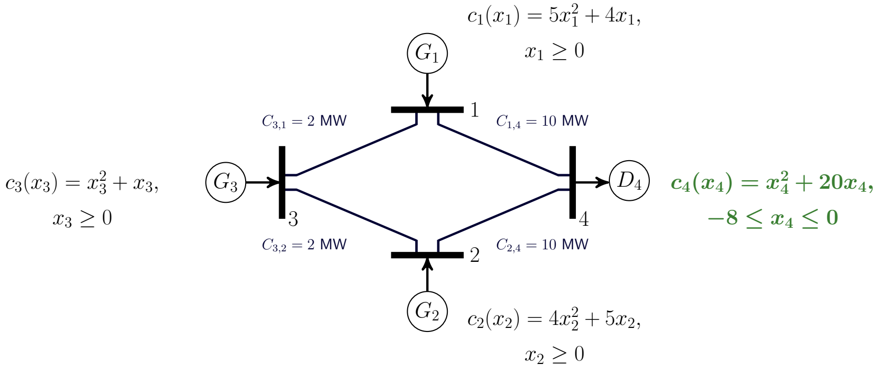


Table: Budget-balance comparison

	Pay-as-bid	LMP	MPCS	VCG
u_{CO}	\$48.3	\$2.8	\$0	-\$34.8

Conclusion

- Summary
  - Studied the VCG mechanism and showed its theoretical virtues
  - Characterized coalition-proof mechanisms as core-selecting
  - Designed coalition-proof mechanisms approximating incentive-compatibility
  - Analyzed budget-balance of the proposed mechanisms
  - Verified with optimal power flow test systems and Swiss reserve market
- Outlook
  - Privacy (bidders might not want to share the true costs...)
  - Learning in a repeated setting
  - Spatial and intertemporal coordination of markets

Thank you for your attention

The results from this talk appear in

- Karaca and Kamgarpour, IEEE CDC 2017
- Karaca and Kamgarpour, IEEE CDC 2018
- Karaca *et al.*, IEEE TAC 2019
- Karaca and Kamgarpour, under review, ArXiv:1811.09646

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