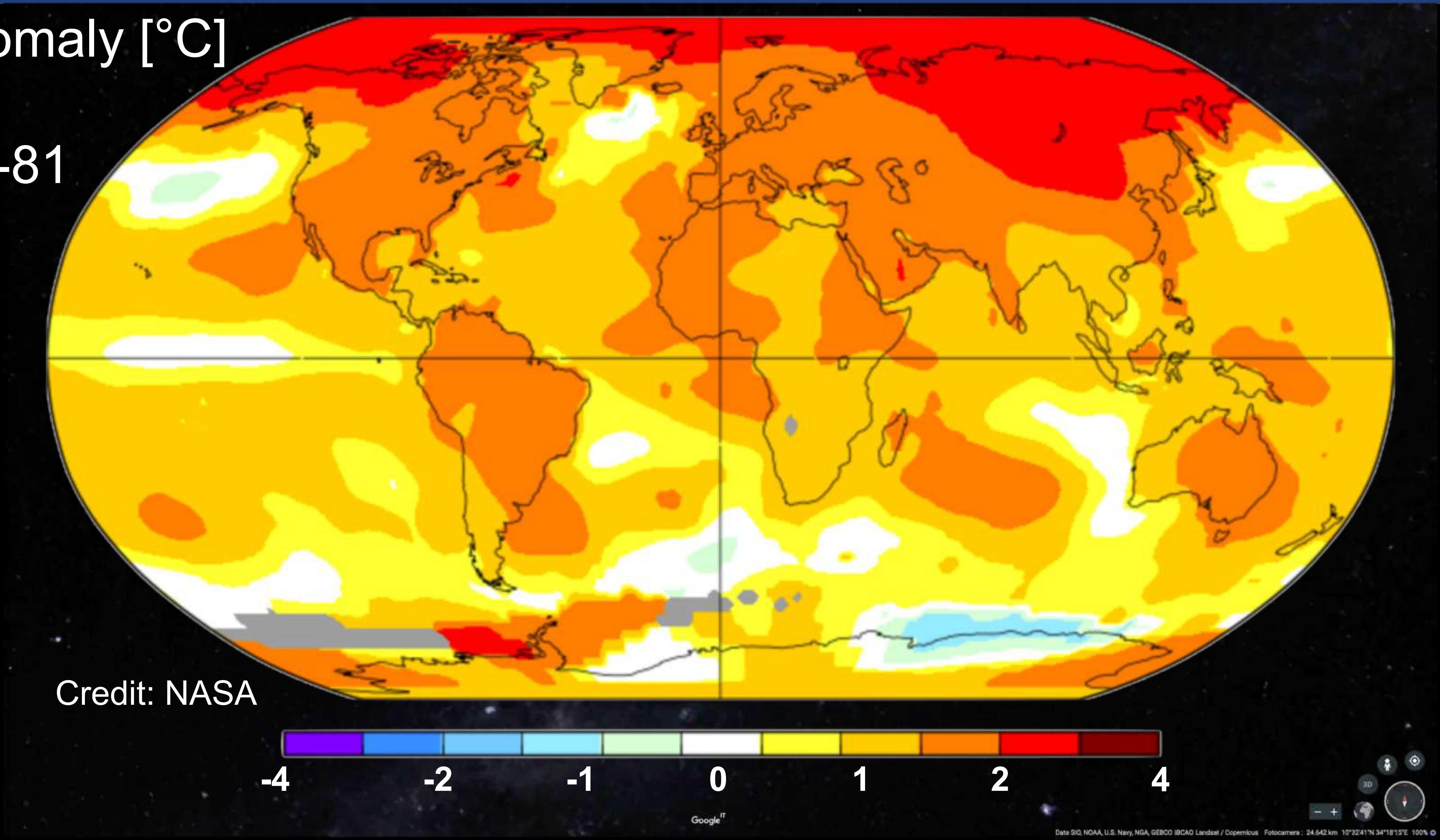
Modelling of thermoacoustic dynamics in gas turbines applications

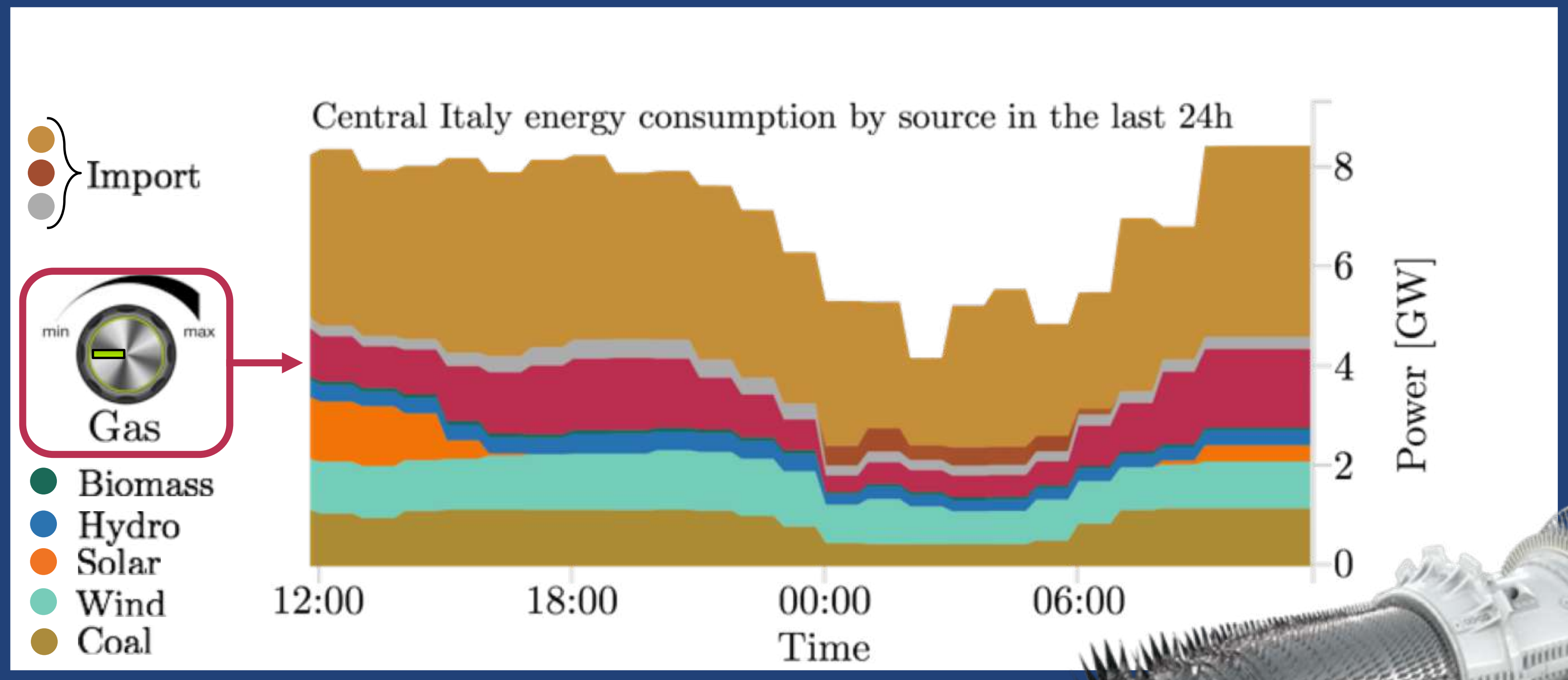
Giacomo Bonciolini

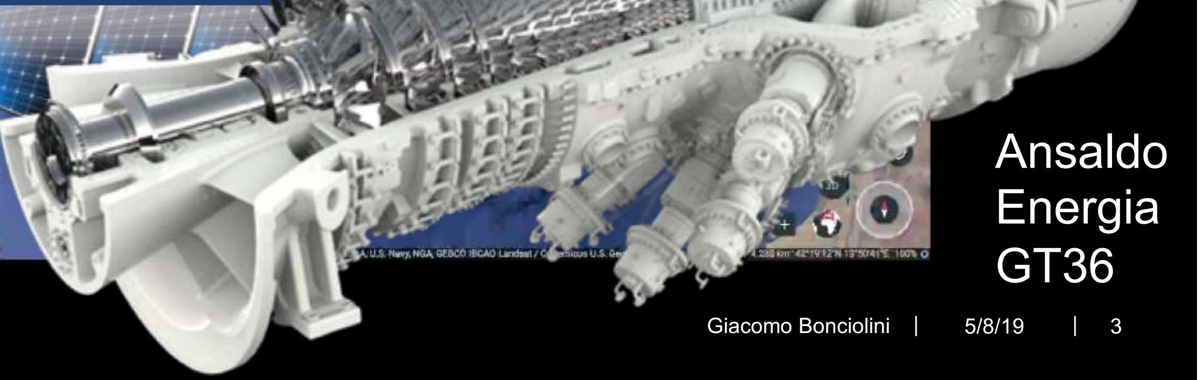
CAPS Laboratory, MAVT department ETH Zürich

T Anomaly [°C]
vs
1950-81

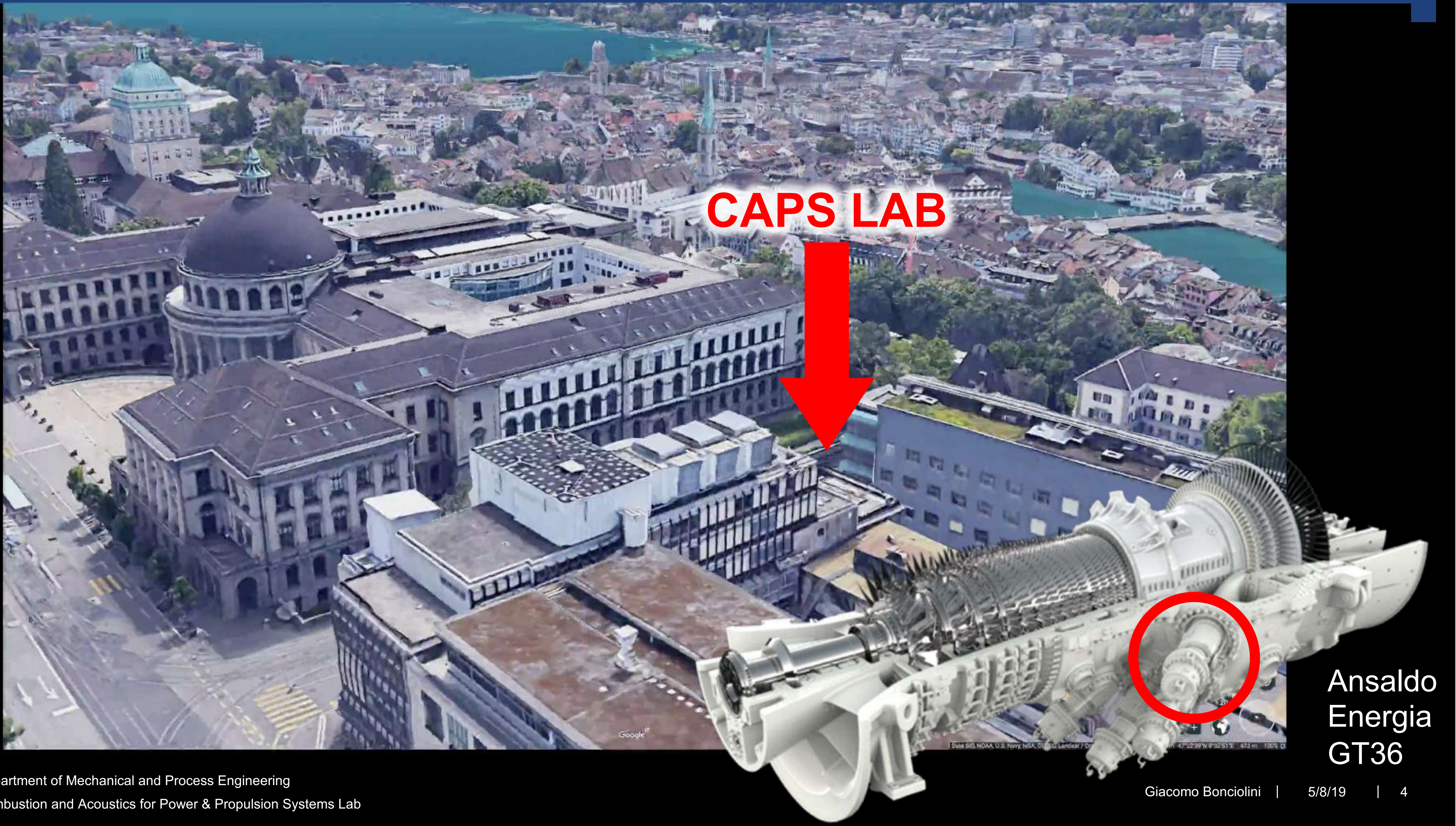


Credit: NASA



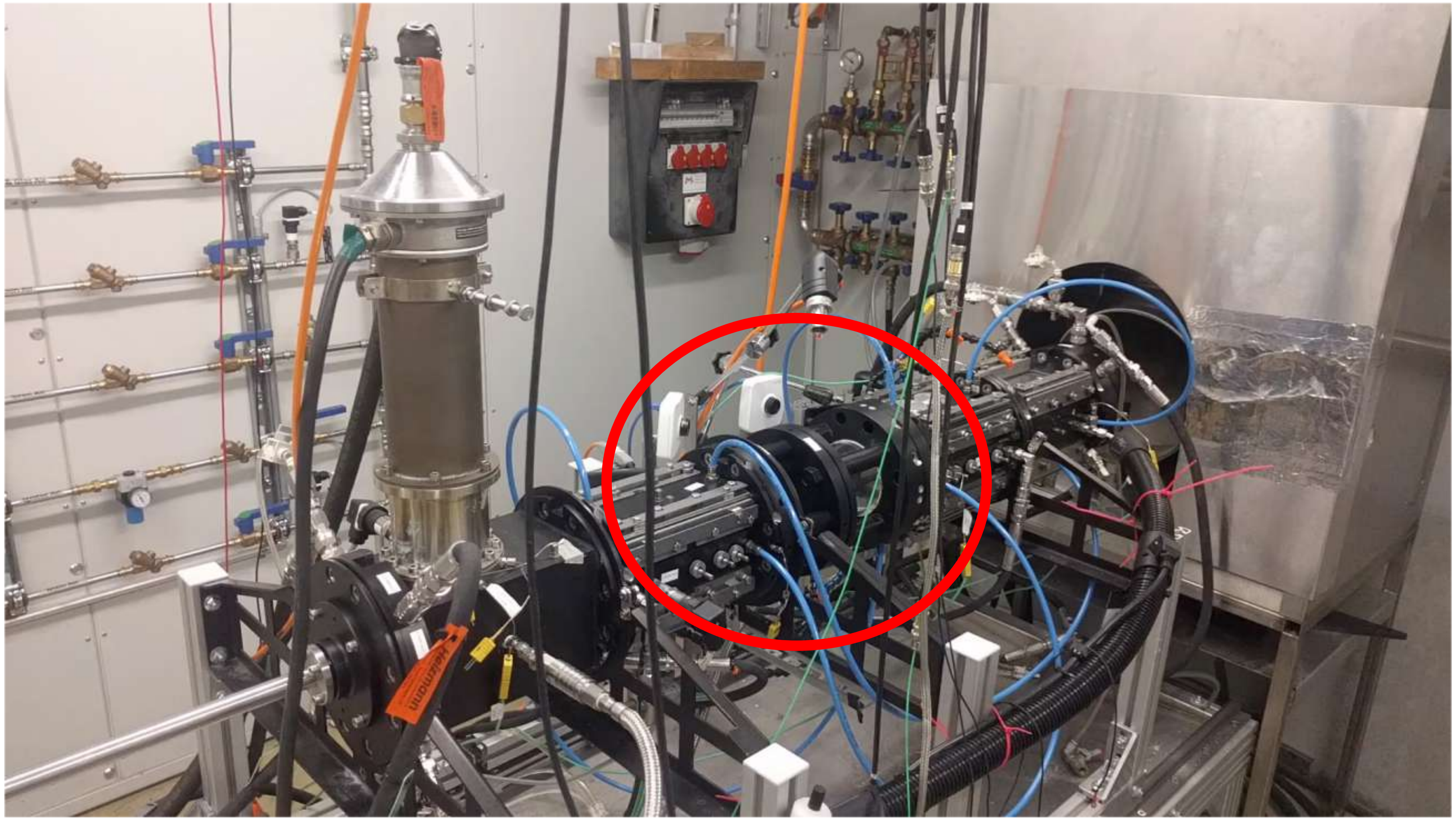


Ansaldo
Energia
GT36

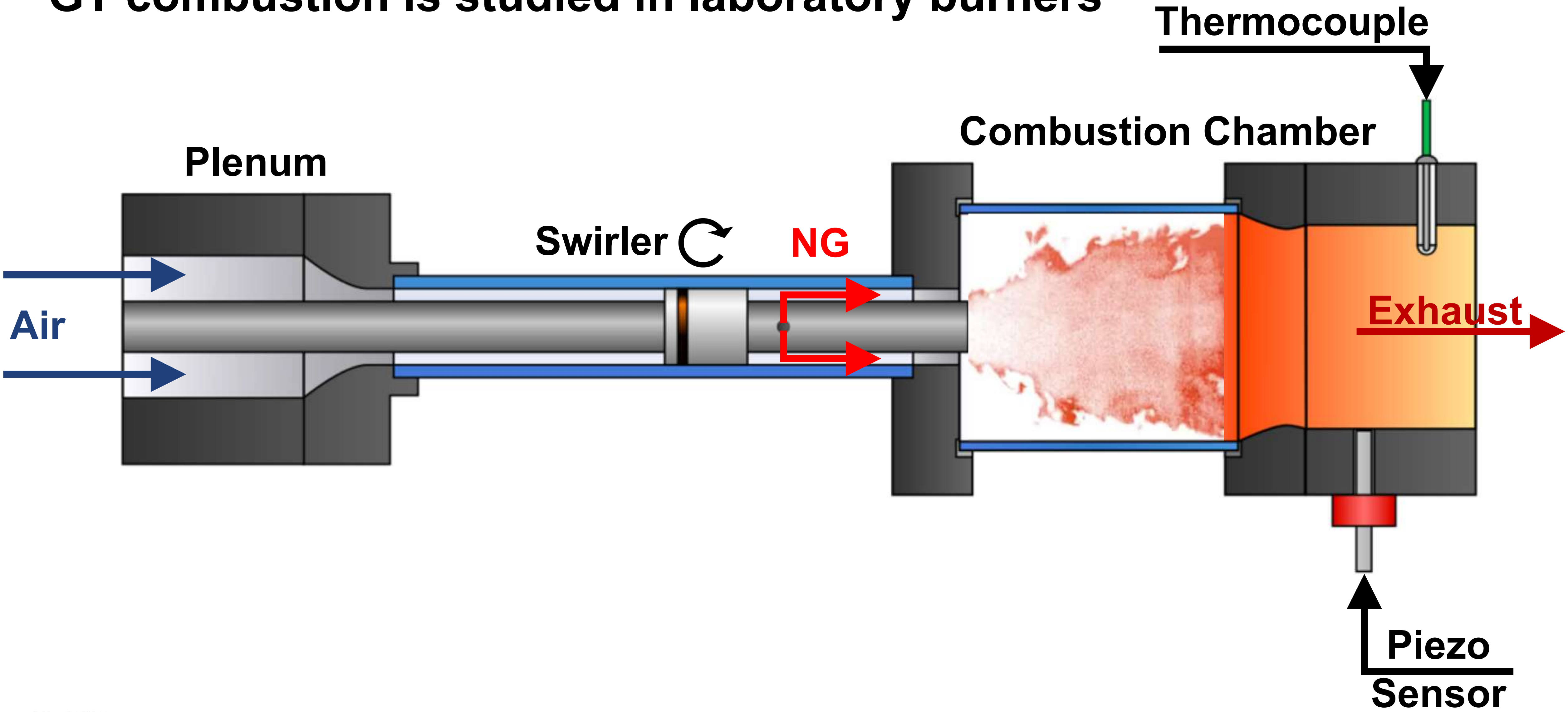


Ansaldo Energia GT36

GT combustion is studied in laboratory burners

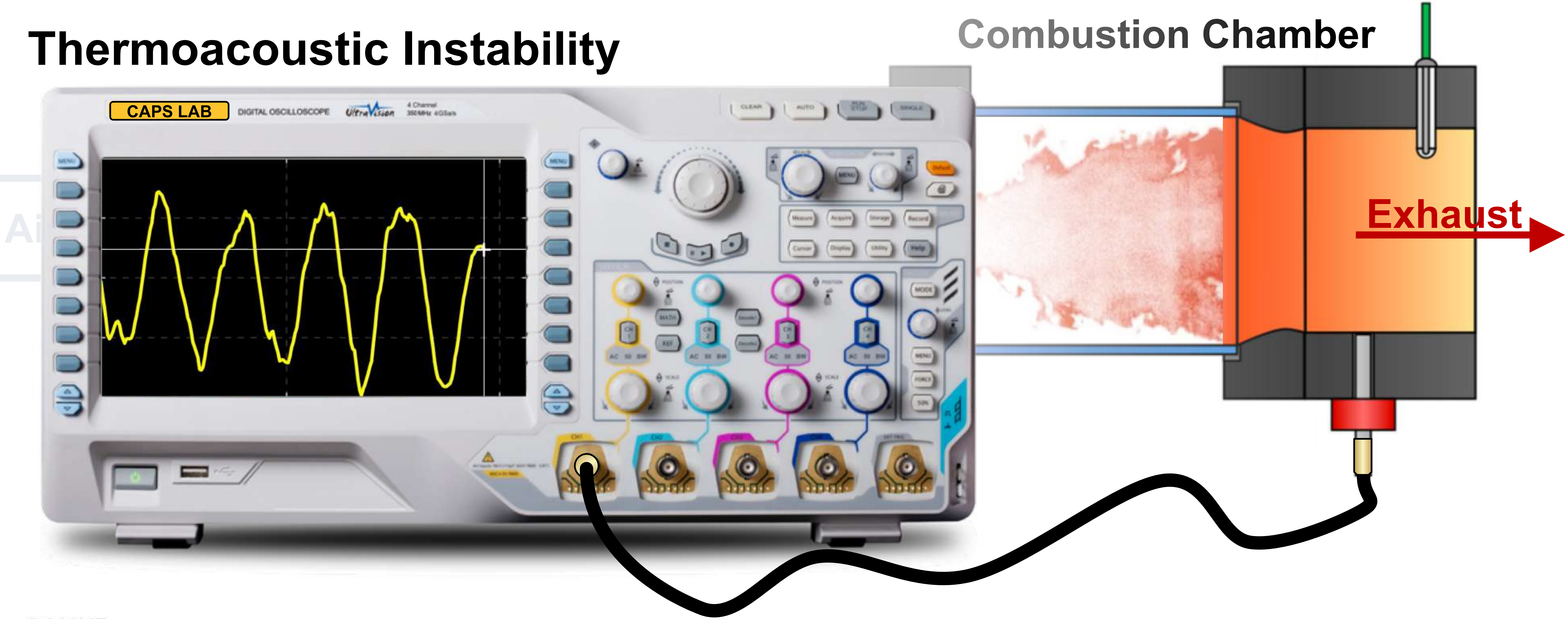


GT combustion is studied in laboratory burners



GT combustion is studied in laboratory burners

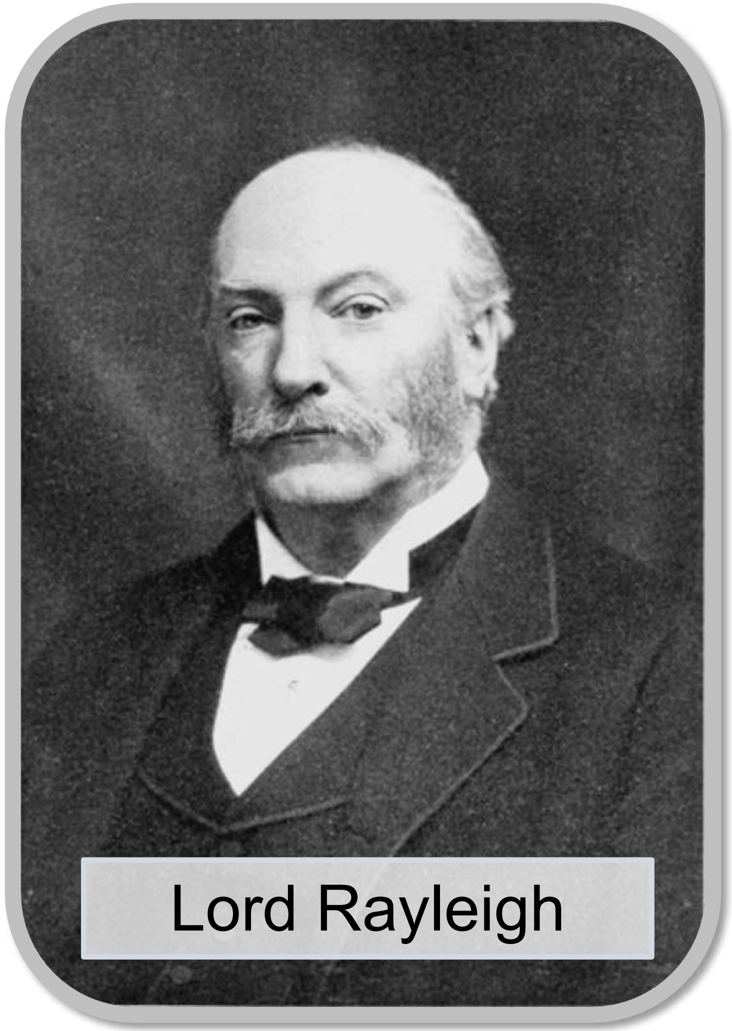
Thermoacoustic Instability



Thermoacoustic coupling first discussed in 18th and 19th century

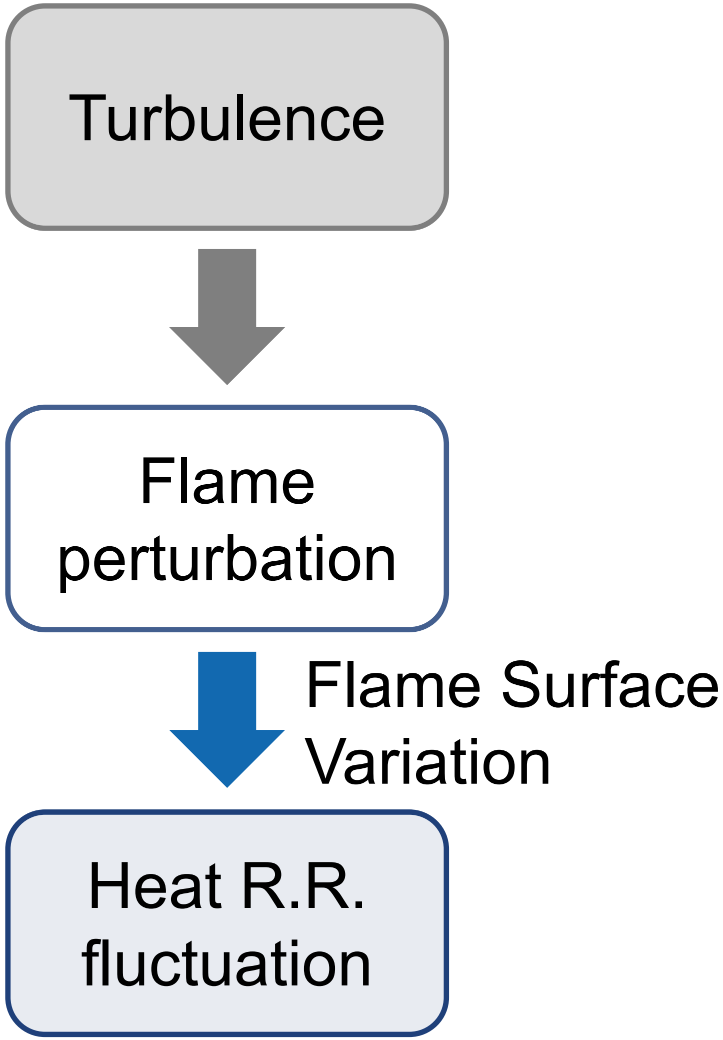
Doctor Higgins' tube

The pyrophone



Lord Rayleigh

Thermoacoustic coupling stems from heat-sound interaction

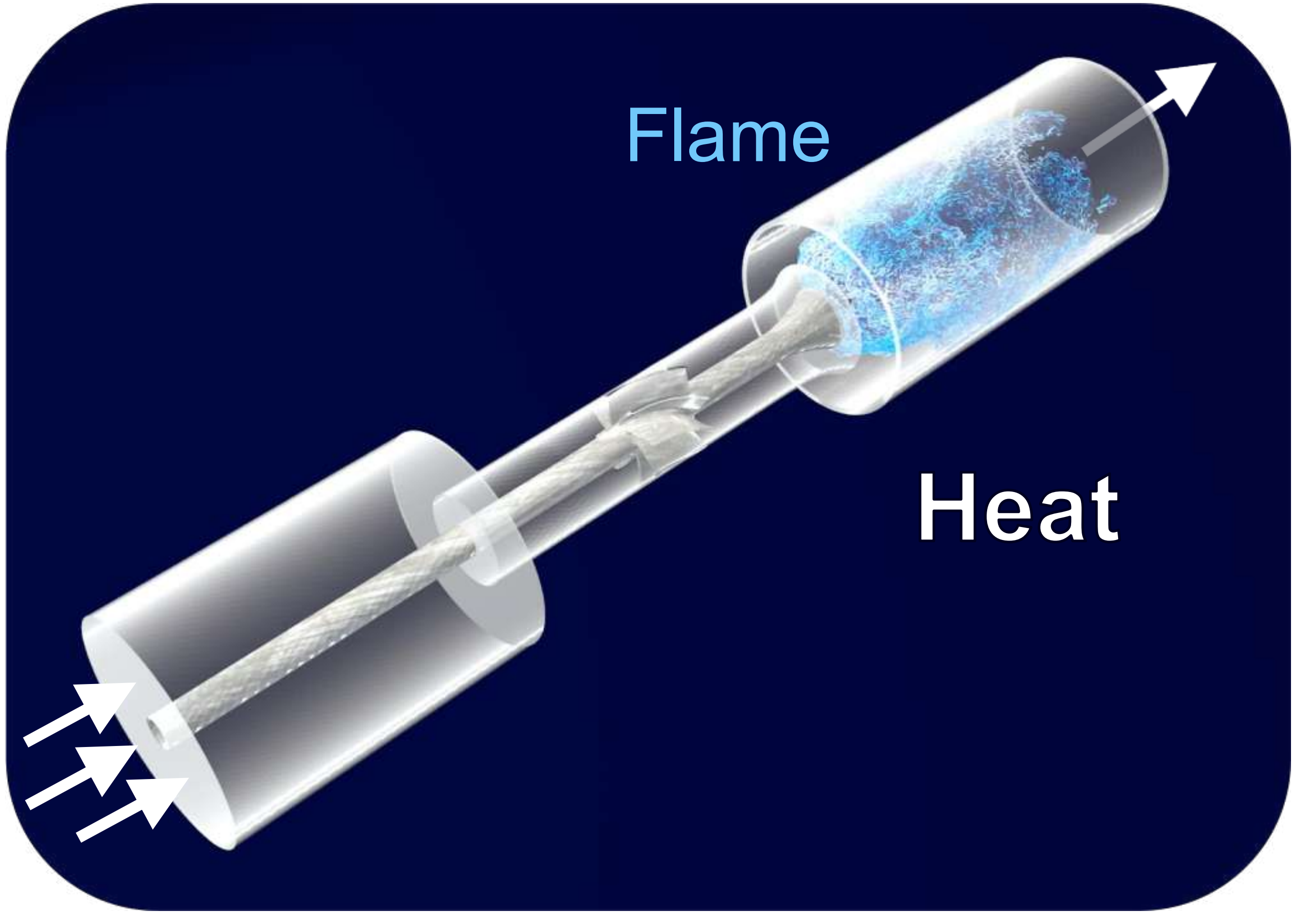


Turbulence

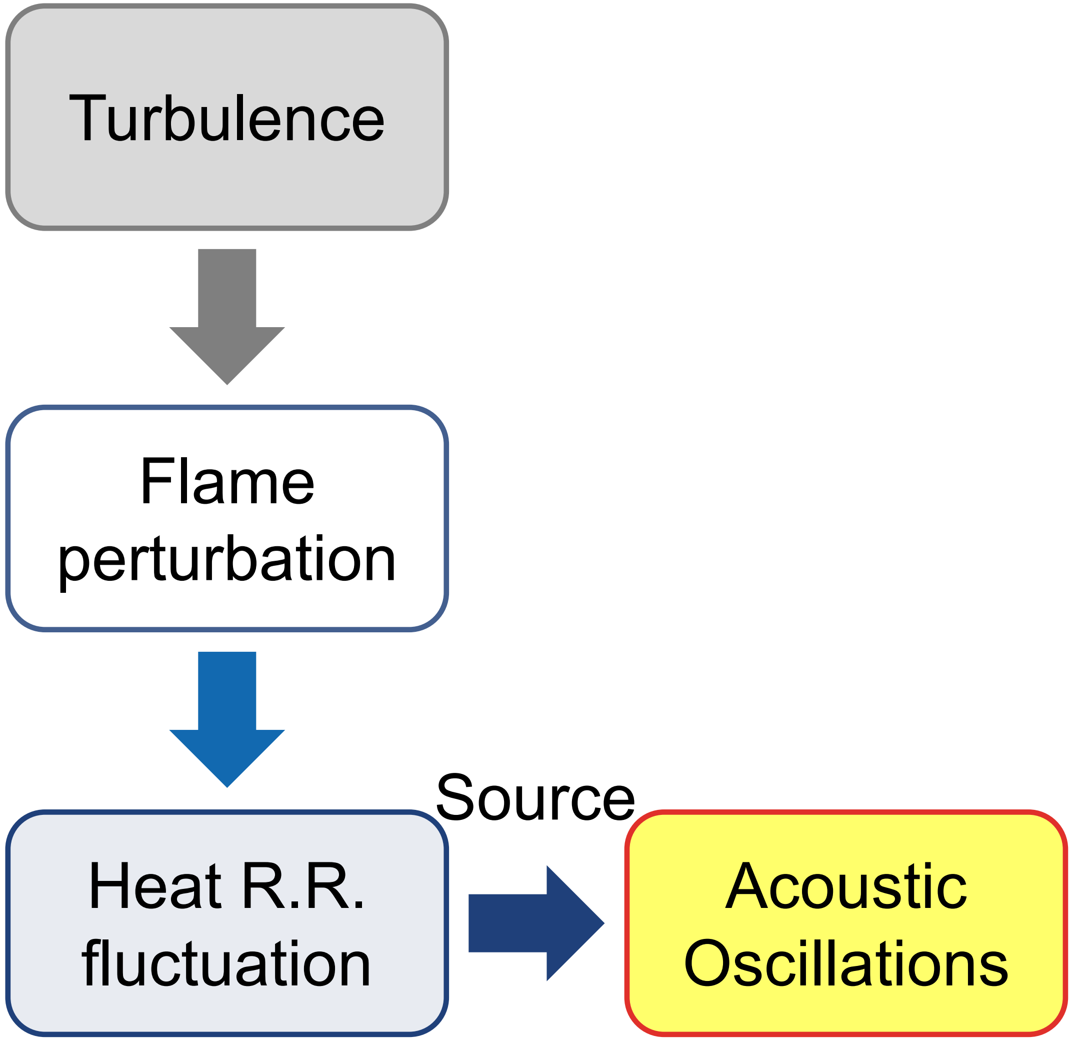
Flame perturbation

Flame Surface Variation

Heat R.R. fluctuation



Thermoacoustic coupling stems from heat-sound interaction



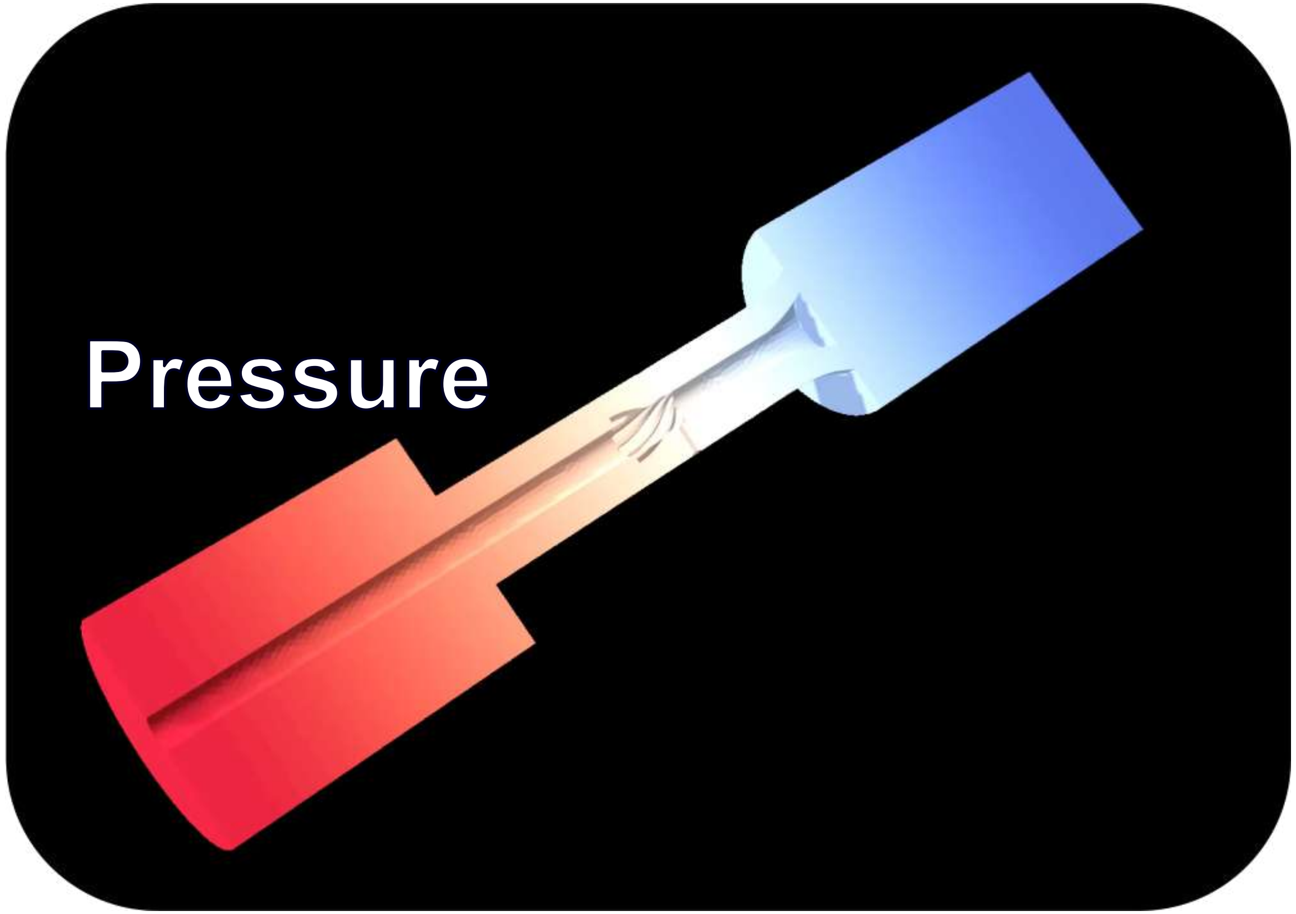
Turbulence

Flame perturbation

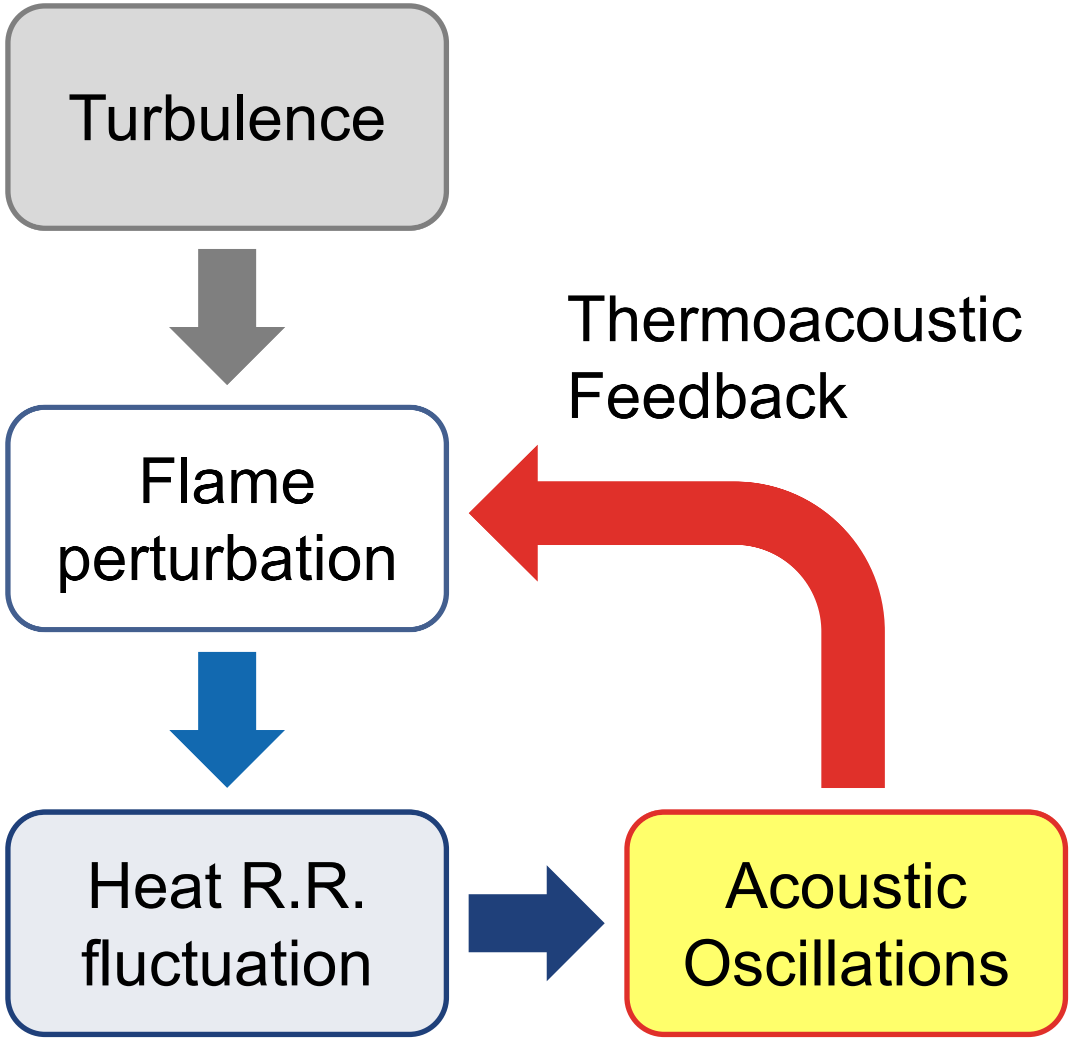
Heat R.R. fluctuation

Source

Acoustic Oscillations



Thermoacoustic coupling stems from heat-sound interaction



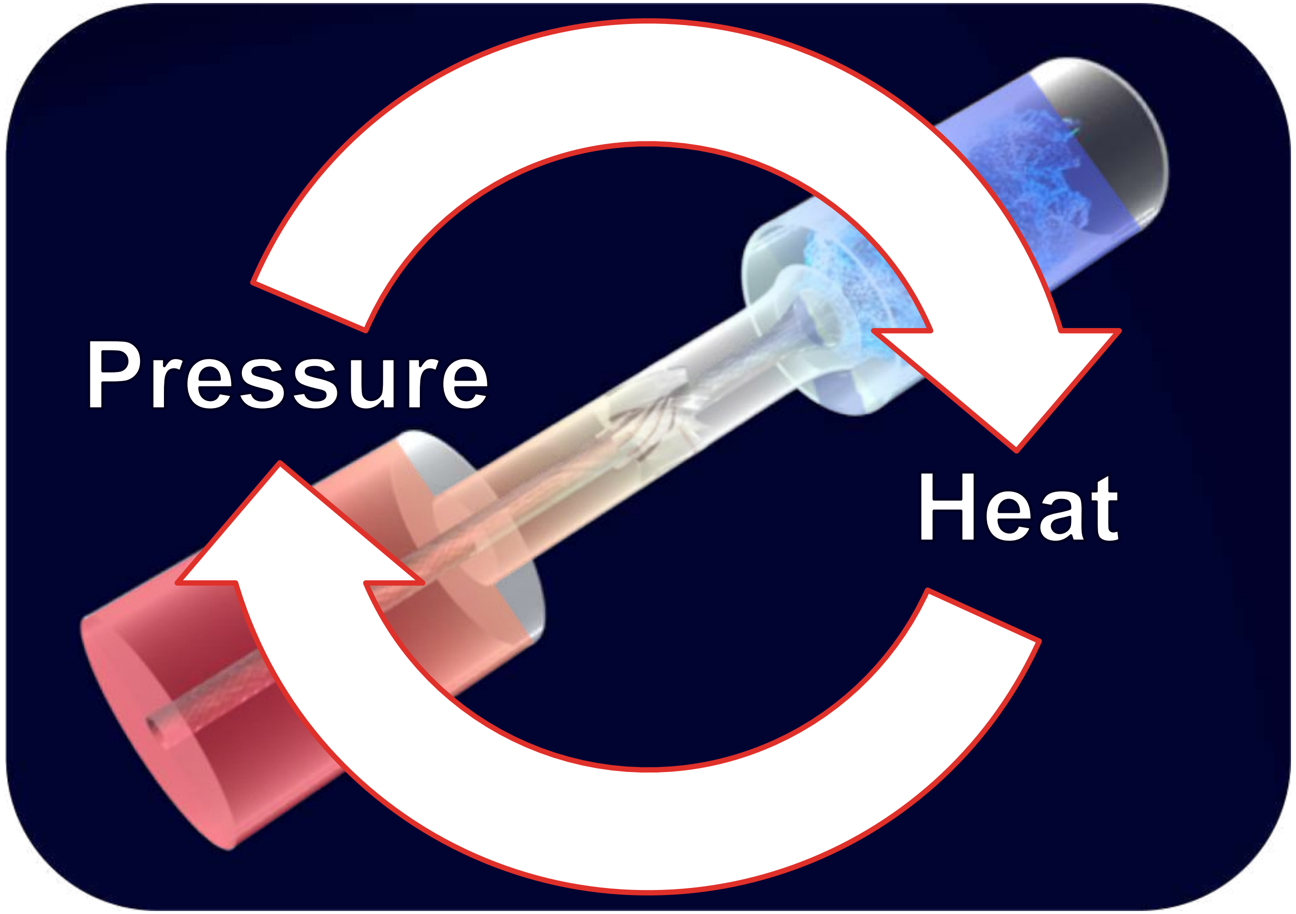
Turbulence

Flame perturbation

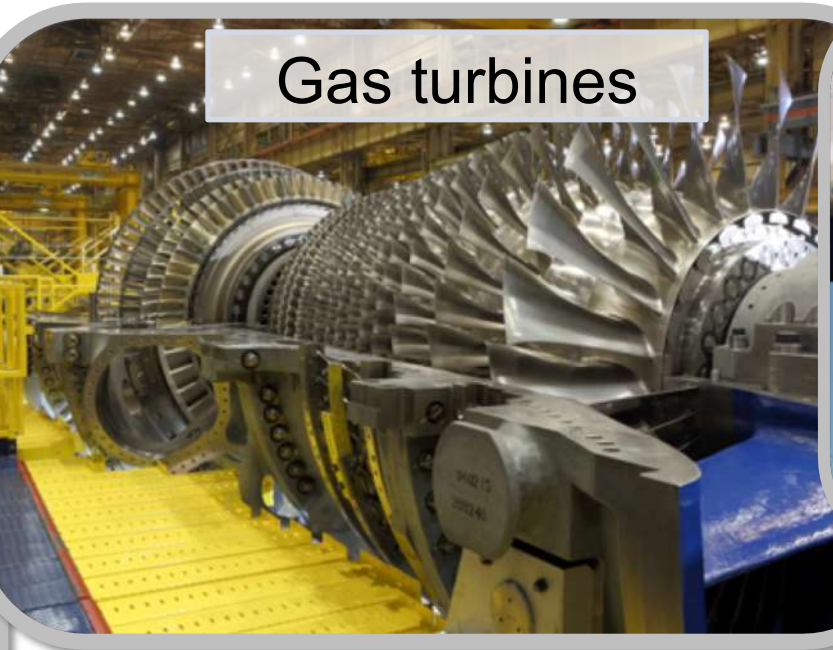
Heat R.R. fluctuation

Acoustic Oscillations

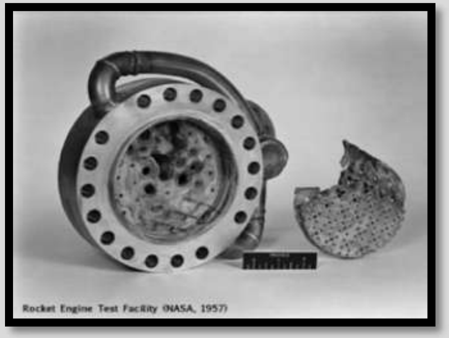
Thermoacoustic Feedback

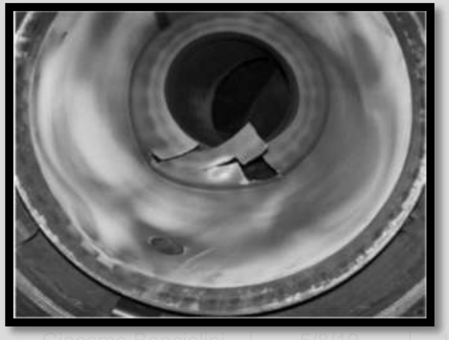


Thermoacoustic coupling concerns today's practical systems

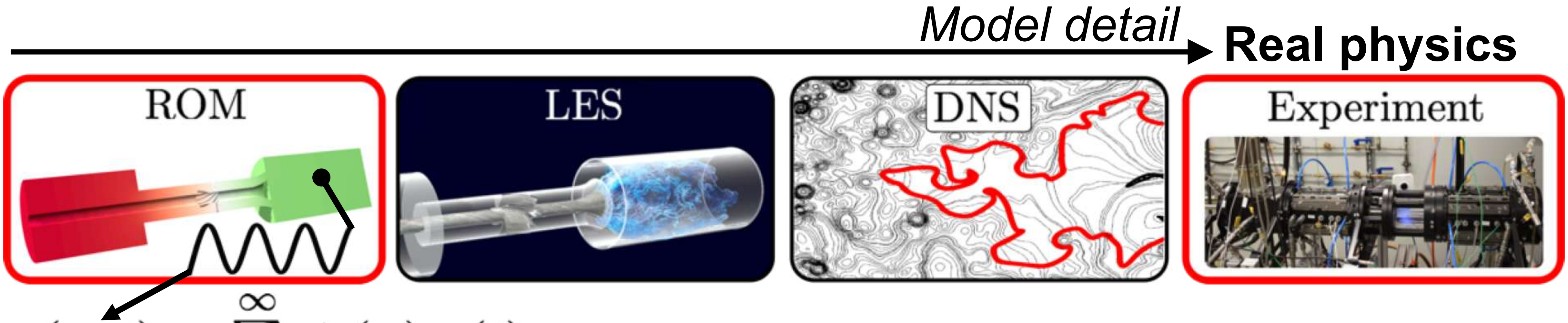


Structural damages


Rocket Engine Test Facility (NASA, 1957)



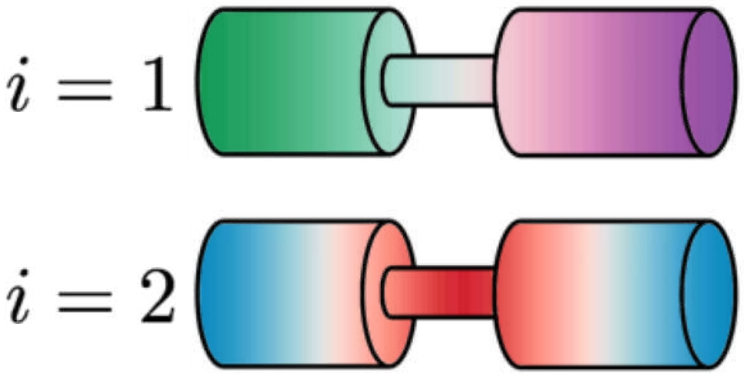
Thermoacoustics can be modelled with different levels of detail



Model detail

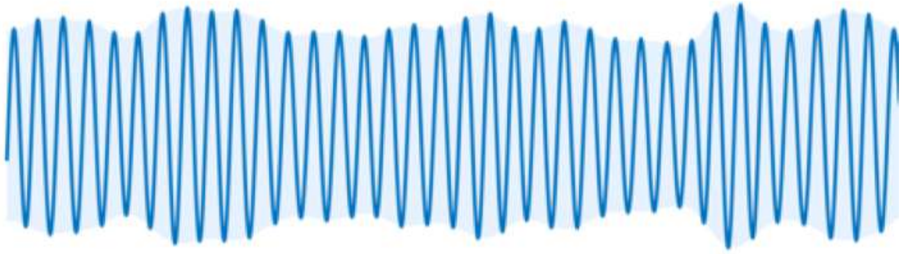
Real physics

$$p(\boldsymbol{x},t) = \sum_{i=1}^{\infty} \psi_i(\boldsymbol{x})\eta_i(t)$$

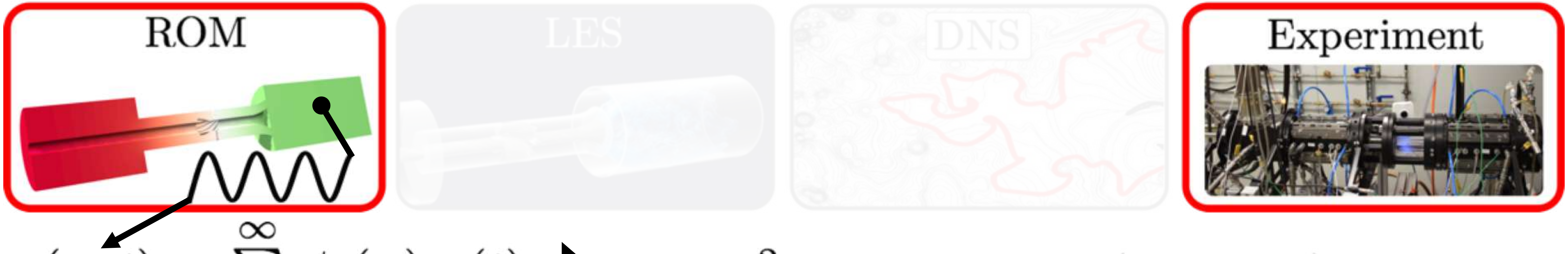


Thermoacoustics can be modelled with different levels of detail

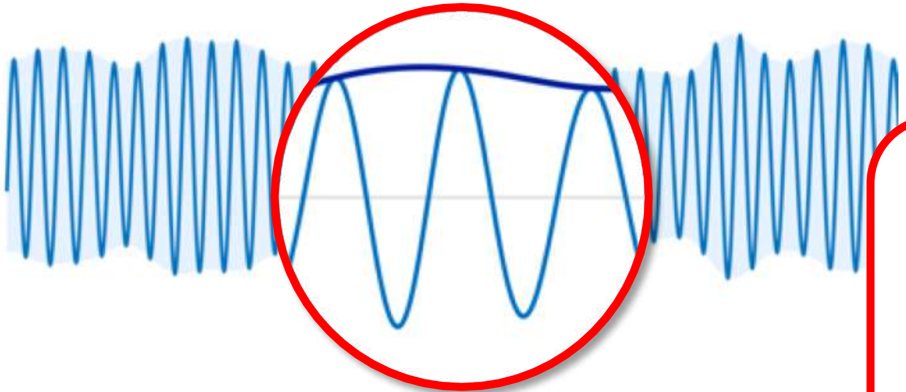
$$p(\boldsymbol{x},t)=\sum_{i=1}^{\infty}\psi_i(\boldsymbol{x})\eta_i(t) \Rightarrow \ddot{\eta}_i+\omega_i^2\eta_i=2\nu_i\dot{\eta}_i-g_i(\eta_j,\dot{\eta}_j,\ldots)_{j=1\ldots\infty}+\xi_i$$



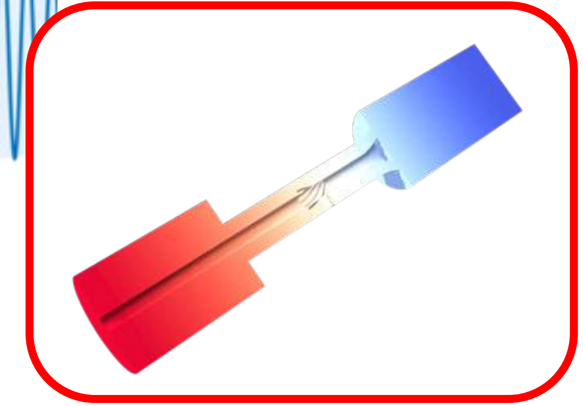
Thermoacoustics can be modelled with different levels of detail



$$p(\boldsymbol{x},t) = \sum_{i=1}^{\infty} \psi_i(\boldsymbol{x})\eta_i(t) \quad \Rightarrow \quad \underline{\ddot{\eta}_i + \omega_i^2\eta_i} = 2\nu_i\dot{\eta}_i - g_i(\eta_j,\dot{\eta}_j,\ldots)_{j=1\ldots\infty} + \xi_i$$

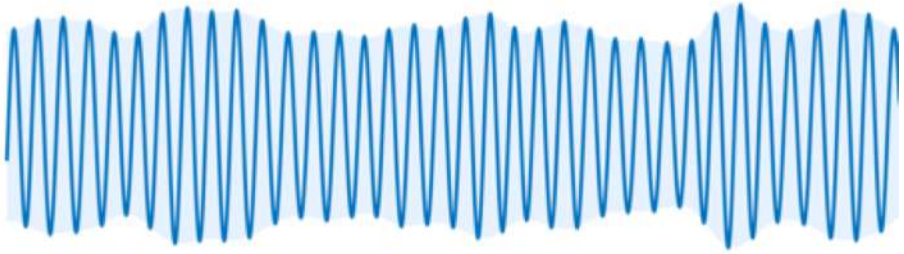


Oscillation

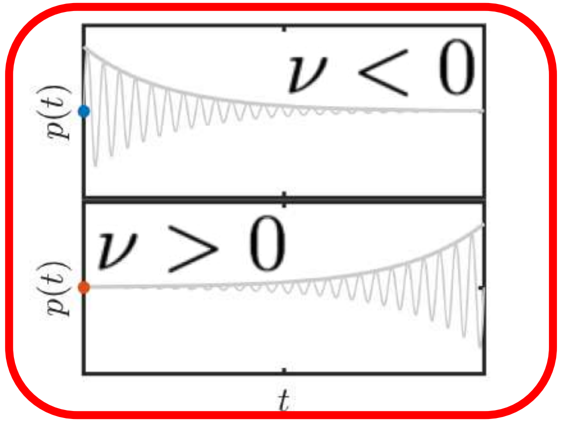


Thermoacoustics can be modelled with different levels of detail

$$p(\boldsymbol{x},t) = \sum_{i=1}^{\infty} \psi_i(\boldsymbol{x})\eta_i(t) \Rightarrow \ddot{\eta}_i + \omega_i^2\eta_i = 2\nu_i\dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j, \ldots)_{j=1\ldots\infty} + \xi_i$$

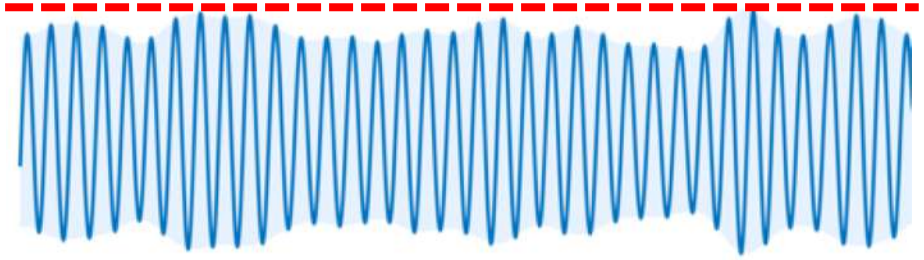


Linear decay/growth



Thermoacoustics can be modelled with different levels of detail

$$p(\boldsymbol{x},t)=\sum_{i=1}^{\infty}\psi_i(\boldsymbol{x})\eta_i(t) \Rightarrow \ddot{\eta}_i+\omega_i^2\eta_i=2\nu_i\dot{\eta}_i-\underbrace{g_i(\eta_j,\dot{\eta}_j,\dots)_{j=1\dots\infty}}+\xi_i$$



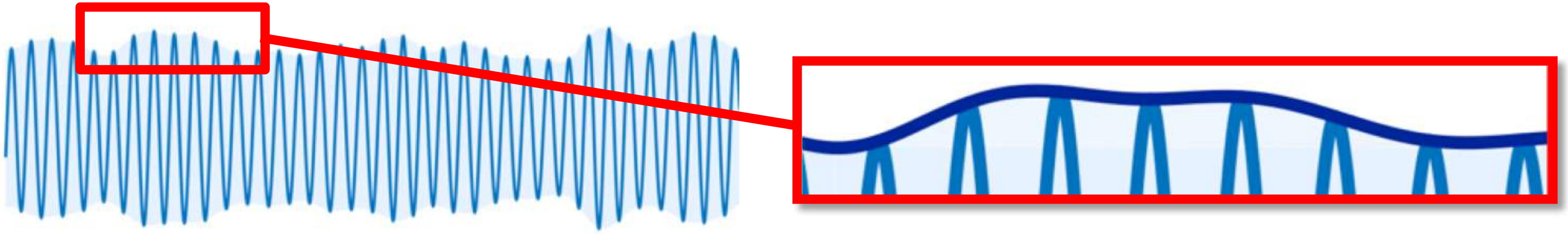
Flame Non-Linear Response

\+

Modes coupling

Thermoacoustics can be modelled with different levels of detail

$$p(\boldsymbol{x},t)=\sum_{i=1}^{\infty}\psi_i(\boldsymbol{x})\eta_i(t) \;\rightarrow\; \ddot{\eta}_i+\omega_i^2\eta_i=2\nu_i\dot{\eta}_i-g_i(\eta_j,\dot{\eta}_j,\ldots)_{j=1\ldots\infty}+\xi_i$$



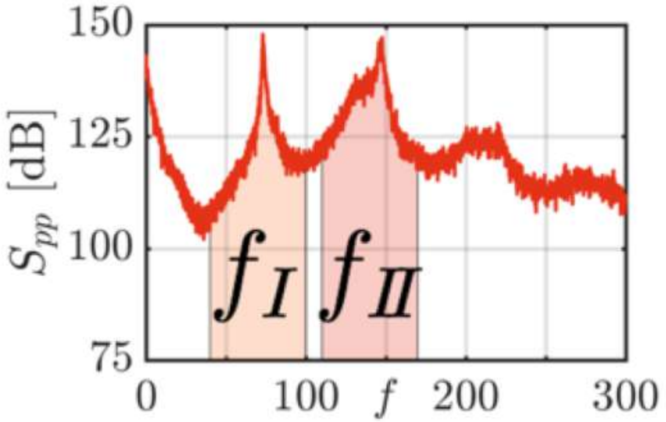
Effect of turbulence:
Random Amplitude

Changing oscillator's elements to reproduce different dynamics

$$p(\boldsymbol{x},t) = \sum_{i=1}^{N} \psi_i(\boldsymbol{x})\eta_i(t)$$

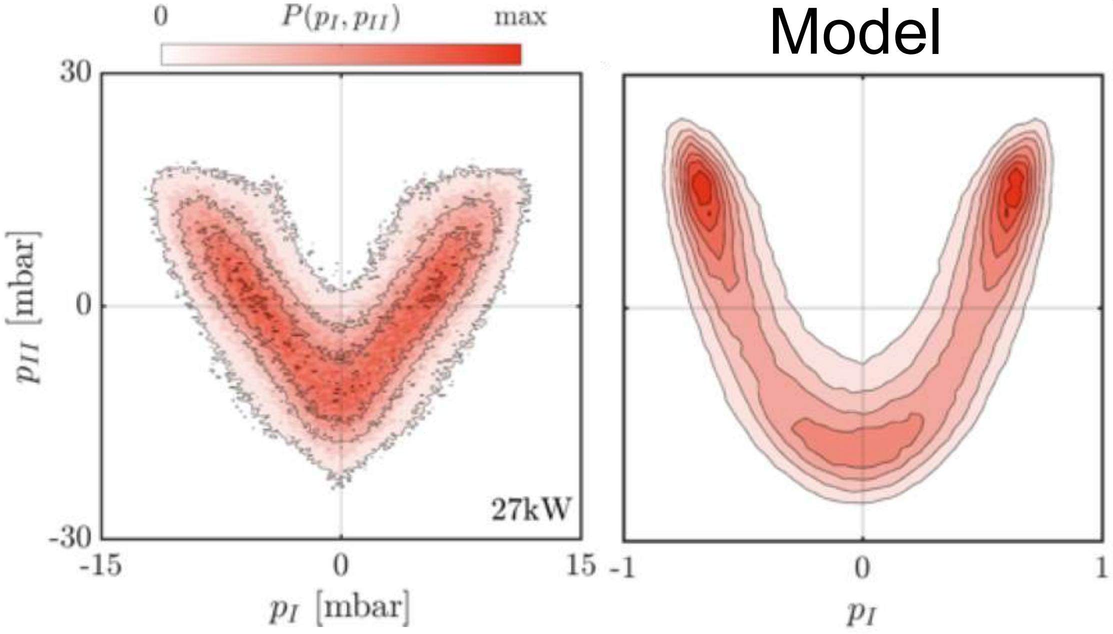
$$\ddot{\eta}_i + \omega_i^2\eta_i = 2\nu_i\dot{\eta}_i - g_i(\eta_j,\dot{\eta}_j) + \xi_i$$

Experiment – 2 Modes

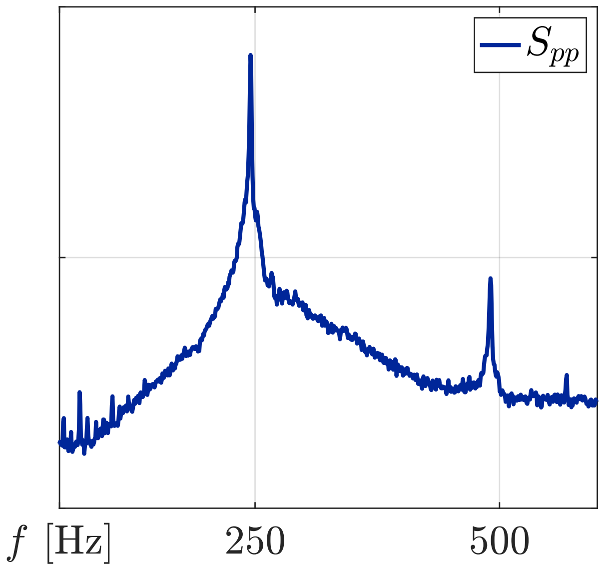


Bonciolini, Noiray, "Synchronization of Thermoacoustic Modes in Sequential Combustors", Journal of Engineering for Gas Turbines and Power (2018).

Model



1 Mode

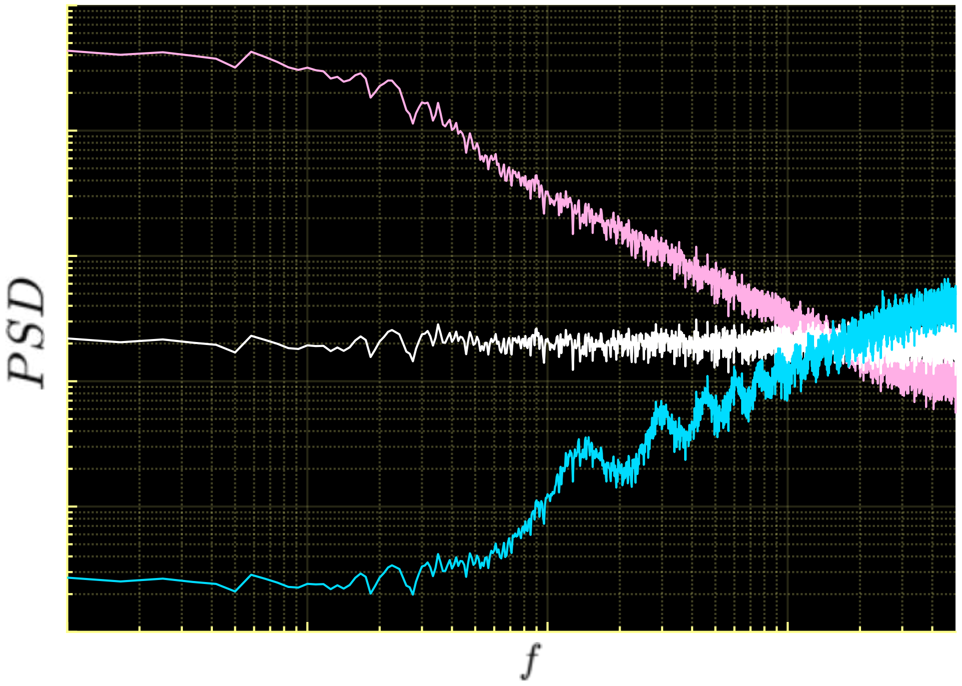


Choice #1: Number of modes

Changing oscillator's elements to reproduce different dynamics

$$\ddot{\eta} + \omega_0^2 \eta = 2\nu\dot{\eta} - g(\eta, \dot{\eta}) + \boxed{\xi}$$

Noise color ⇔ Spectral content



🔊 **White Noise**
constant

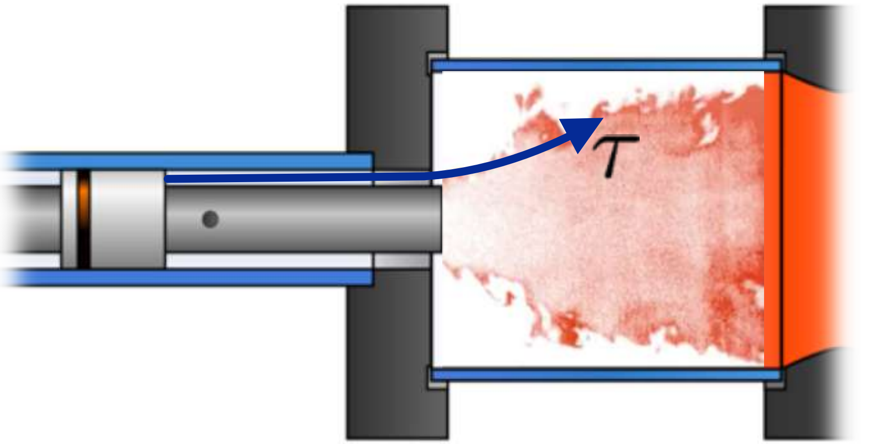
OK

Bonciolini, Boujo, Noiray, "Output-only parameter identification of a colored-noise-driven Van-der-Pol oscillator: Thermoacoustic instabilities as an example", Physical Review E, (2017).

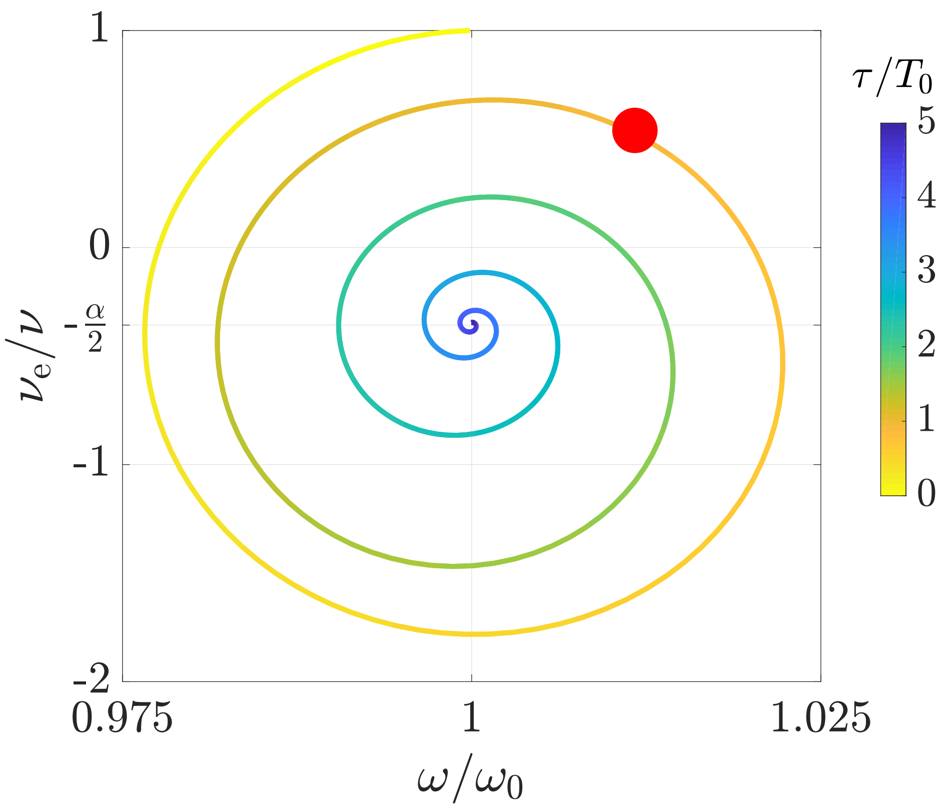
Choice #2: Noise color

Changing oscillator’s elements to reproduce different dynamics

$$\ddot{\eta} + \omega_0^2 \eta = \underline{2\nu\dot{\eta} - g(\eta, \dot{\eta})} + \xi$$



Perturbations - Time delay τ



Effective LGR

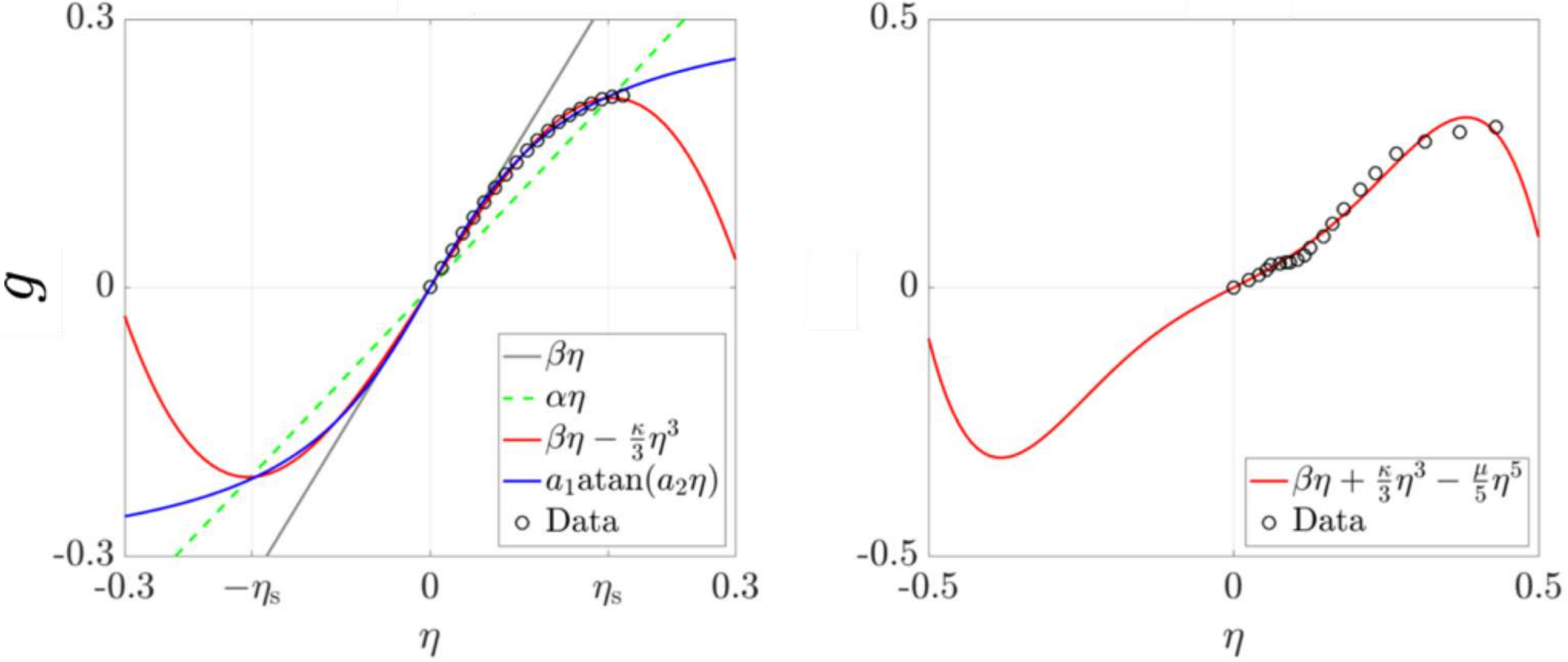
$\nu_e \quad \cancel{\tau}$

Bonciolini, Bourquard, Noiray, “Effect of flame response delay and nonlinearity on the modelling of thermoacoustic instabilities”, in preparation

Choice #3: Flame Response Delay

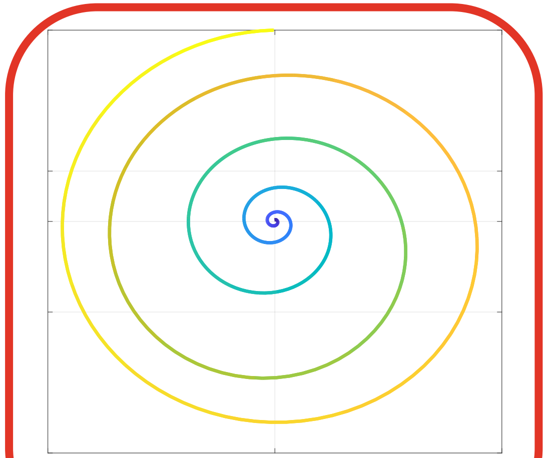
Changing oscillator's elements to reproduce different dynamics

$$\ddot{\eta} + \omega_0^2 \eta = 2\nu\dot{\eta} - \underline{g(\eta, \dot{\eta})} + \xi$$

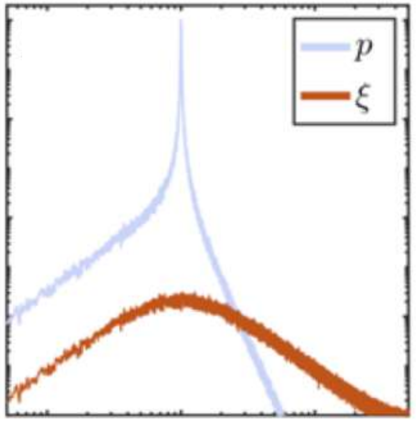


Choice #4: Nonlinearity

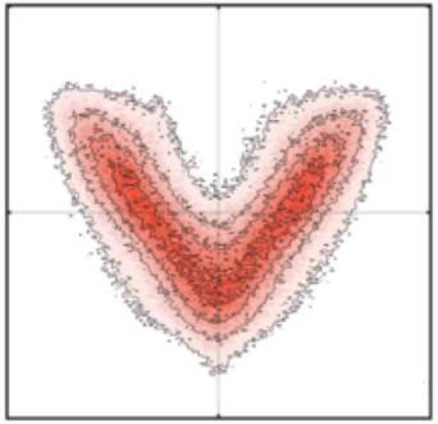
Model Elements



Delay - NL



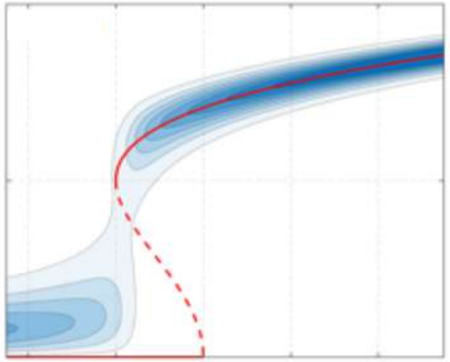
Noise Color



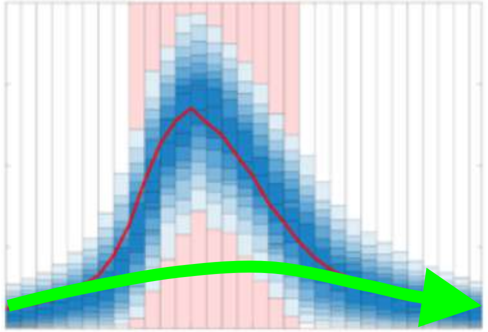
Two Modes

My PhD

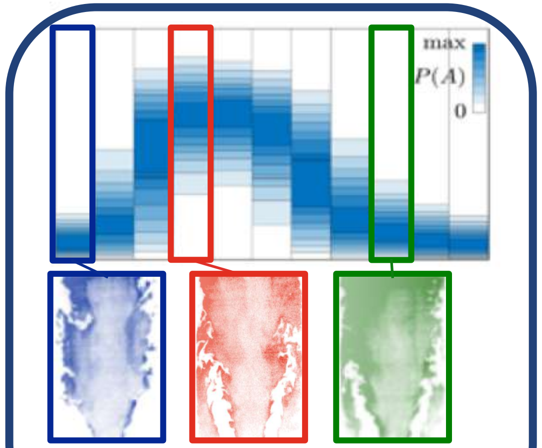
Transients



Subcritical Tipping

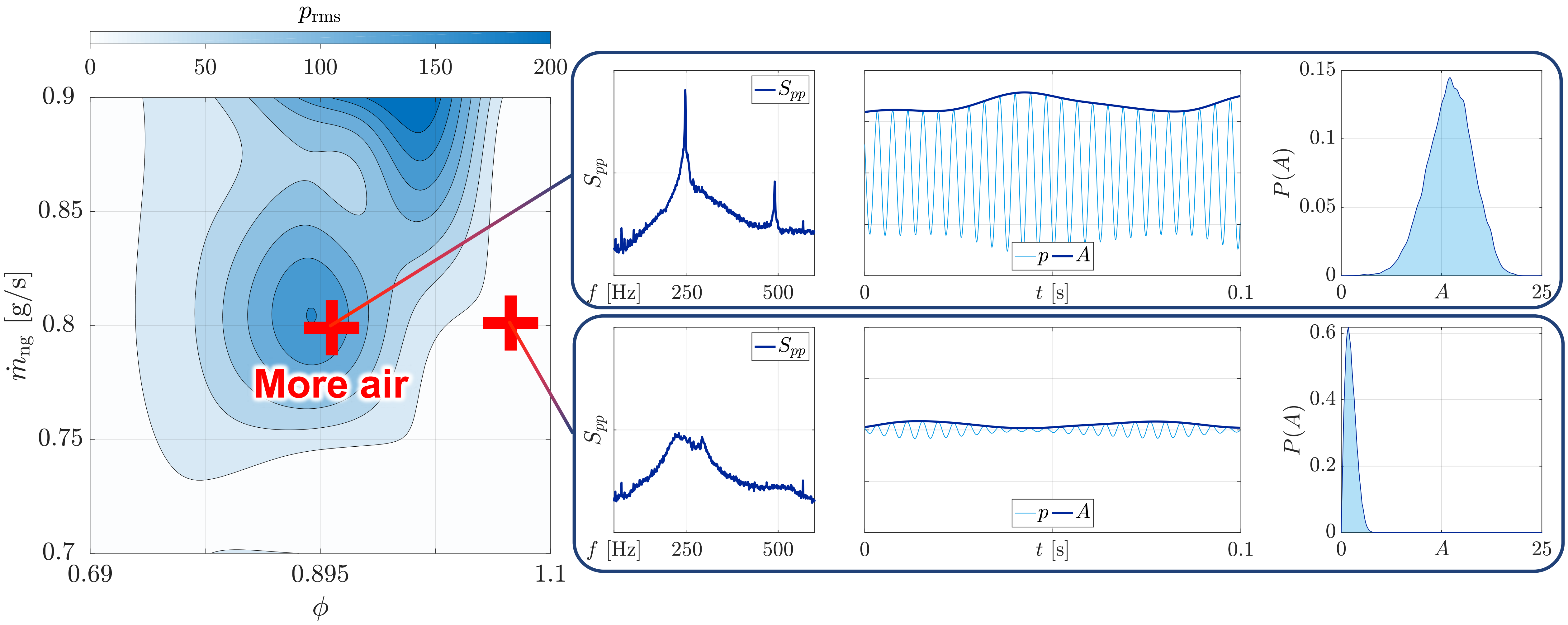


Bifurcation Dodge

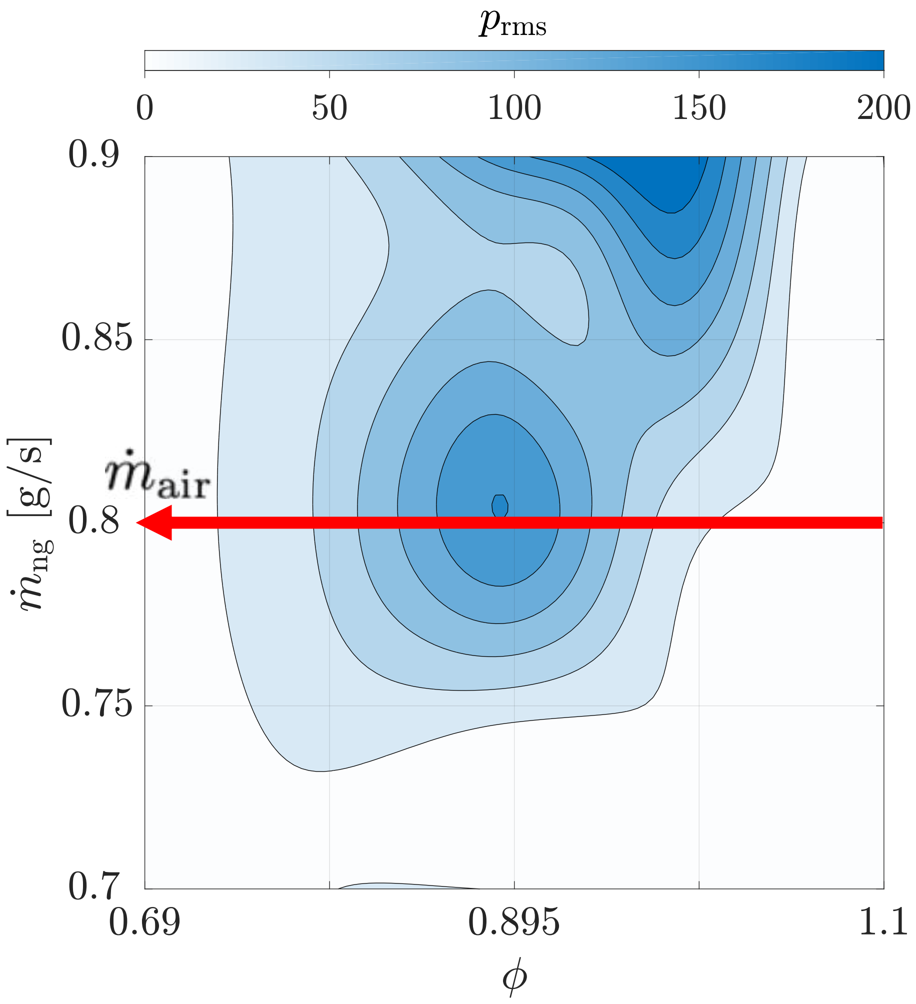


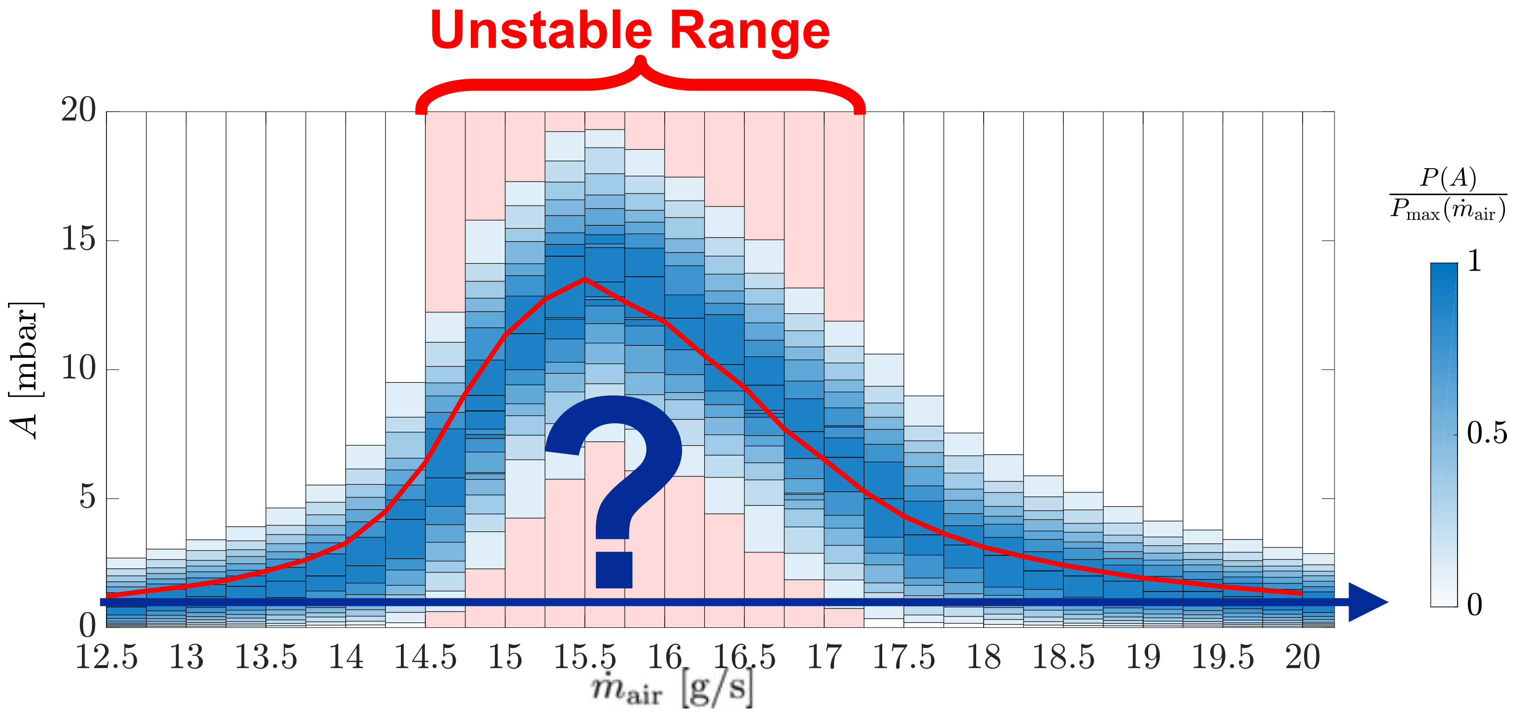
Thermal Inertia

Thermoacoustics depends on operating conditions



Thermoacoustics depends on operating conditions

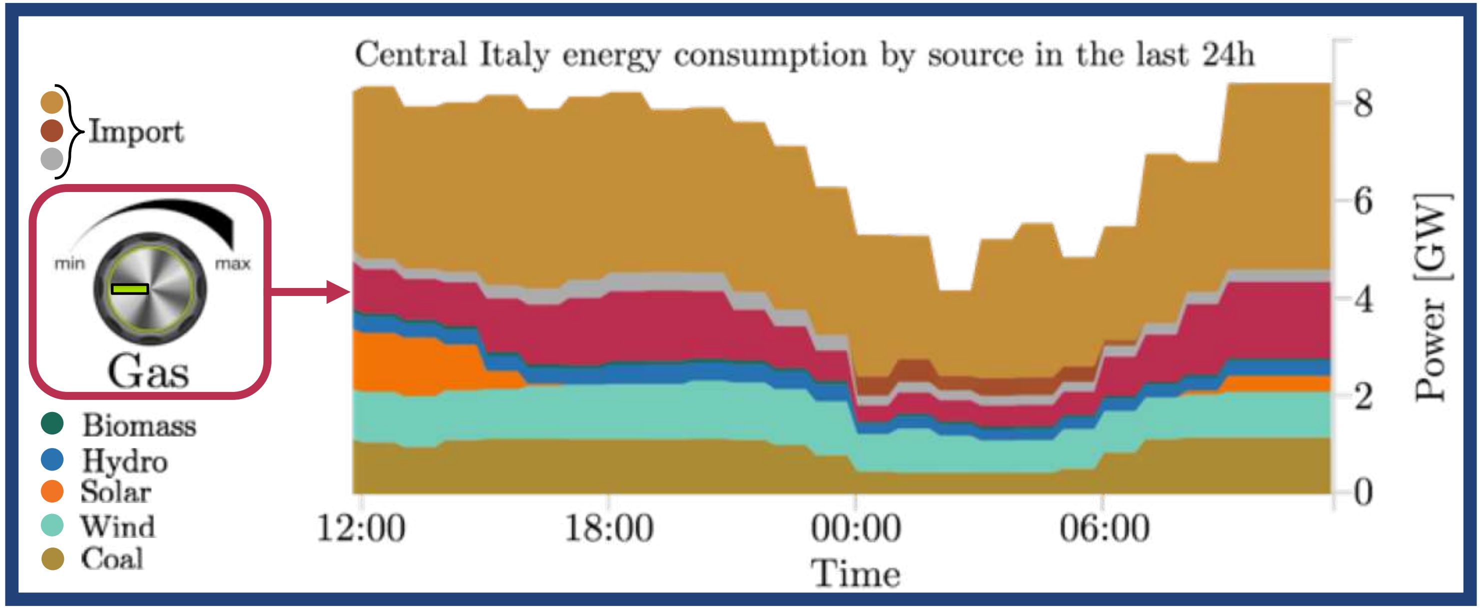




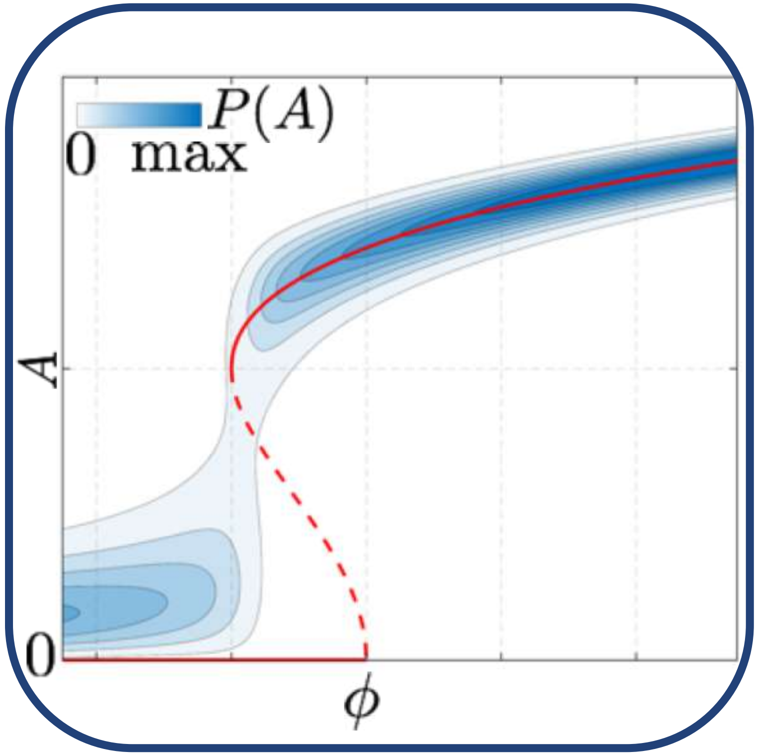
- Bifurcation: dynamics change with one parameter

- **Transient dynamics through the bifurcation?**

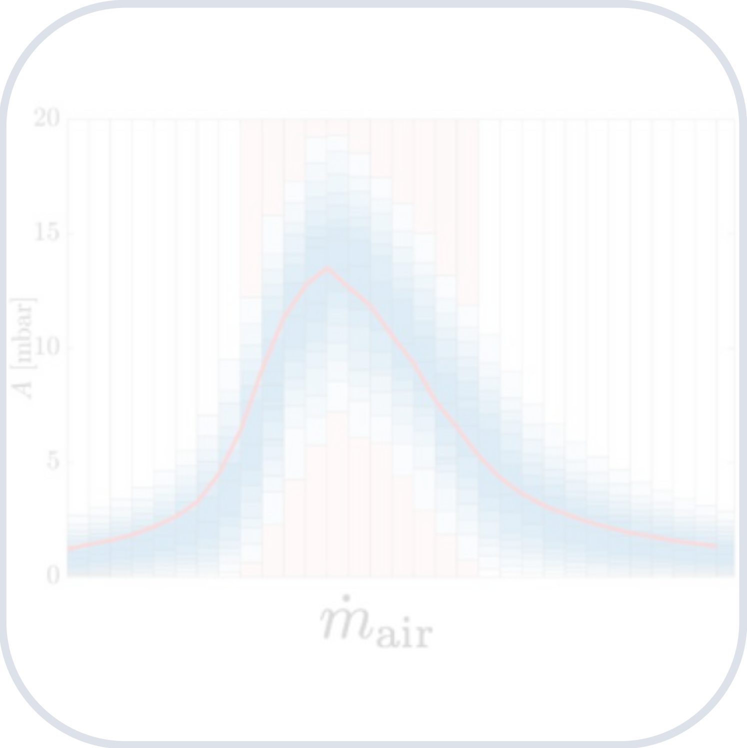
Transient Thermoacoustics is a relevant problem for Gas Turbines



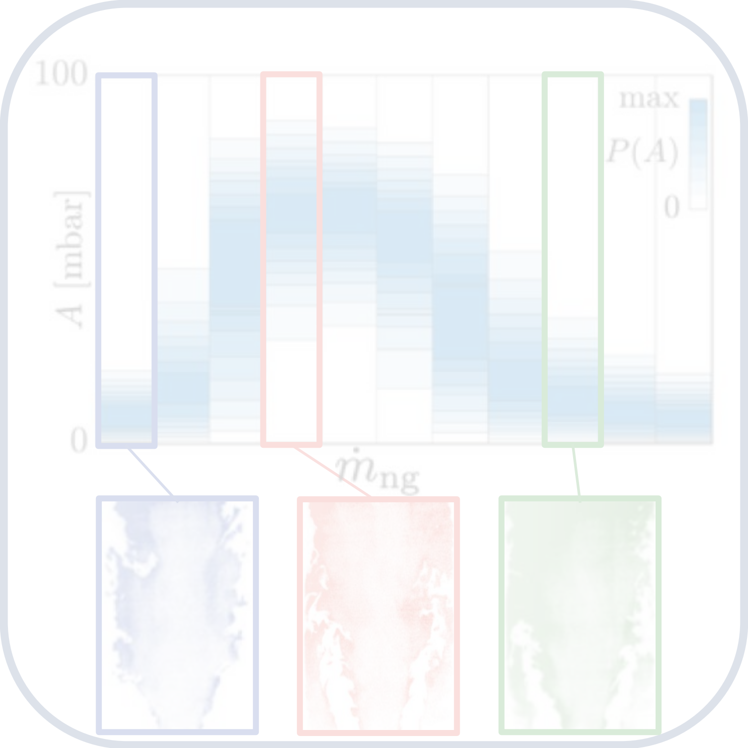
The transient thermoacoustics trilogy



Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation
Royal Society Open Science

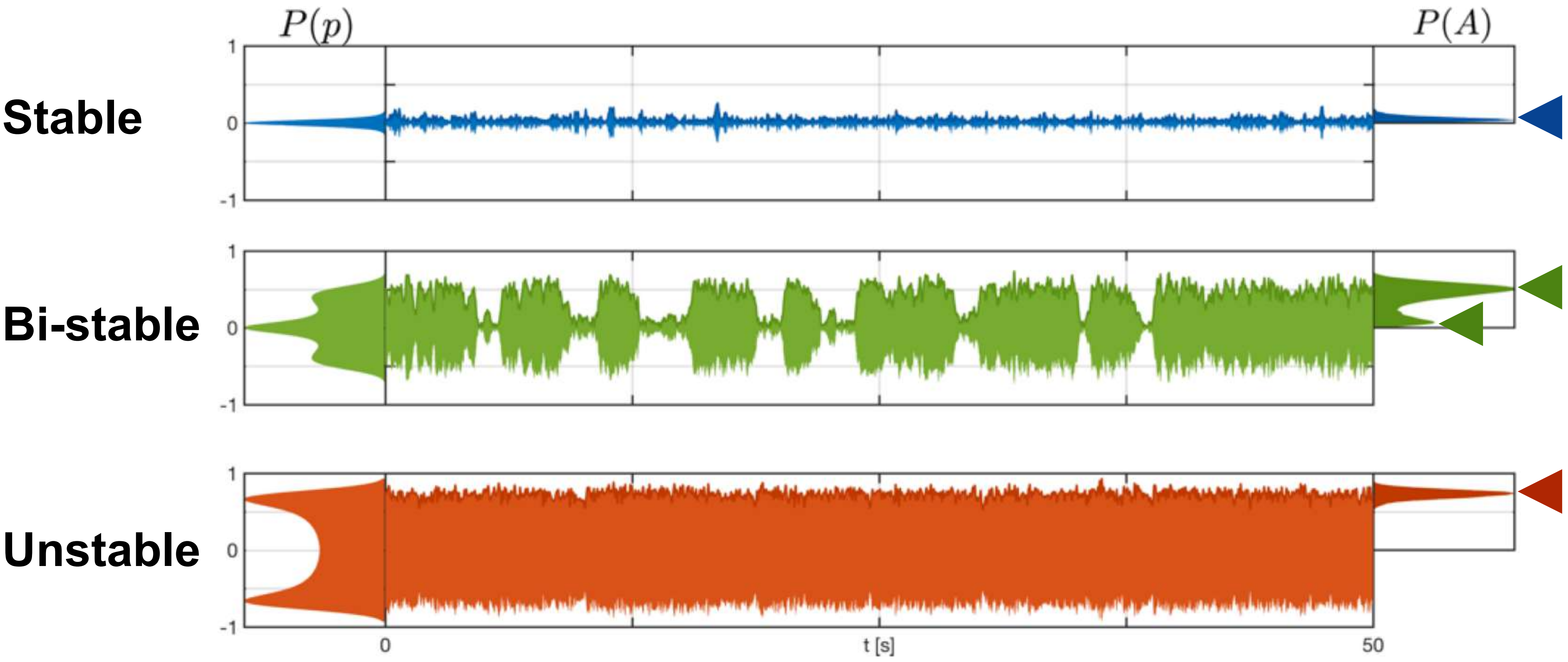


Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation
Nonlinear Dynamics

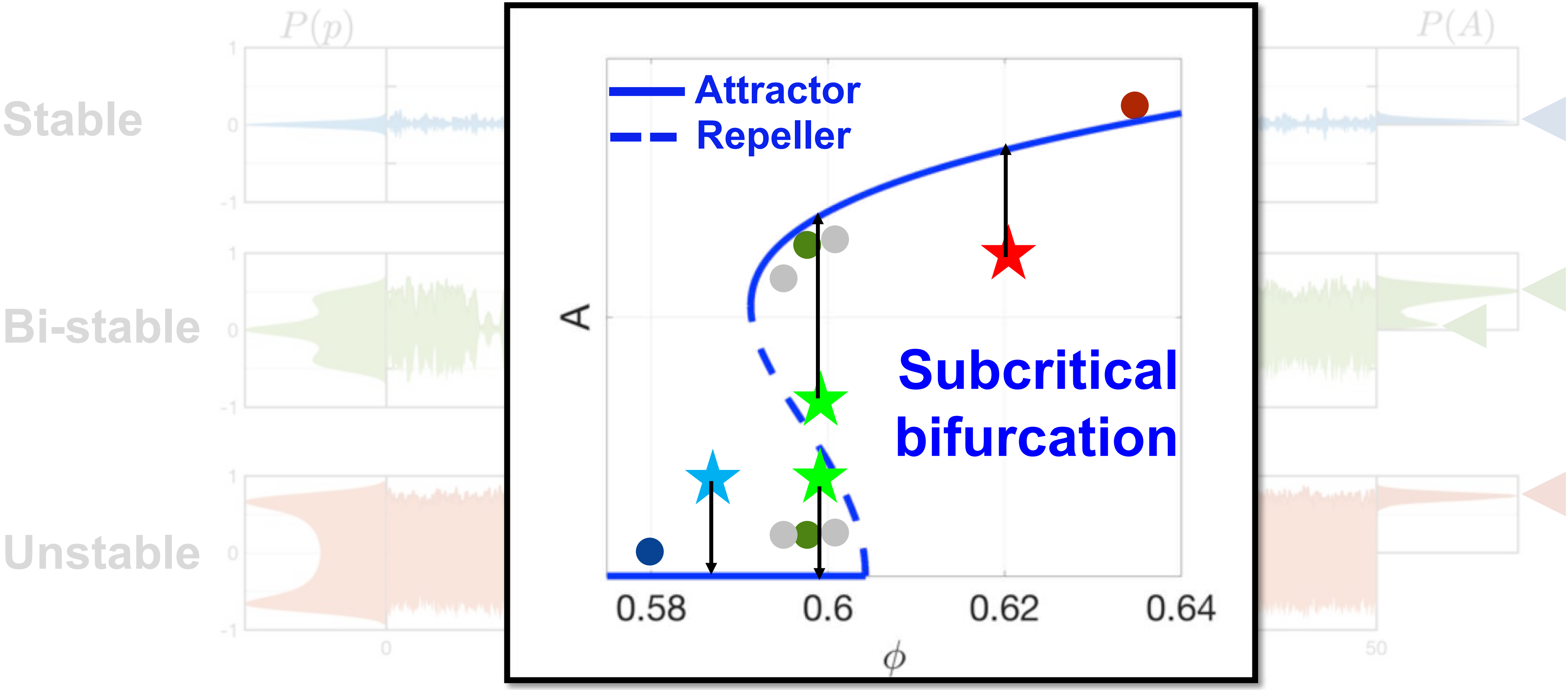


Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame
Proceedings of the Combustion Institute

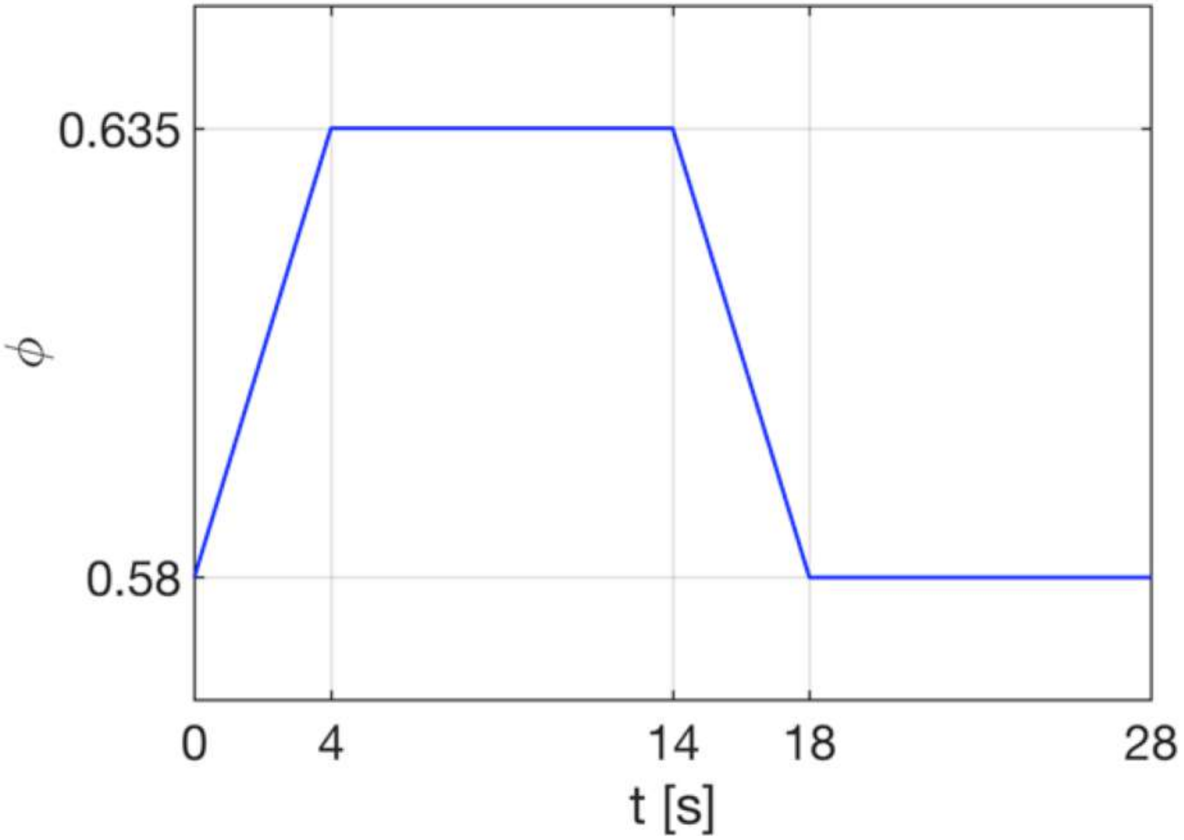
The dynamics changes for different fuel/air ratios



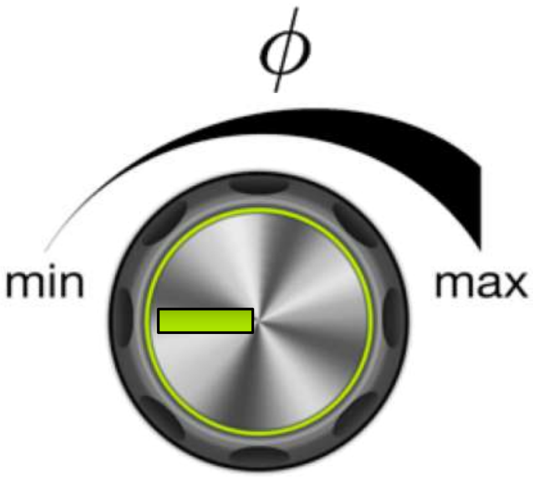
The dynamics changes for different fuel/air ratios



Transient dynamics is explored by ramping the fuel/air ratio

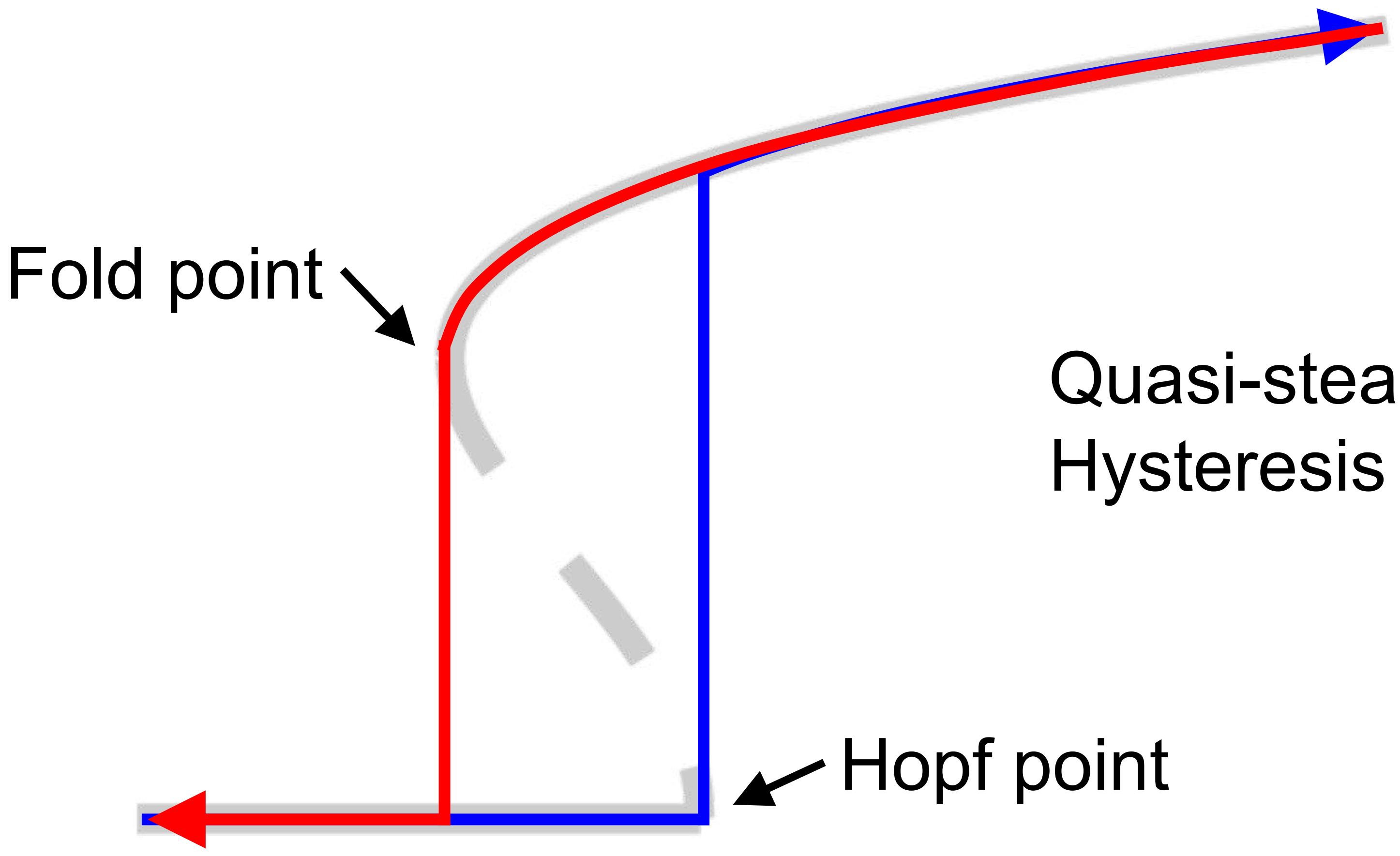


$$\phi = \frac{(\dot m_{\mathrm{CH_4}}/\dot m_{\mathrm{air}})}{(\dot m_{\mathrm{CH_4}}/\dot m_{\mathrm{air}})_{\mathrm{stoich.}}}$$



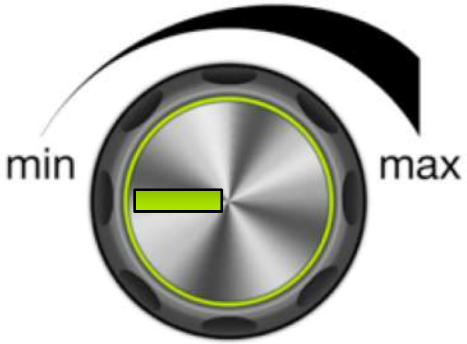
This cycle is repeated 100 times

Transient dynamics is explored by ramping the fuel/air ratio

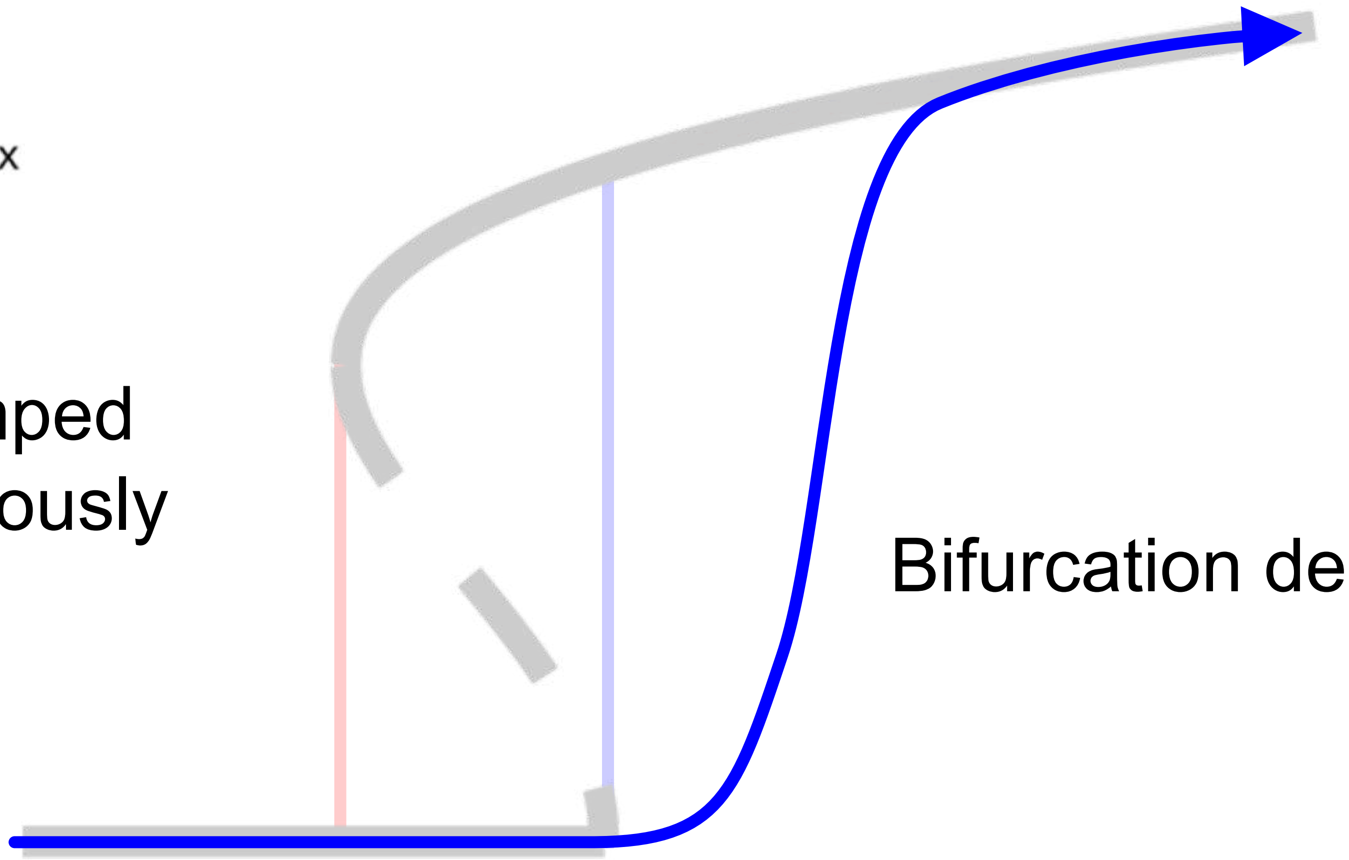


Quasi-steady ramps:
Hysteresis

Transient dynamics is explored by ramping the fuel/air ratio

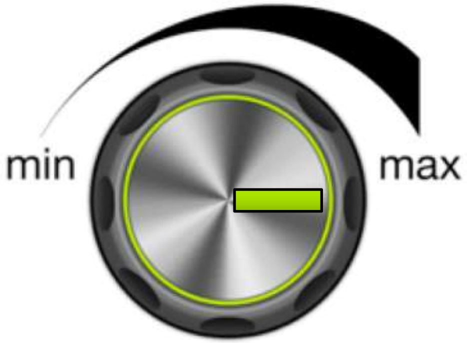


Fuel/air ratio ramped **up**&down continuously

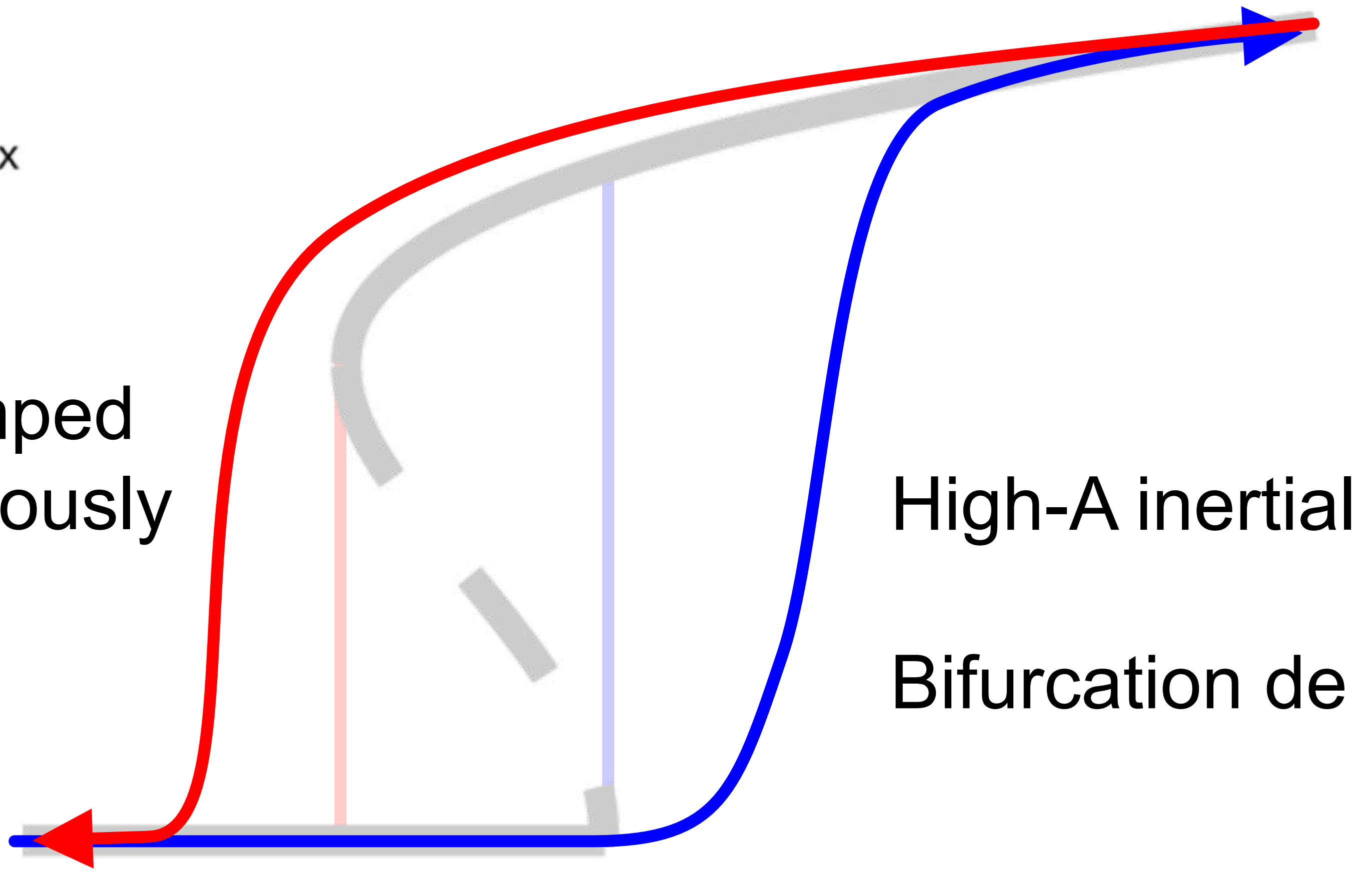


Bifurcation delay

Transient dynamics is explored by ramping the fuel/air ratio



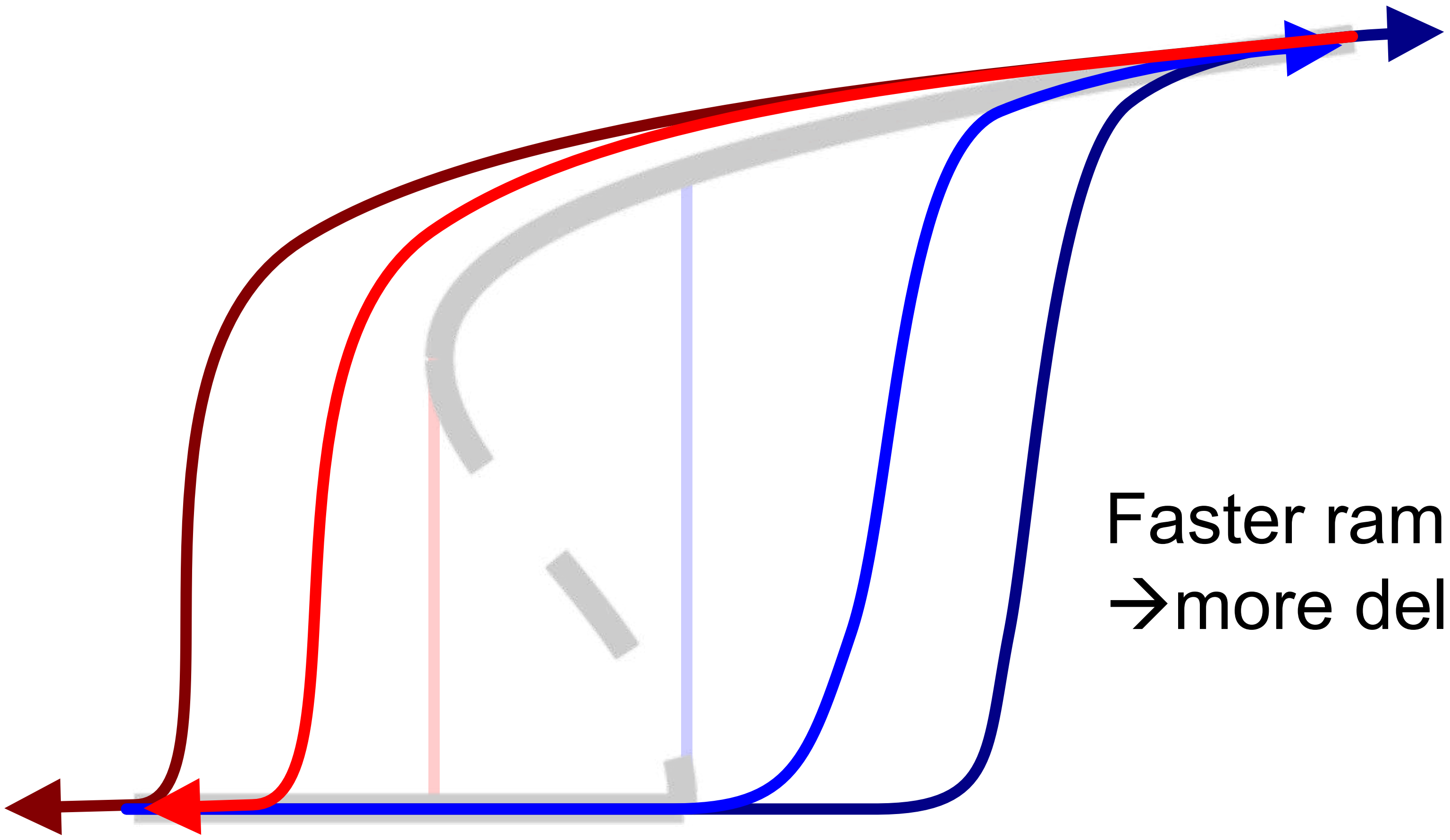
Fuel/air ratio ramped up&**down** continuously



High-A inertial effects

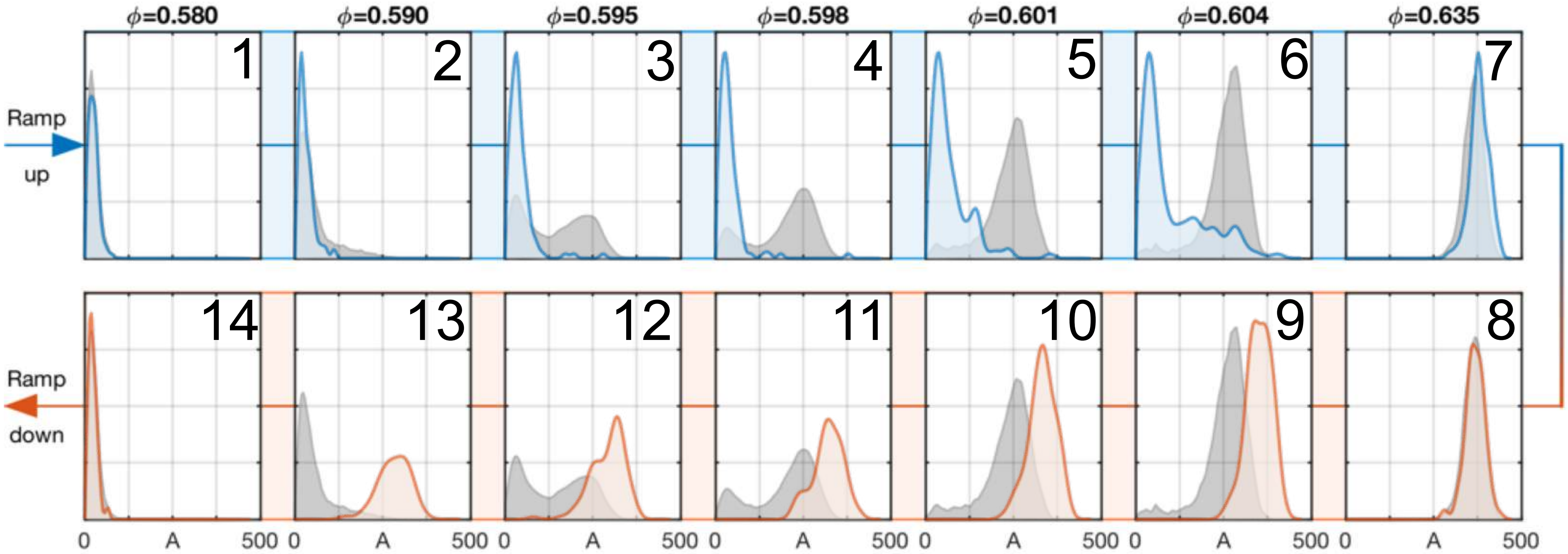
Bifurcation delay

Tipping delay depends on the ramp rate



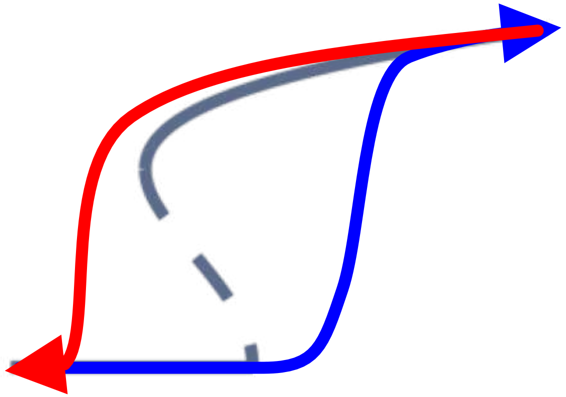
Faster ramp
→more delay

Tipping delay is observed experimentally



No bistability → Hysteresis

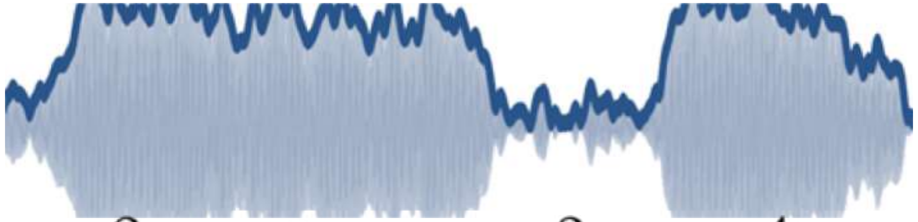
Bifurcation delay



▨ Stationary

◩ Ramp

The model can reproduce the thermoacoustic bifurcation



$$\ddot{p} + \omega_0^2 p = [2\nu + \kappa p^2 - \underline{\gamma p^4}]\dot{p} + \xi$$

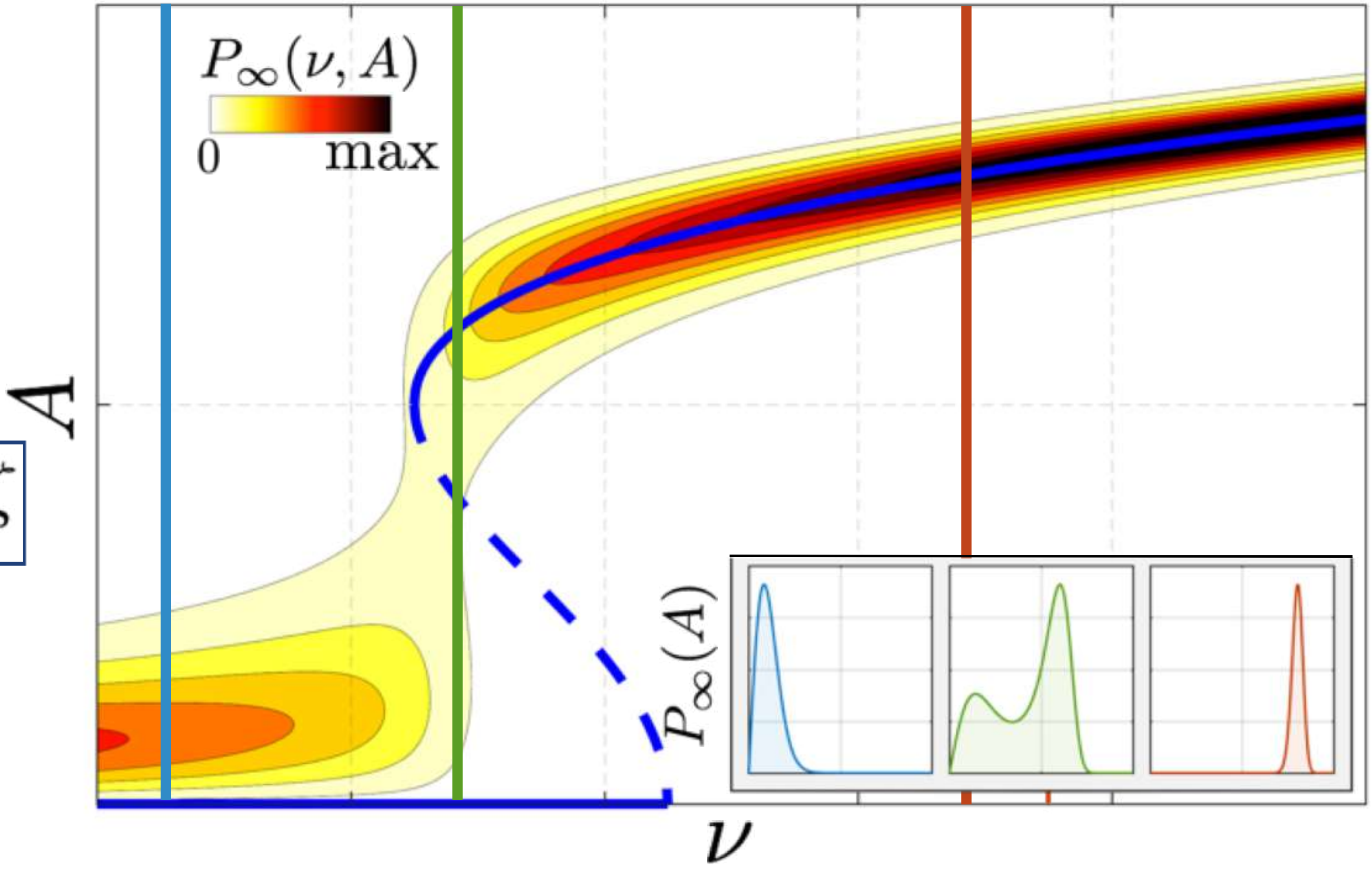
5th order

Envelope

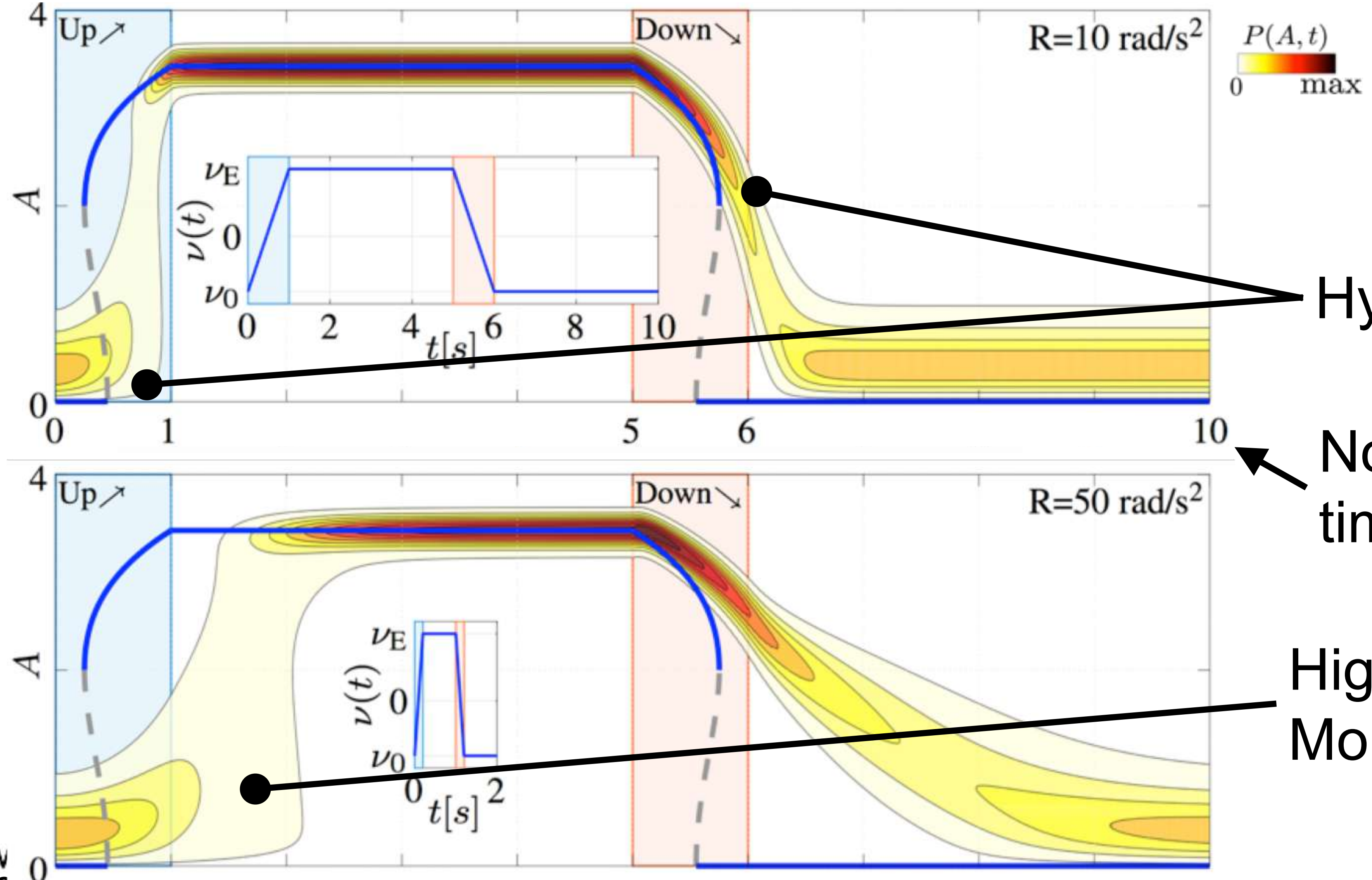
$$\boxed{\dot{A} = \nu A + \frac{\kappa}{8}A^3 - \frac{\gamma}{32}A^5 + \frac{\Gamma}{4\omega_0^2 A} + \zeta}$$

Fokker-Planck eq.

$$\boxed{\frac{\partial P}{\partial t} = -\frac{\partial}{\partial A}[\mathcal{F}(A,t)P] + \frac{\Gamma}{4\omega_0^2}\frac{\partial^2 P}{\partial A^2}}$$



The model can reproduce the transient dynamics



Hysteresis

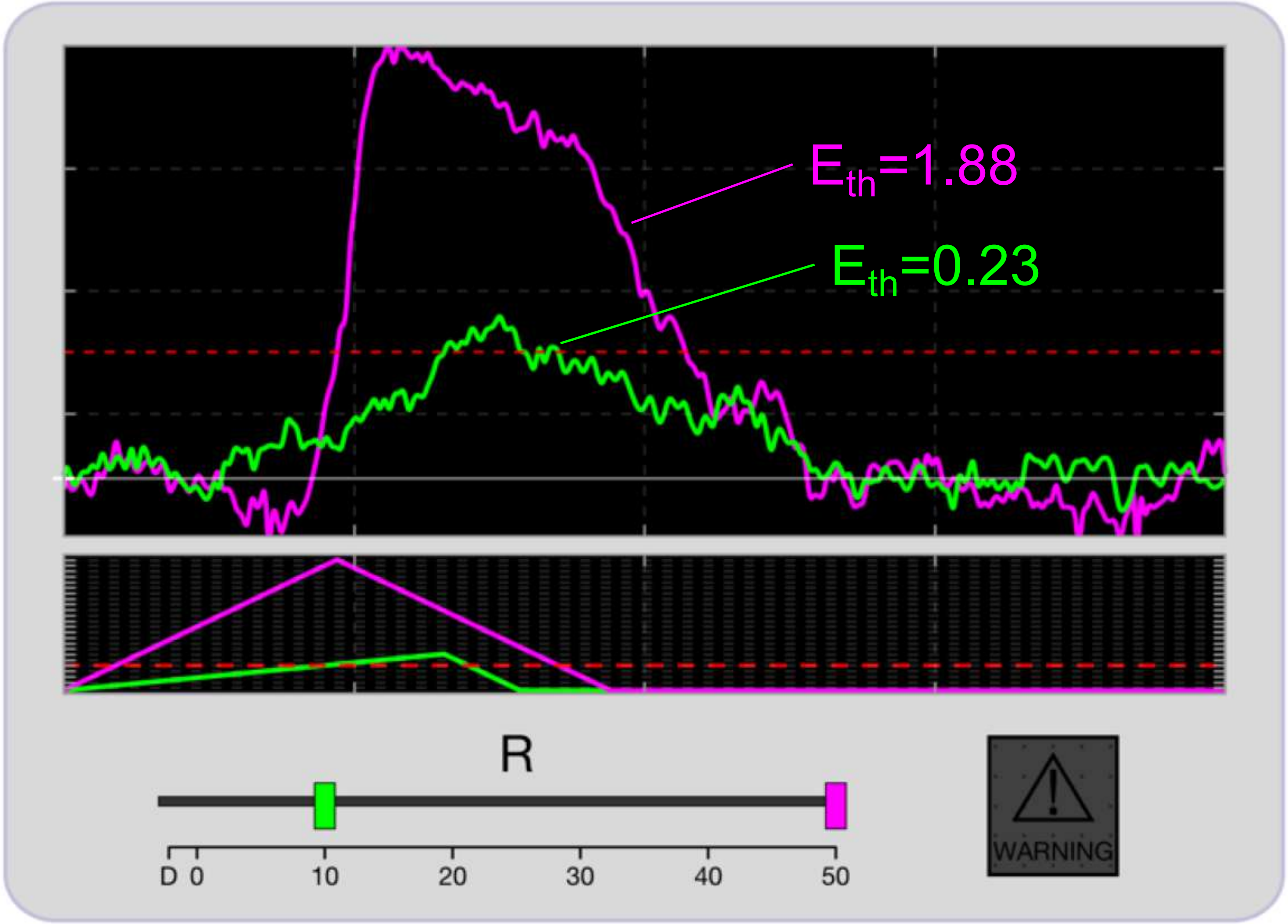
Normalized time

Higher R→ More delay

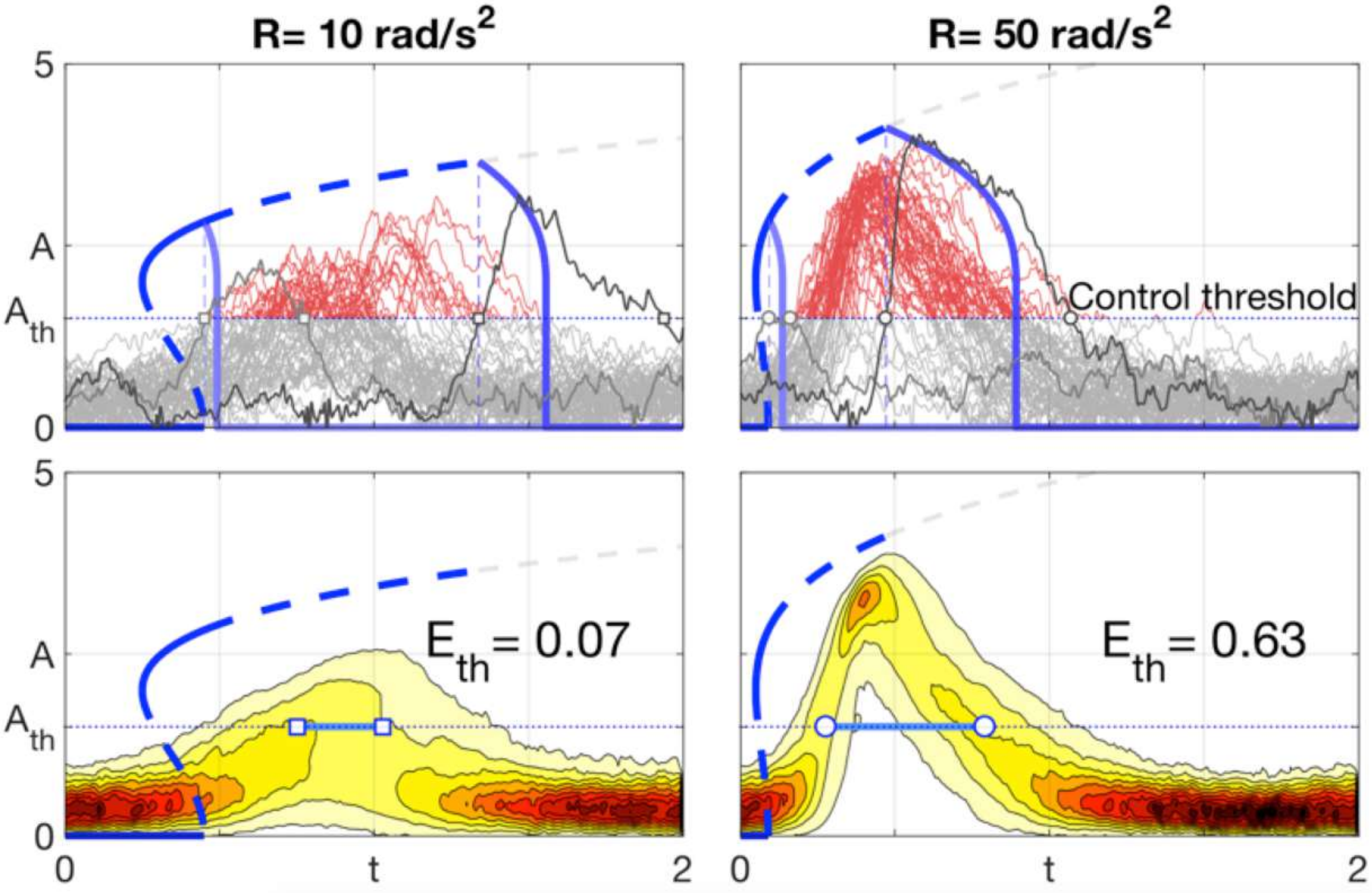
Tipping delay is a risk

Tipping delay is a risk

Tipping delay is a risk

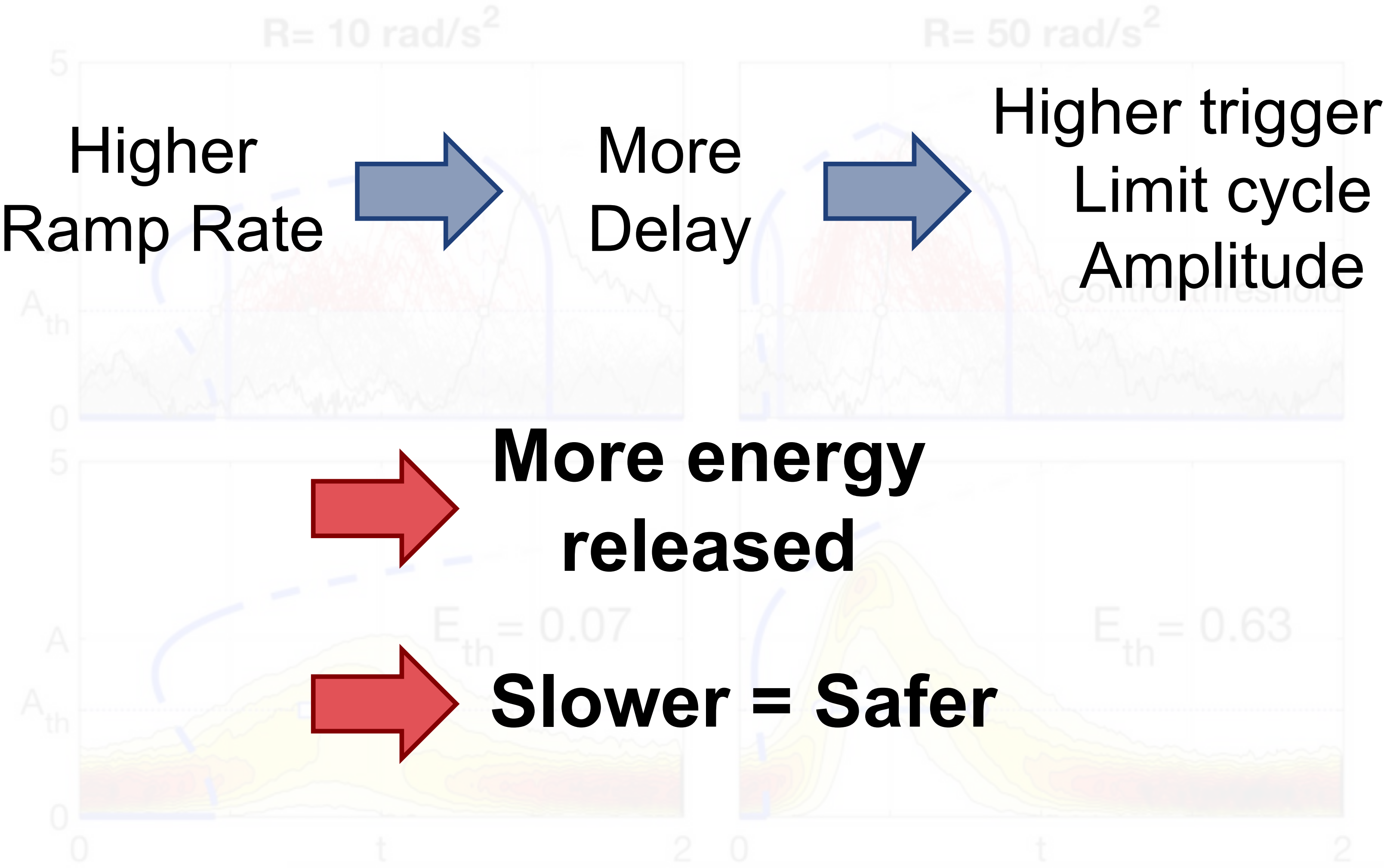


Tipping delay is a risk



5000 realizations of the process

Tipping delay is a risk



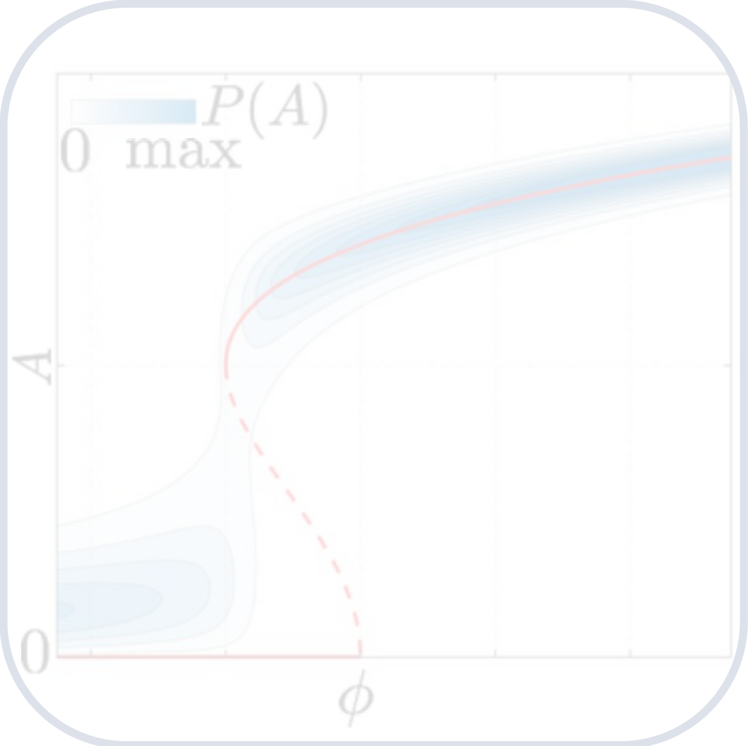
5000 realizations of the process

Higher Ramp Rate → More Delay → Higher triggered Limit cycle Amplitude

→ **More energy released**

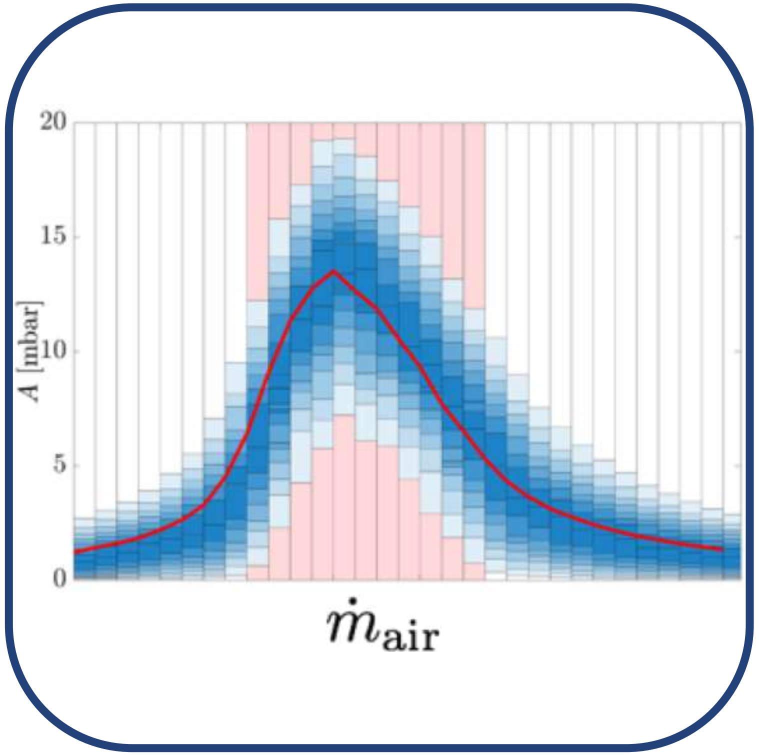
→ **Slower = Safer**

The transient thermoacoustics trilogy



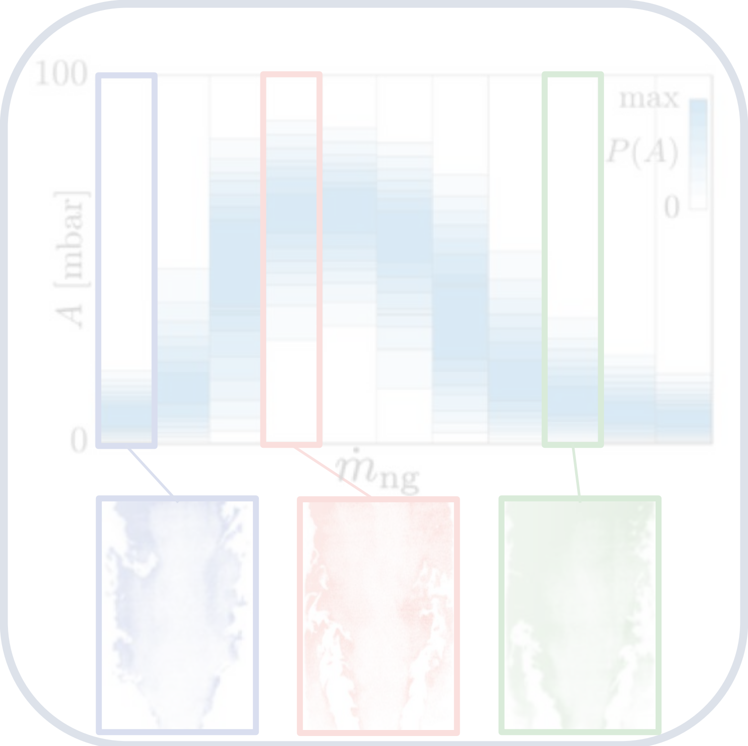
Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation

Royal Society Open Science



Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation

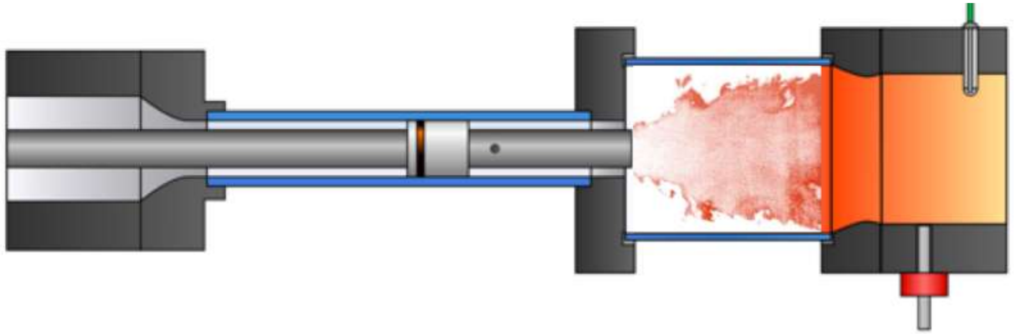
Nonlinear Dynamics

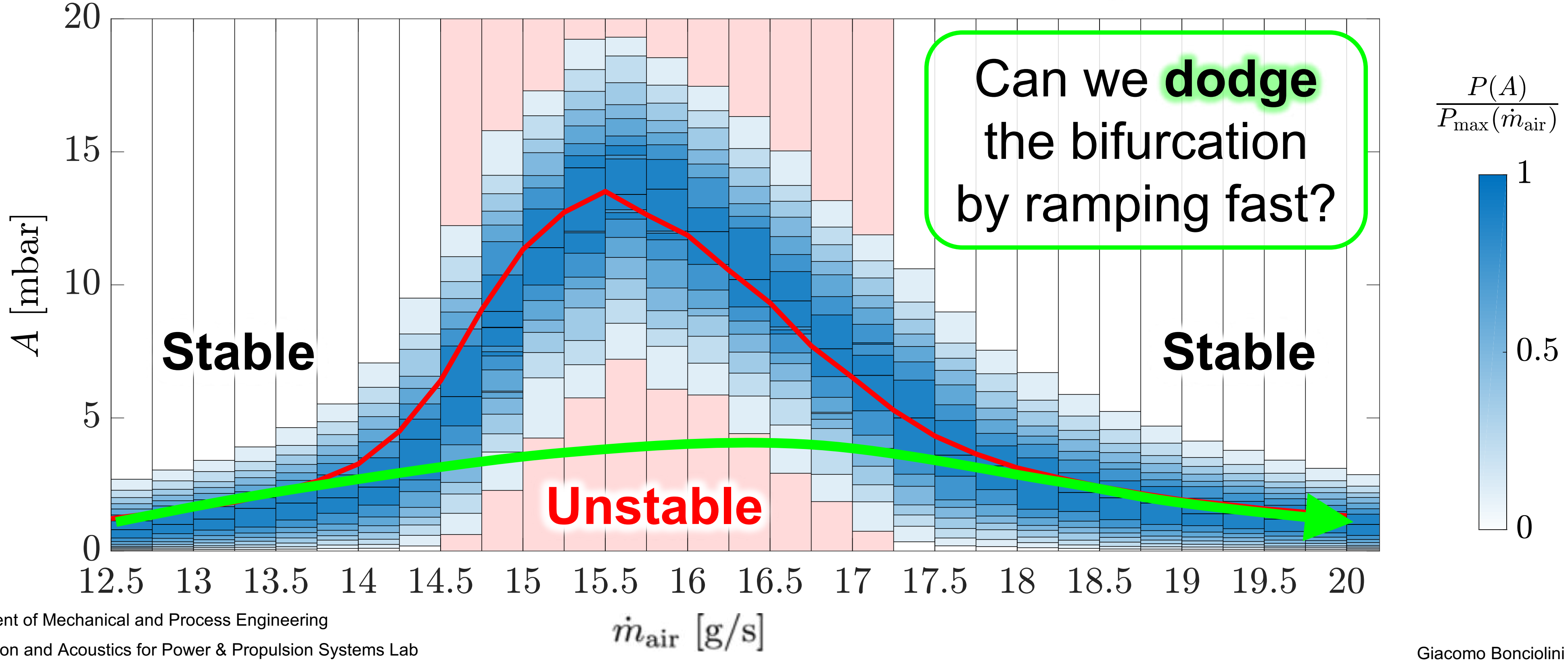


Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame

Proceedings of the Combustion Institute

Two consecutive and mirrored supercritical bifurcations

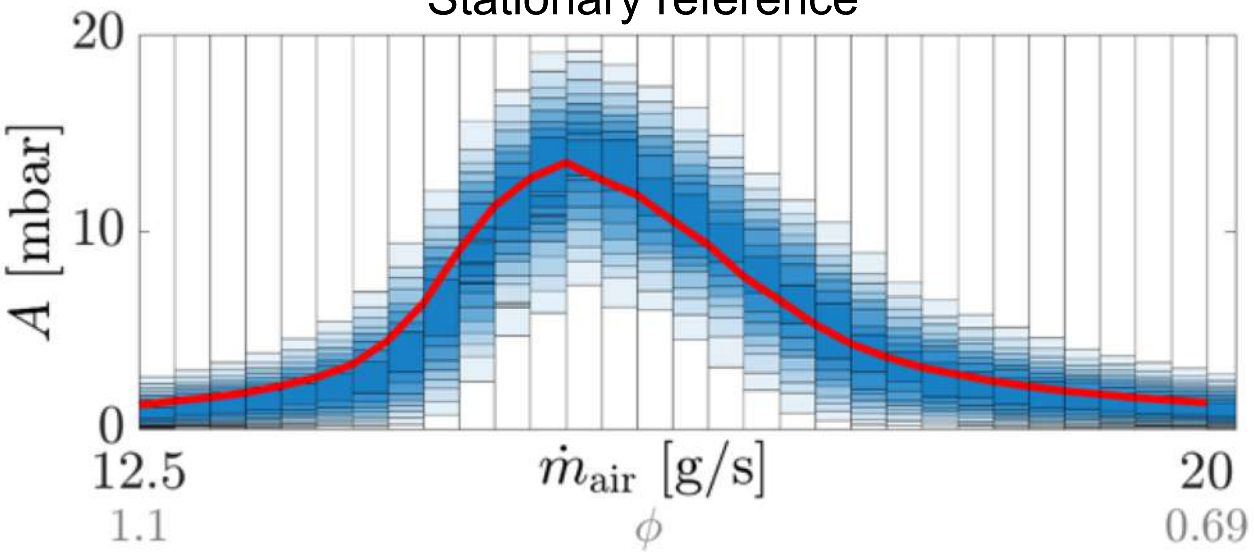


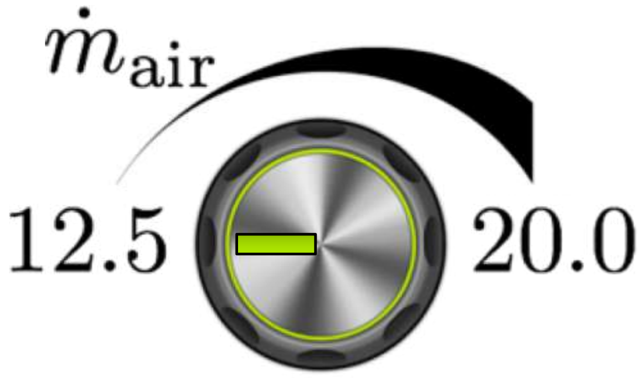


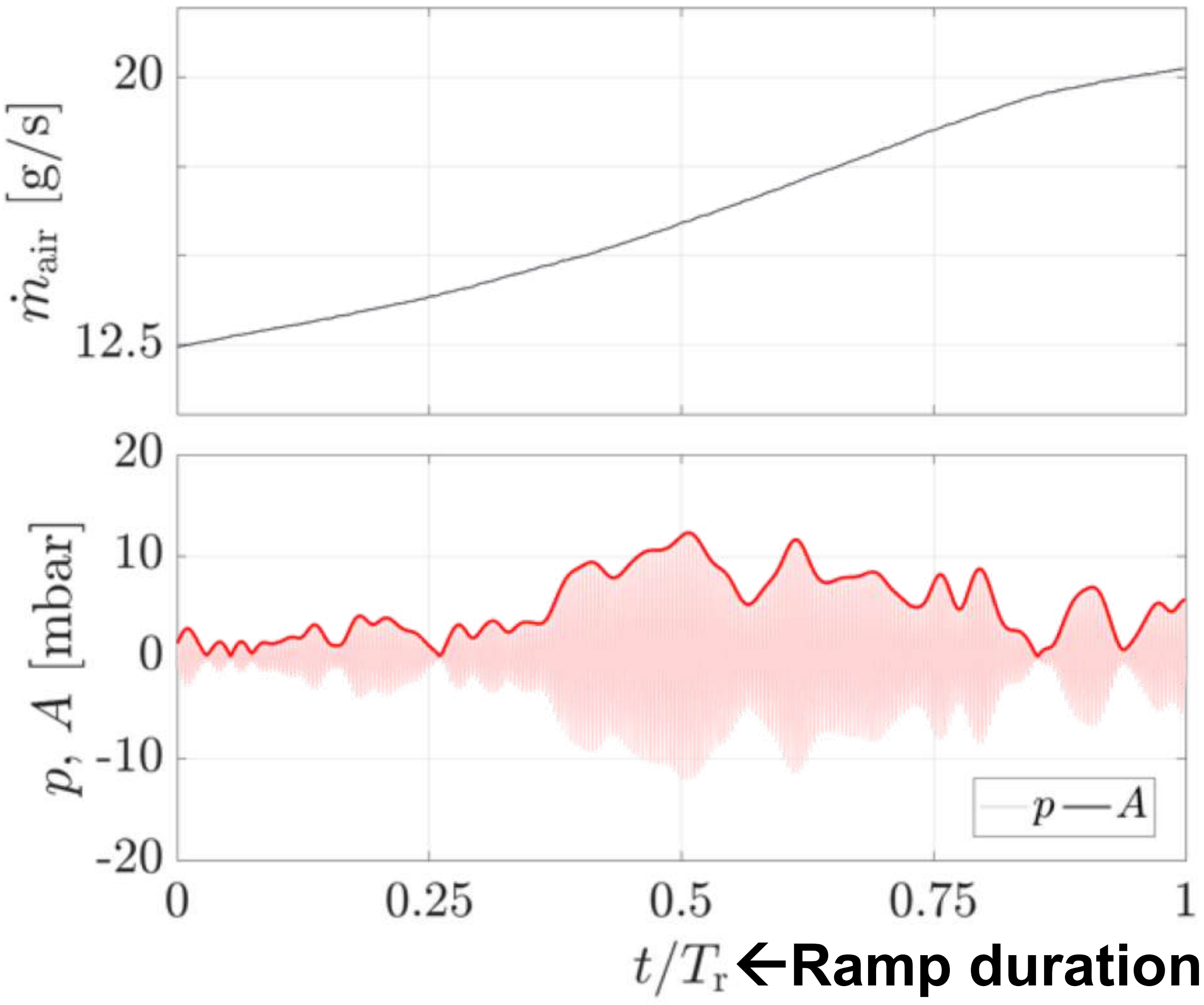
Can we **dodge** the bifurcation by ramping fast?

Transient dynamics is explored with multiple ramp experiments

Stationary reference



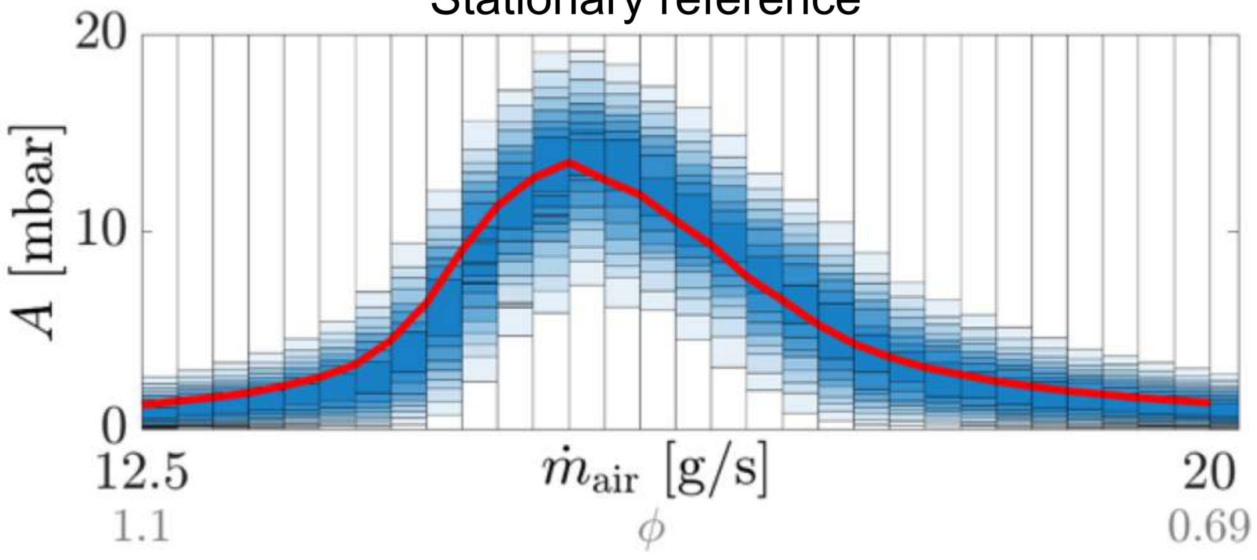


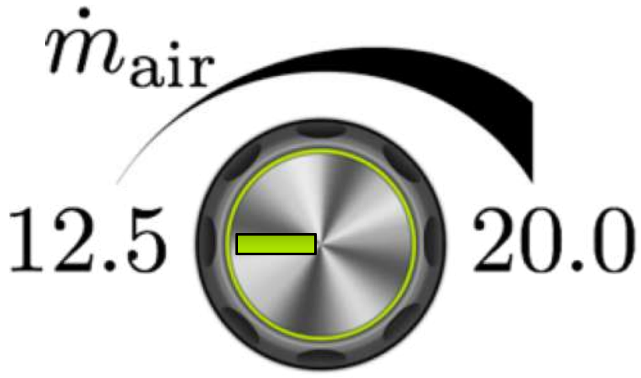


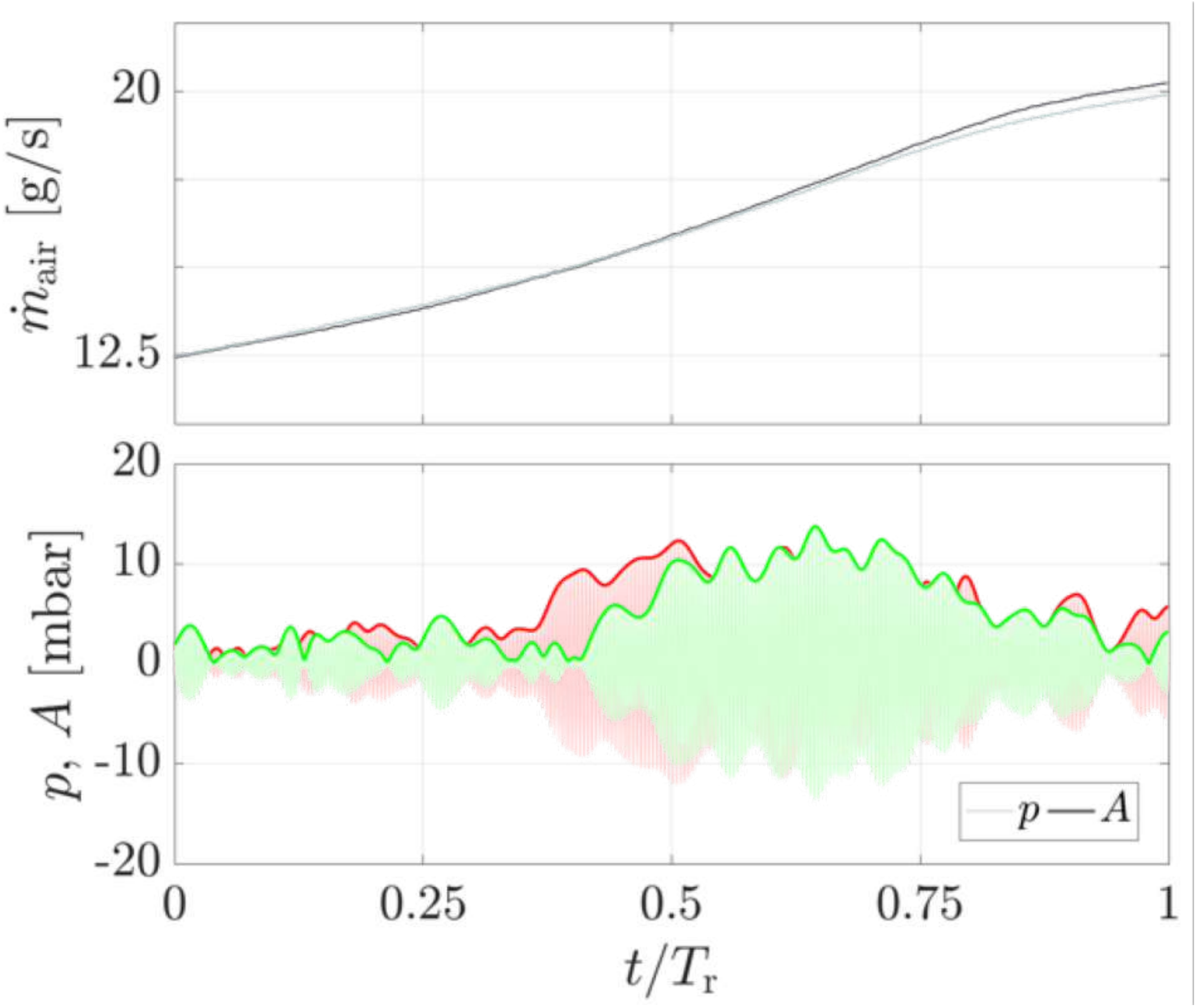
←**Ramp duration**

Transient dynamics is explored with multiple ramp experiments

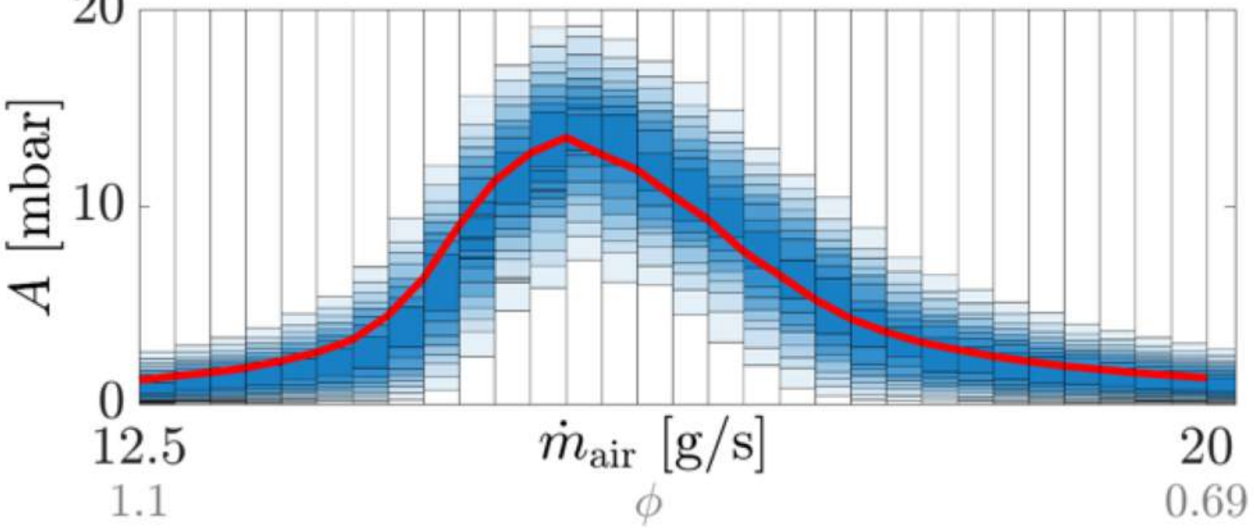
Stationary reference



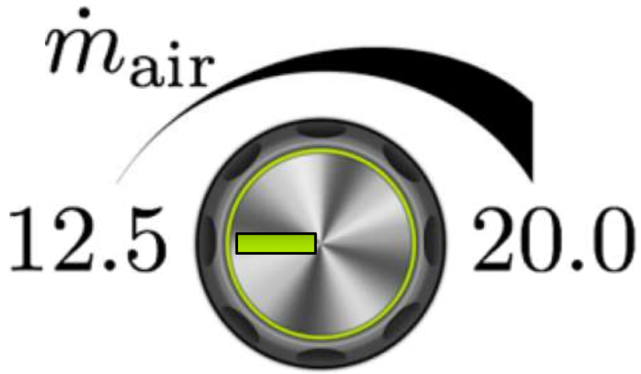


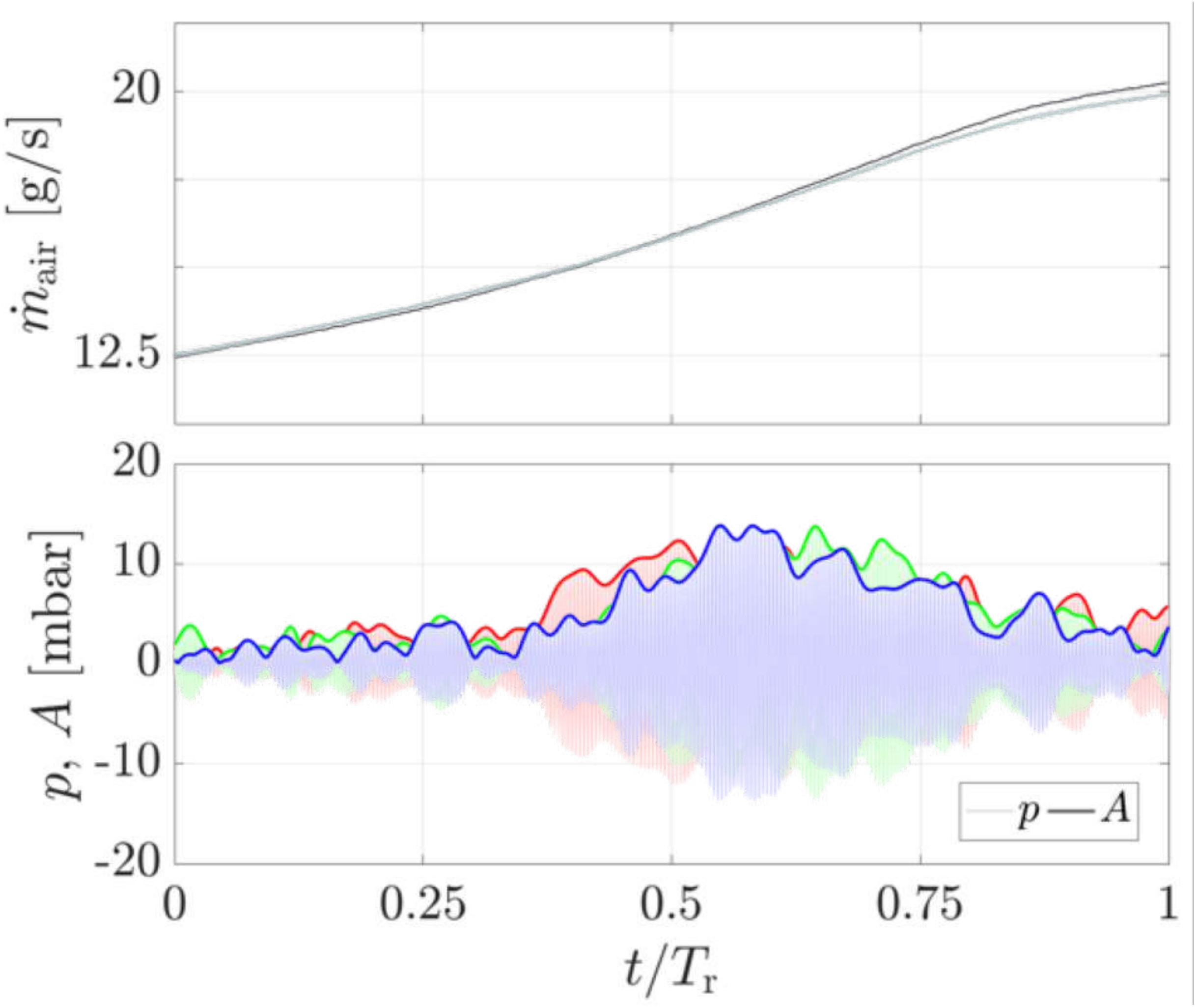


Transient dynamics is explored with multiple ramp experiments

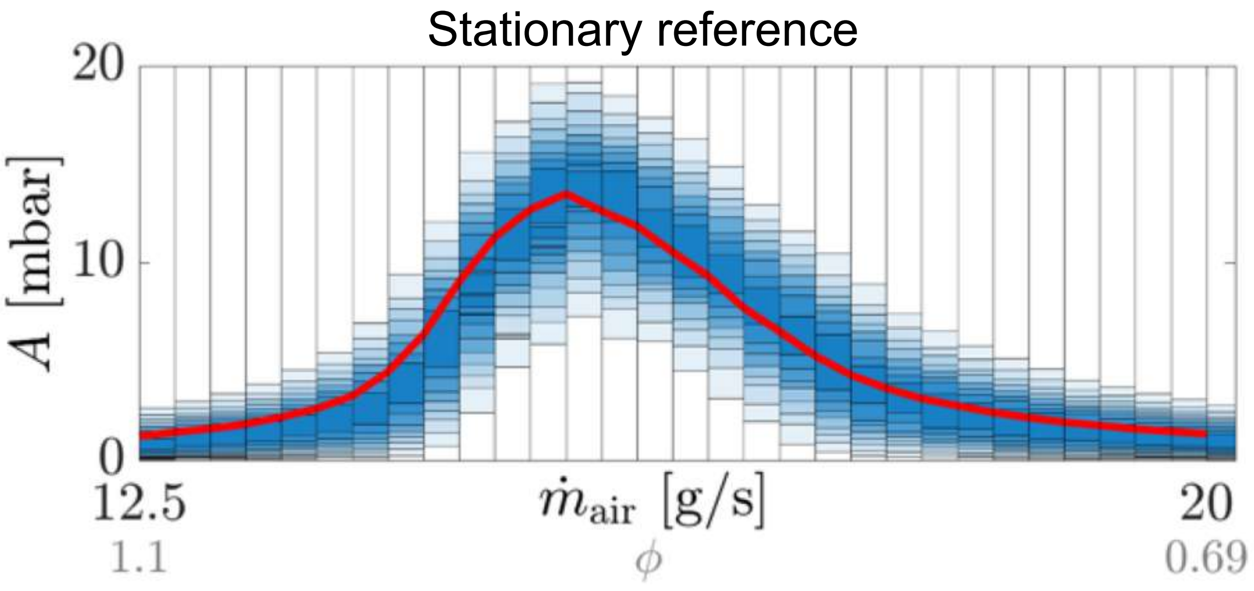


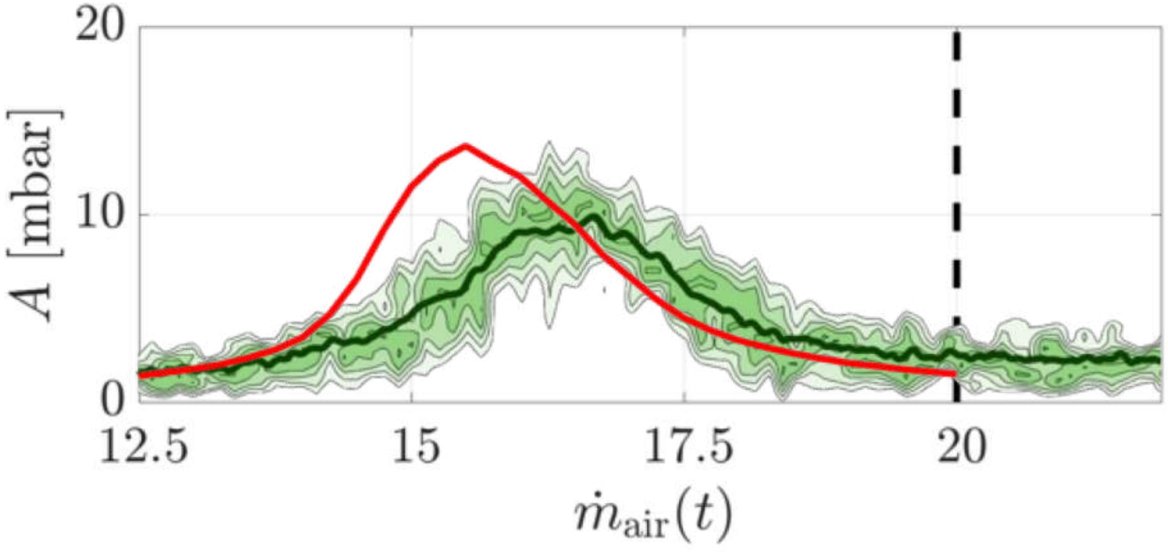
Stationary reference

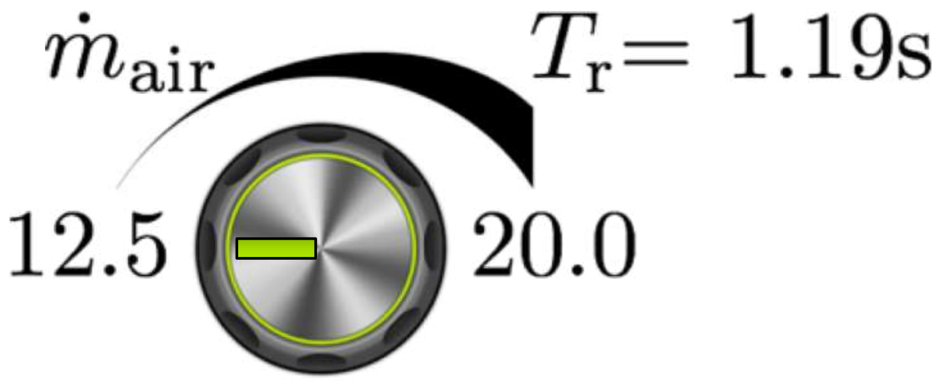




Transient statistics depends on the ramp time

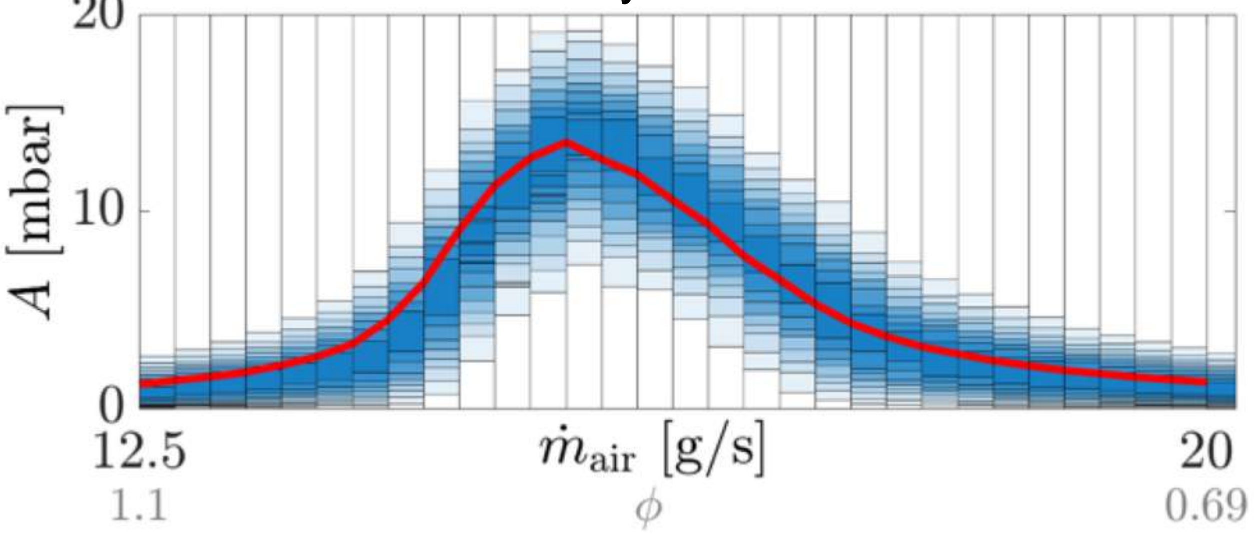

Stationary reference

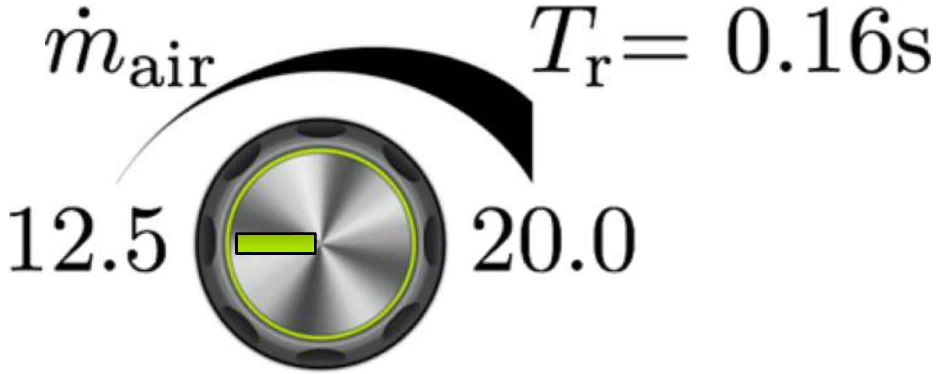


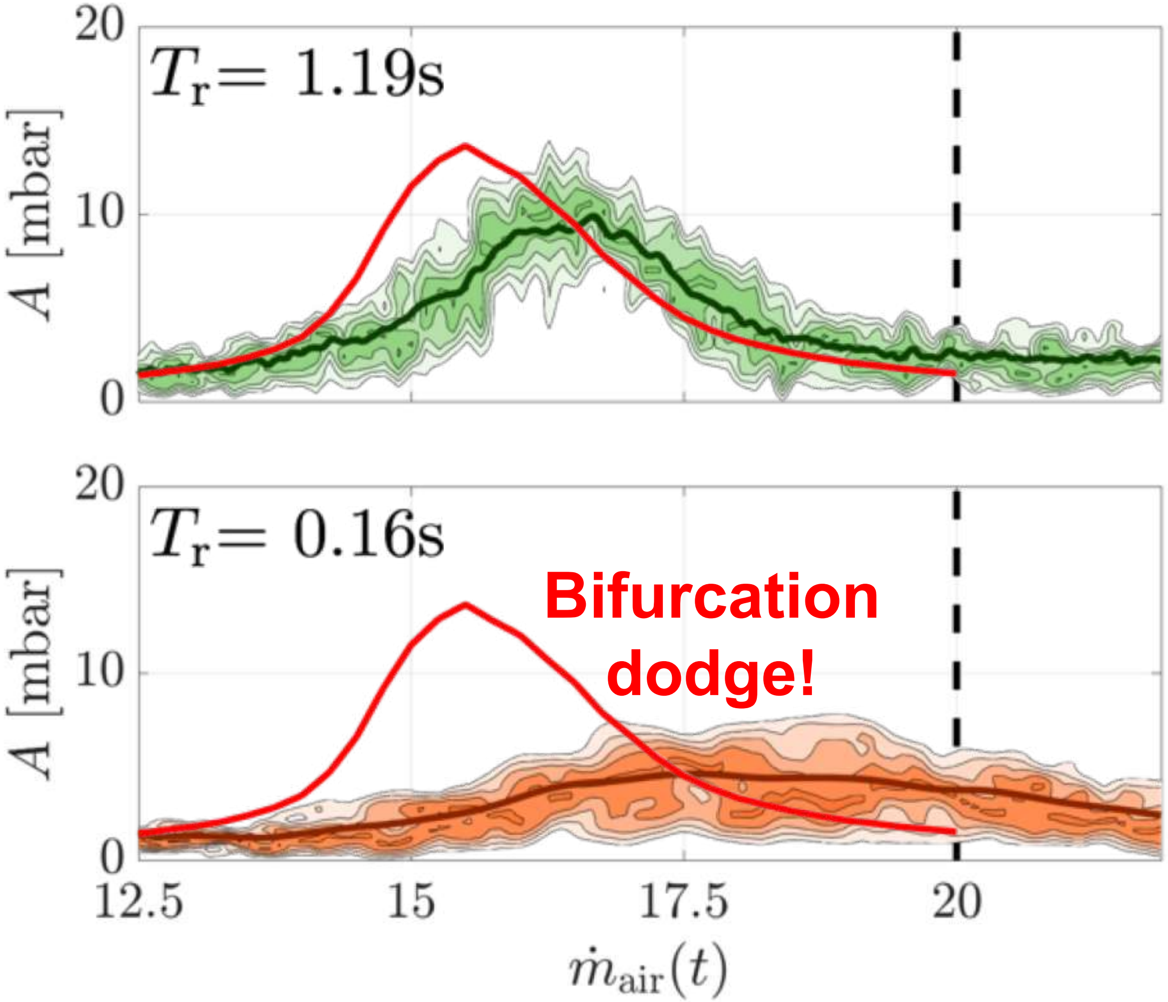


Transient statistics depends on the ramp time

Stationary reference

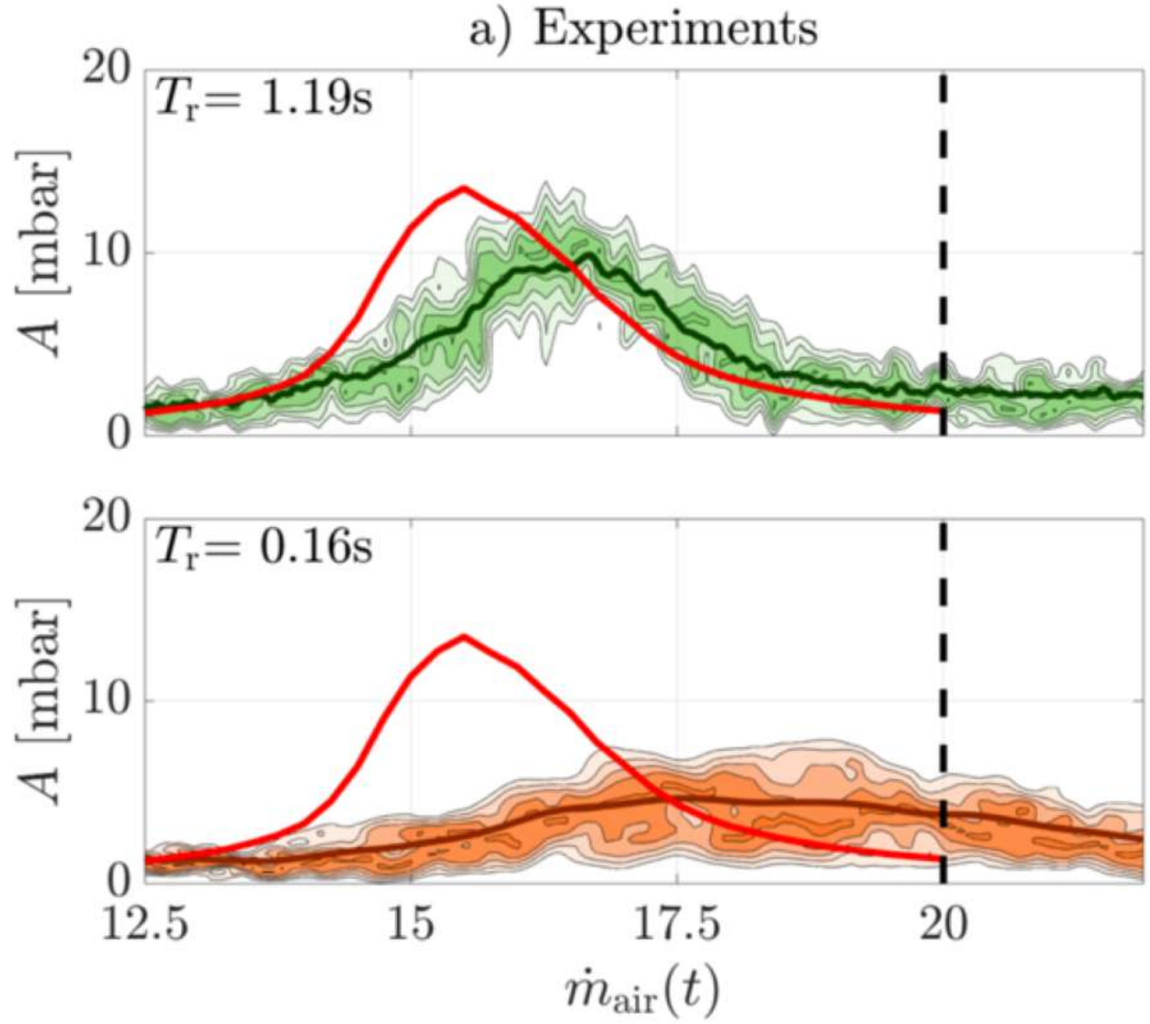






Bifurcation dodge!

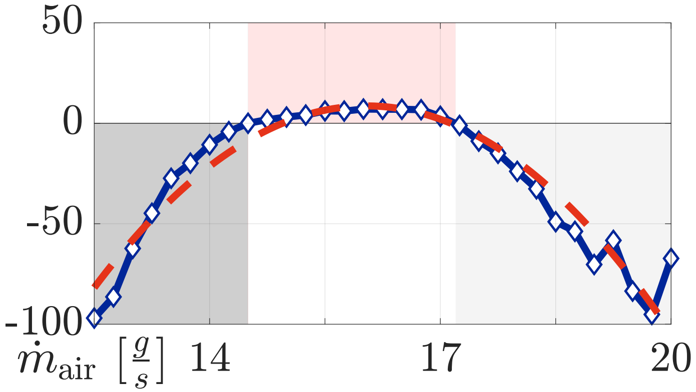
The model can reproduce the observed transient dynamics



Van der Pol Model

$$\ddot{p} + \omega_0^2 p = 2\nu(t)\dot{p} - \kappa p^2 \dot{p} +$$

Identification

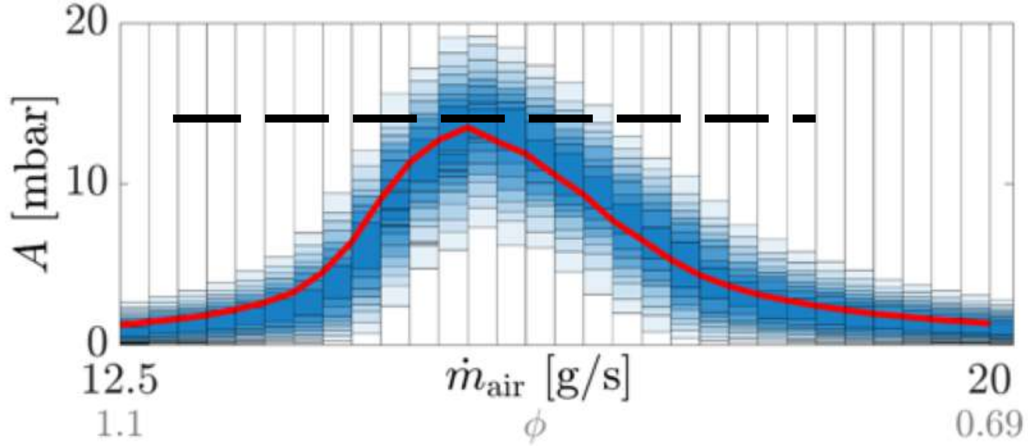


$$\nu(t) = n_1 \cos\left[n_2 \dot{m}_{air}(t)\right] + n_3$$

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial A}\left[\mathcal{F}(A,t)P\right] + \frac{\Gamma}{4\omega_0^2}\frac{\partial^2 P}{\partial A^2}$$

The model can be used for preliminary risk estimation

Stationary mapping

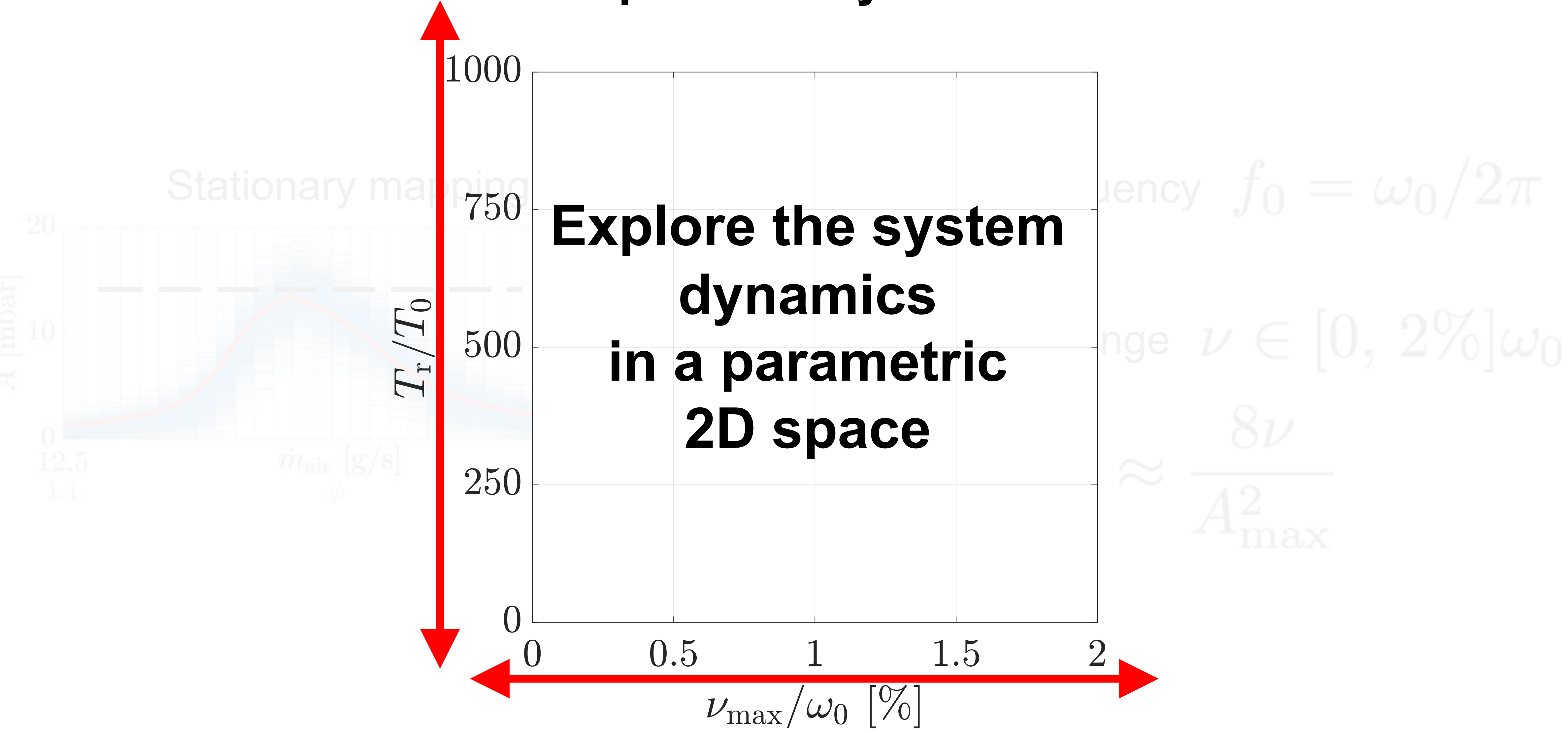


→Oscillation frequency $f_0 = \omega_0 / 2\pi$

→Growth Rate range $\nu \in [0,\ 2\%]\omega_0$

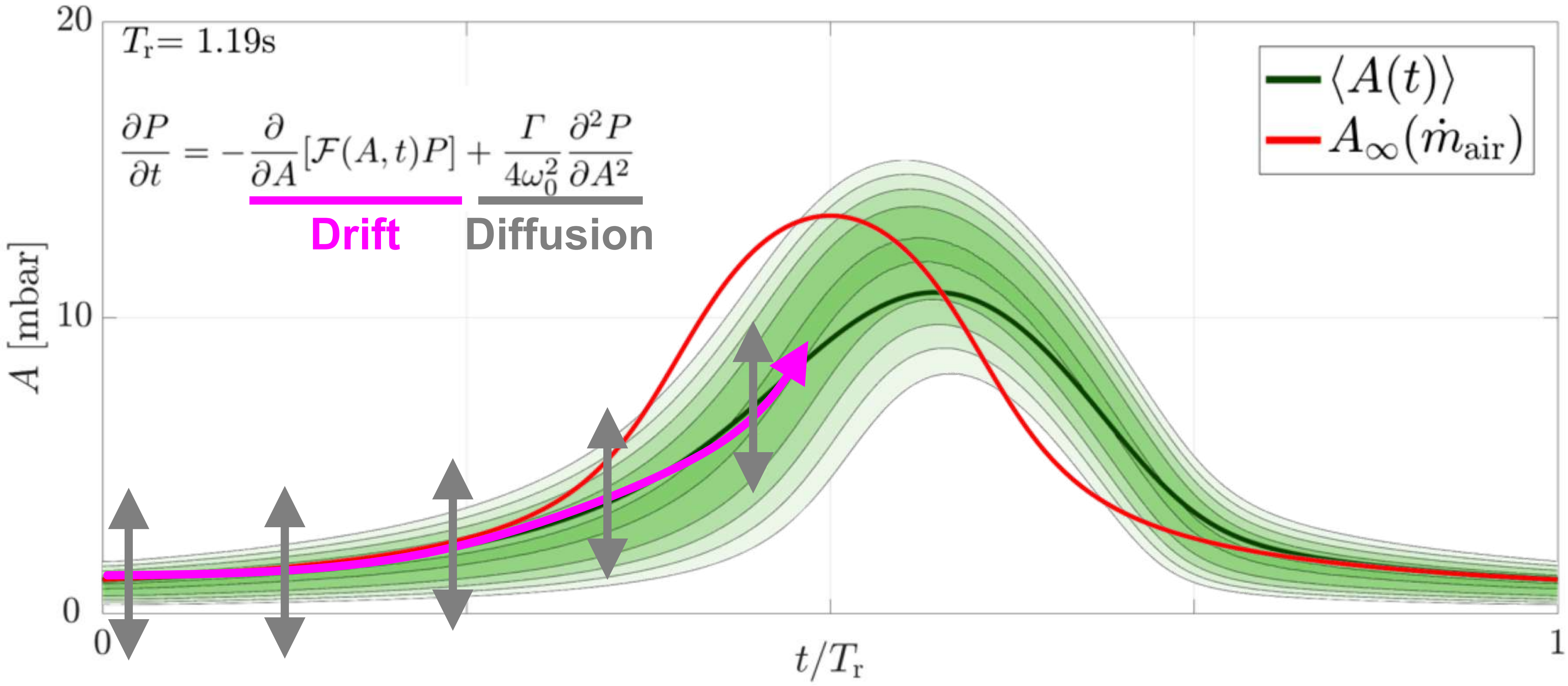
→Max Amp → $\kappa \approx \dfrac{8\nu}{A_{\mathrm{max}}^2}$

The model can be used for preliminary risk estimation



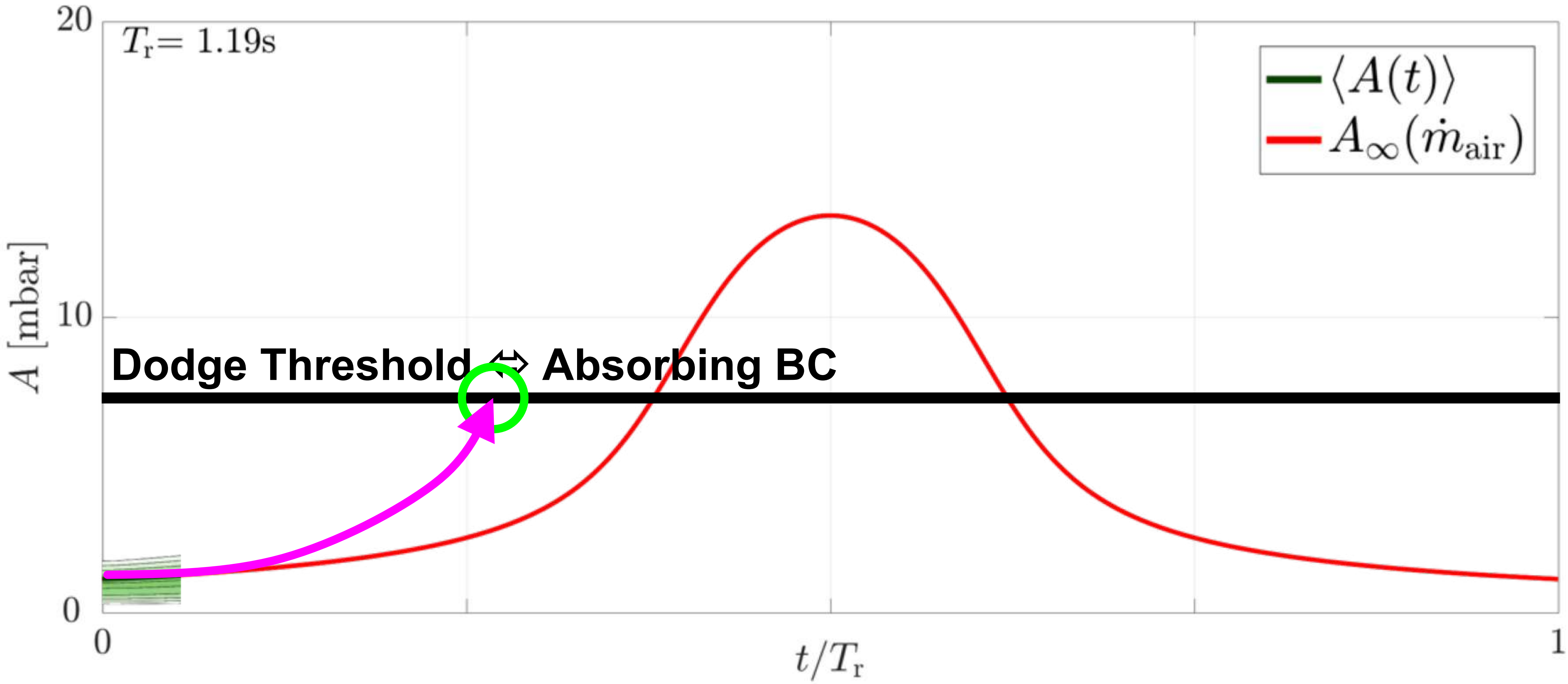
Explore the system dynamics in a parametric 2D space

The model can predict the dodge probability

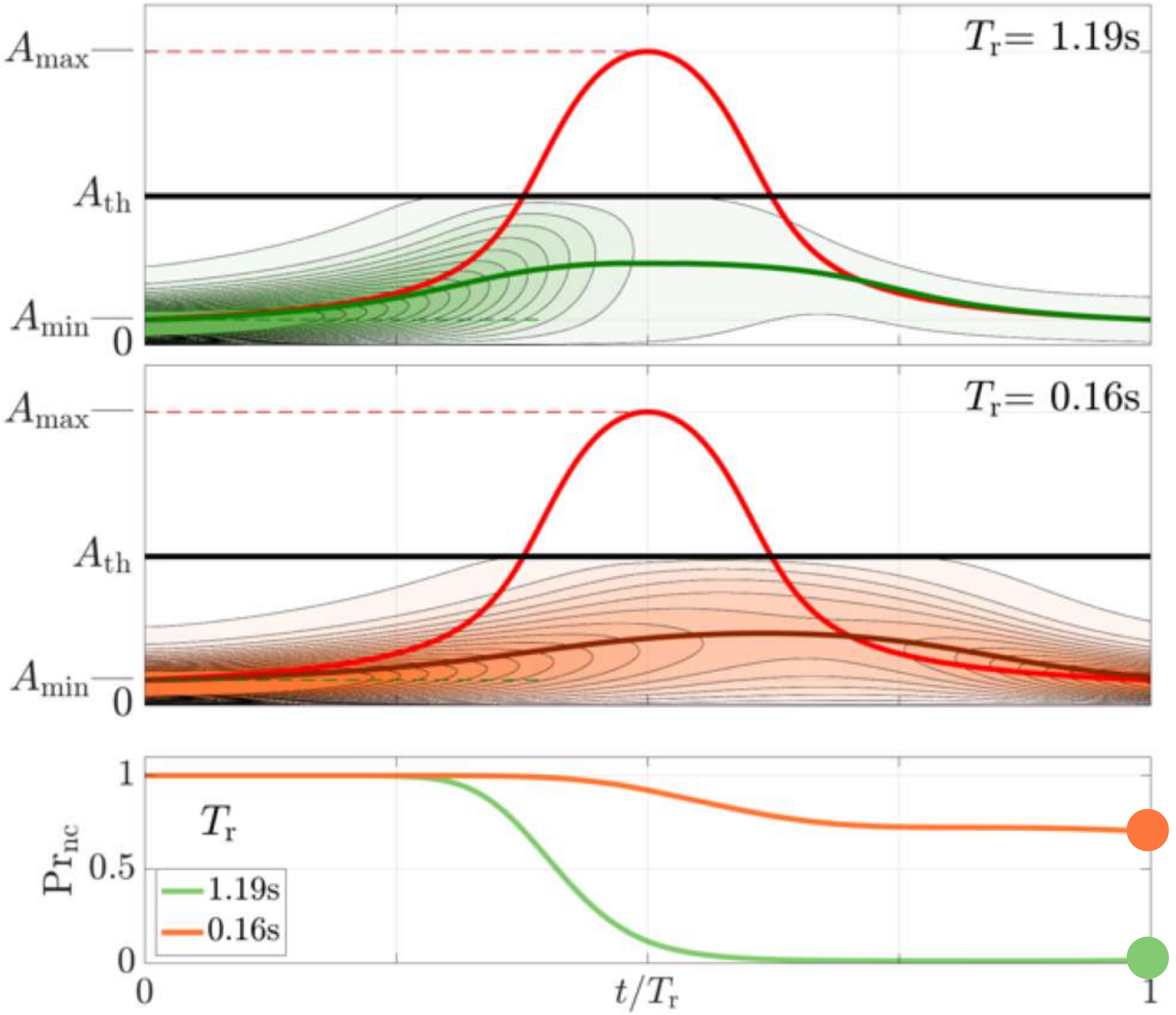


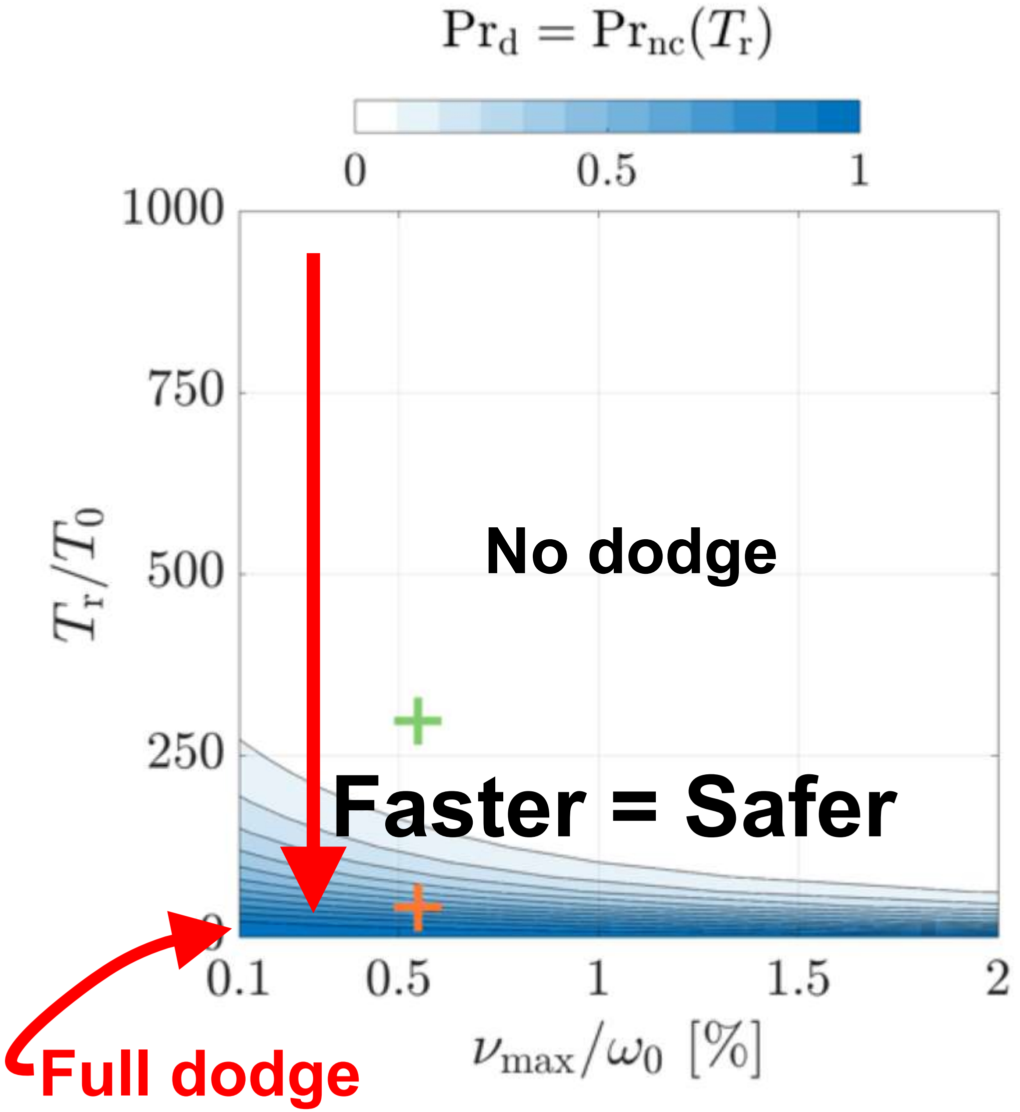
$$\frac{\partial P}{\partial t} = \underbrace{-\frac{\partial}{\partial A}[\mathcal{F}(A,t)P]}_{\text{Drift}} + \underbrace{\frac{\Gamma}{4\omega_0^2}\frac{\partial^2 P}{\partial A^2}}_{\text{Diffusion}}$$

The model can predict the dodge probability



The model can predict the dodge probability



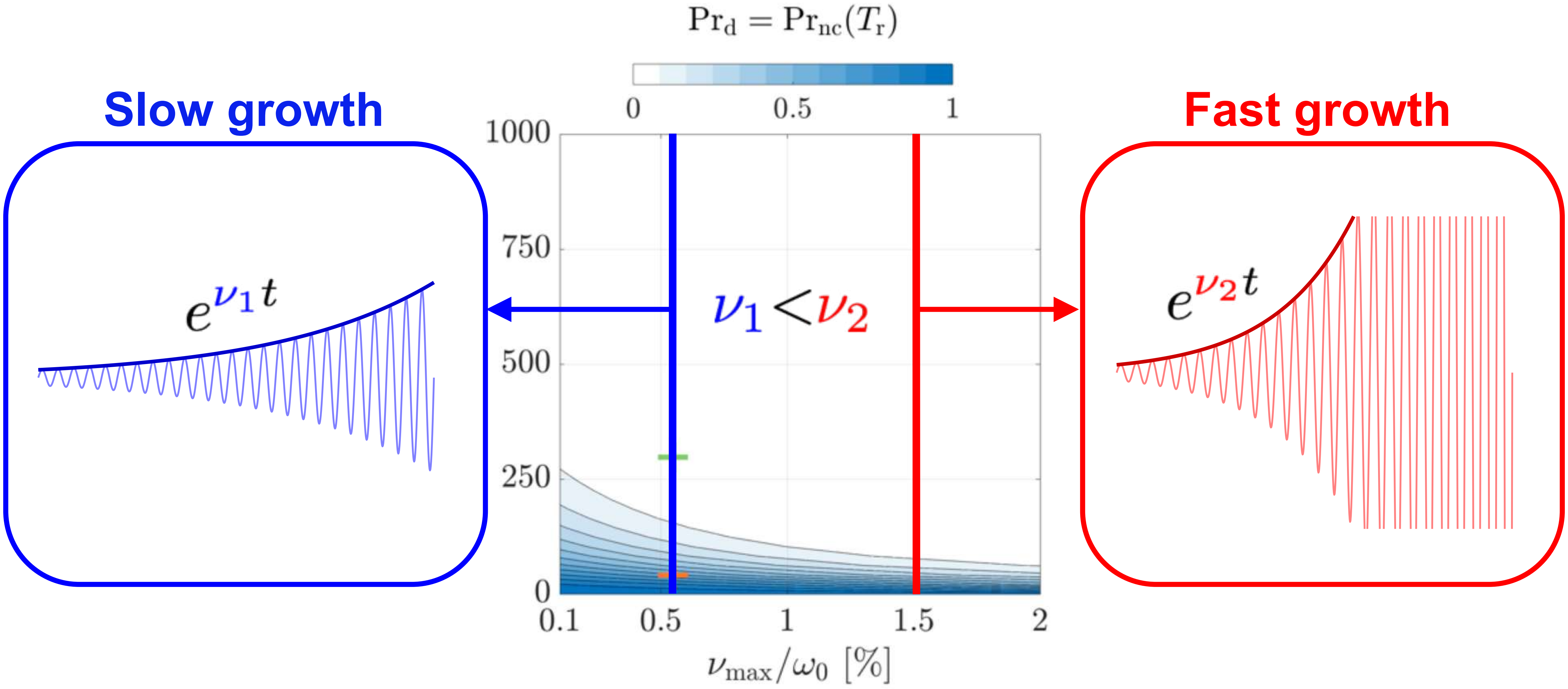


No dodge

Faster = Safer

Full dodge

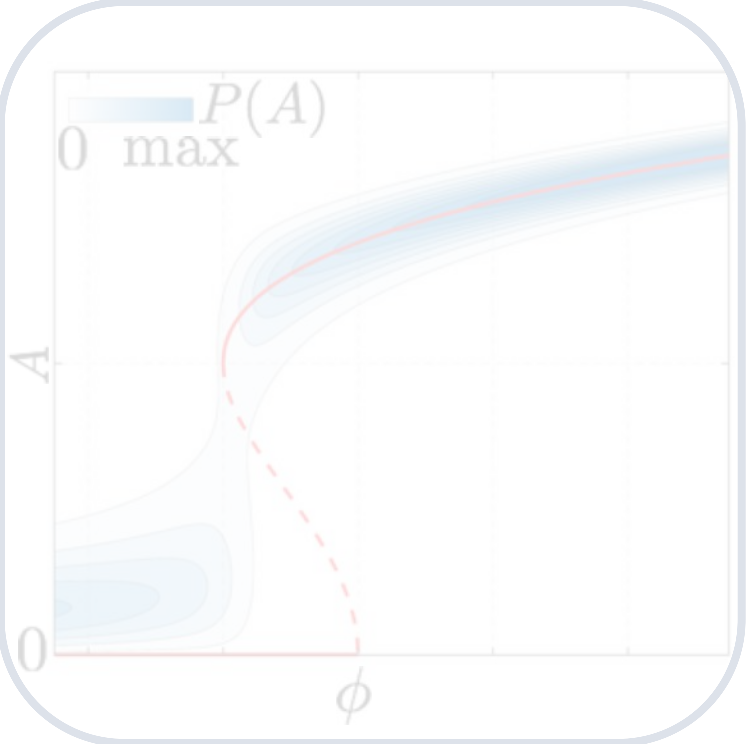
A fast growth is harder to dodge



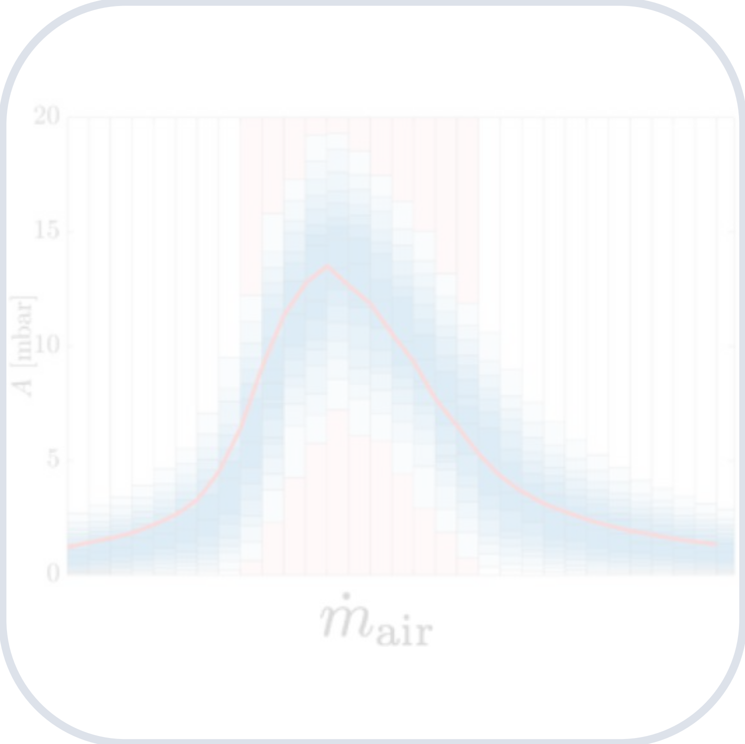
Slow growth

Fast growth

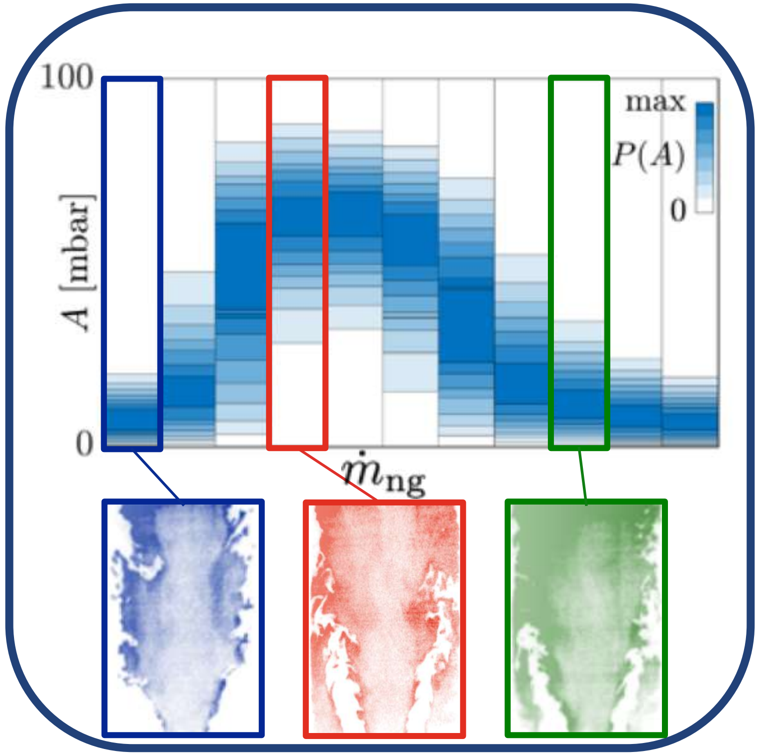
The transient thermoacoustics trilogy



Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation
Royal Society Open Science



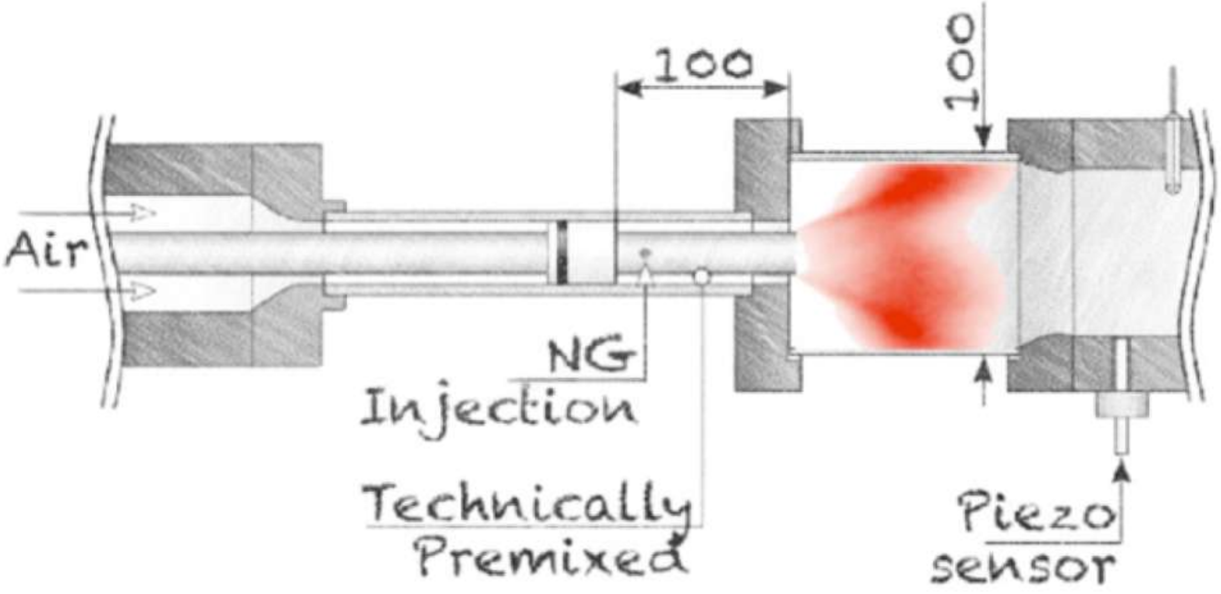
Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation
Nonlinear Dynamics



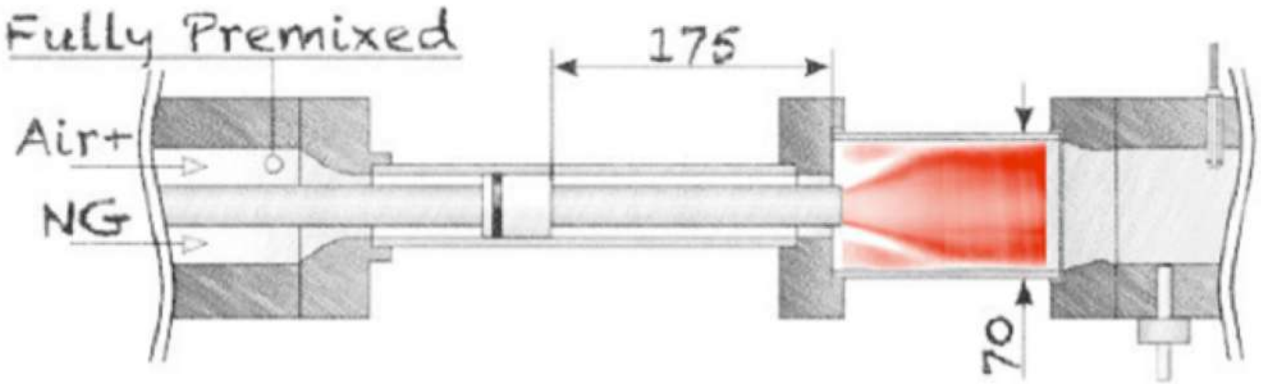
Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame
Proceedings of the Combustion Institute

The configuration is modified

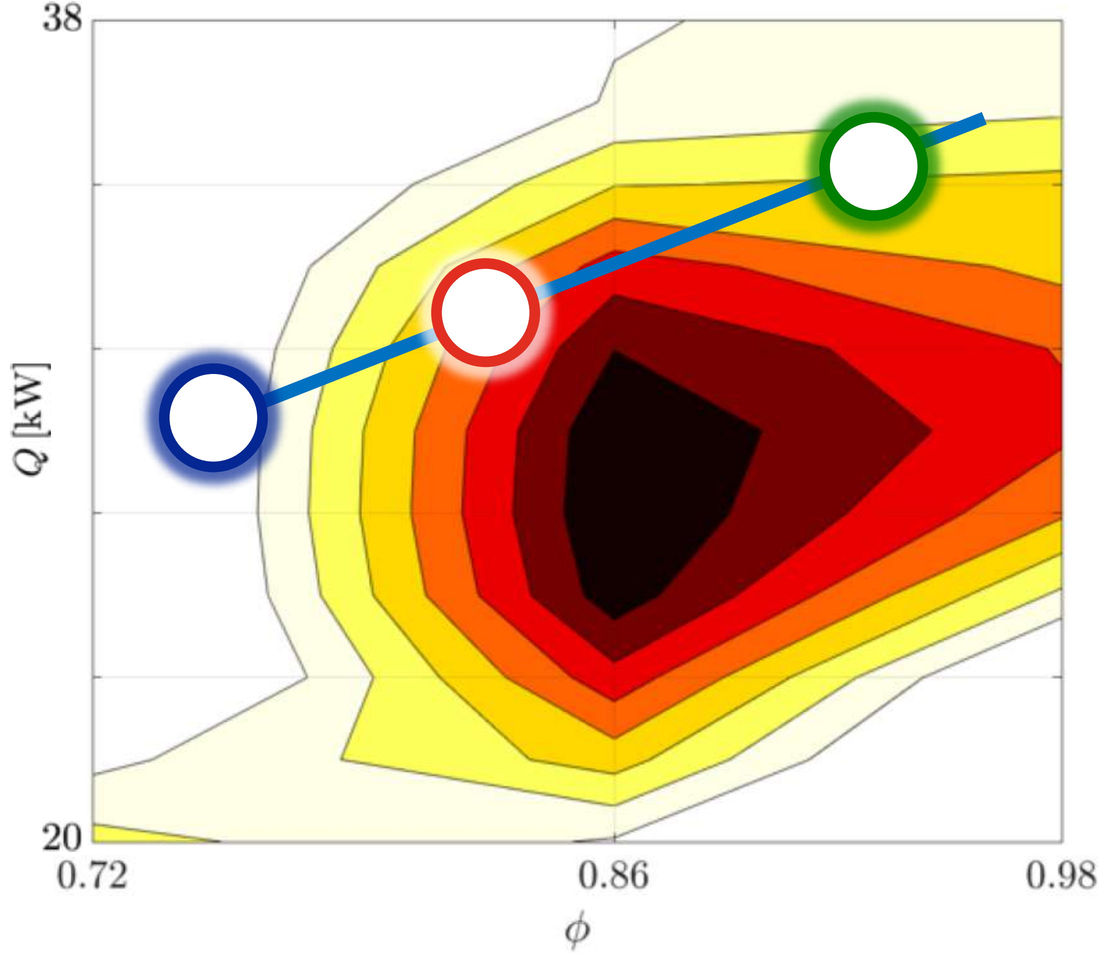
Previous

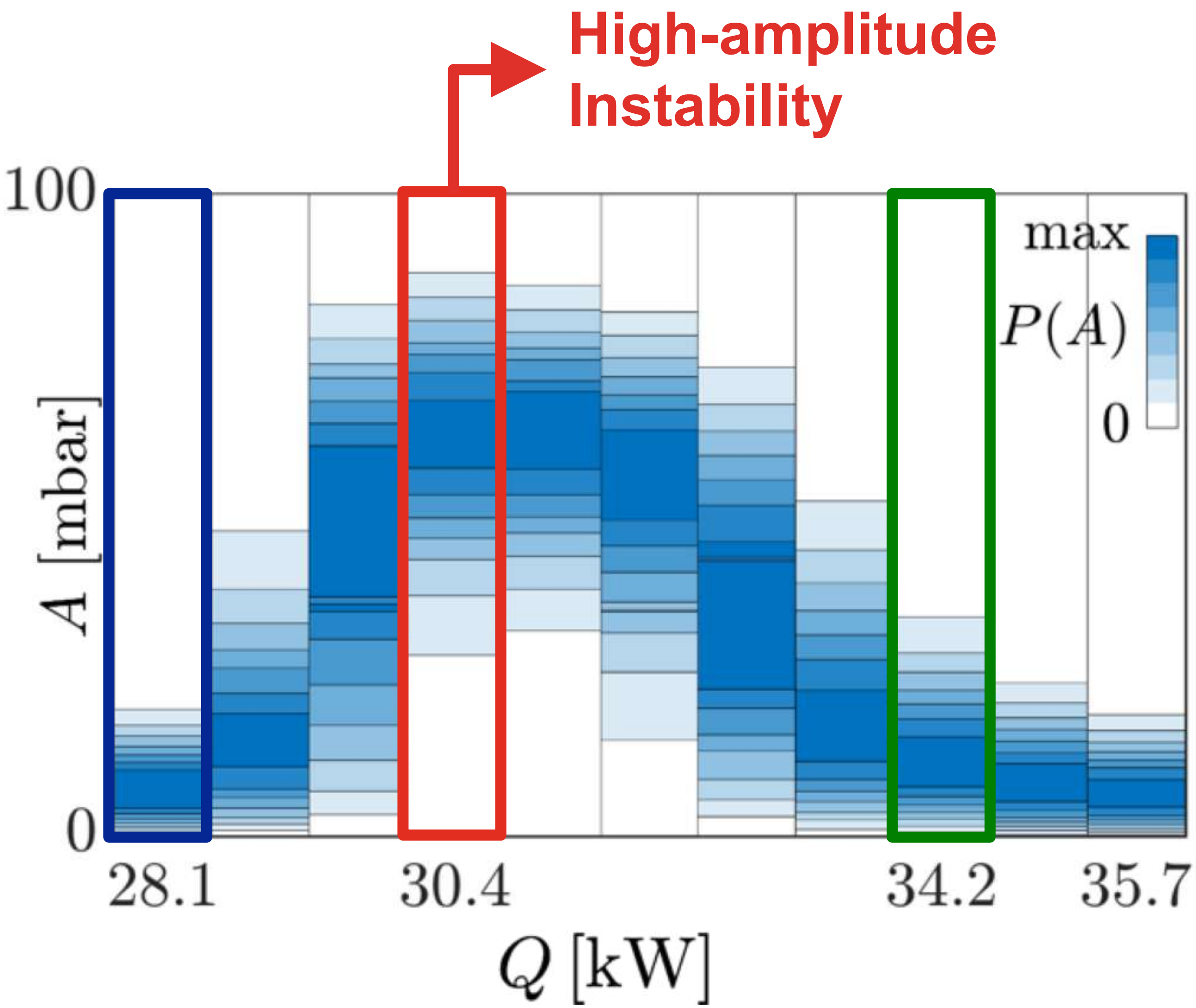


Current

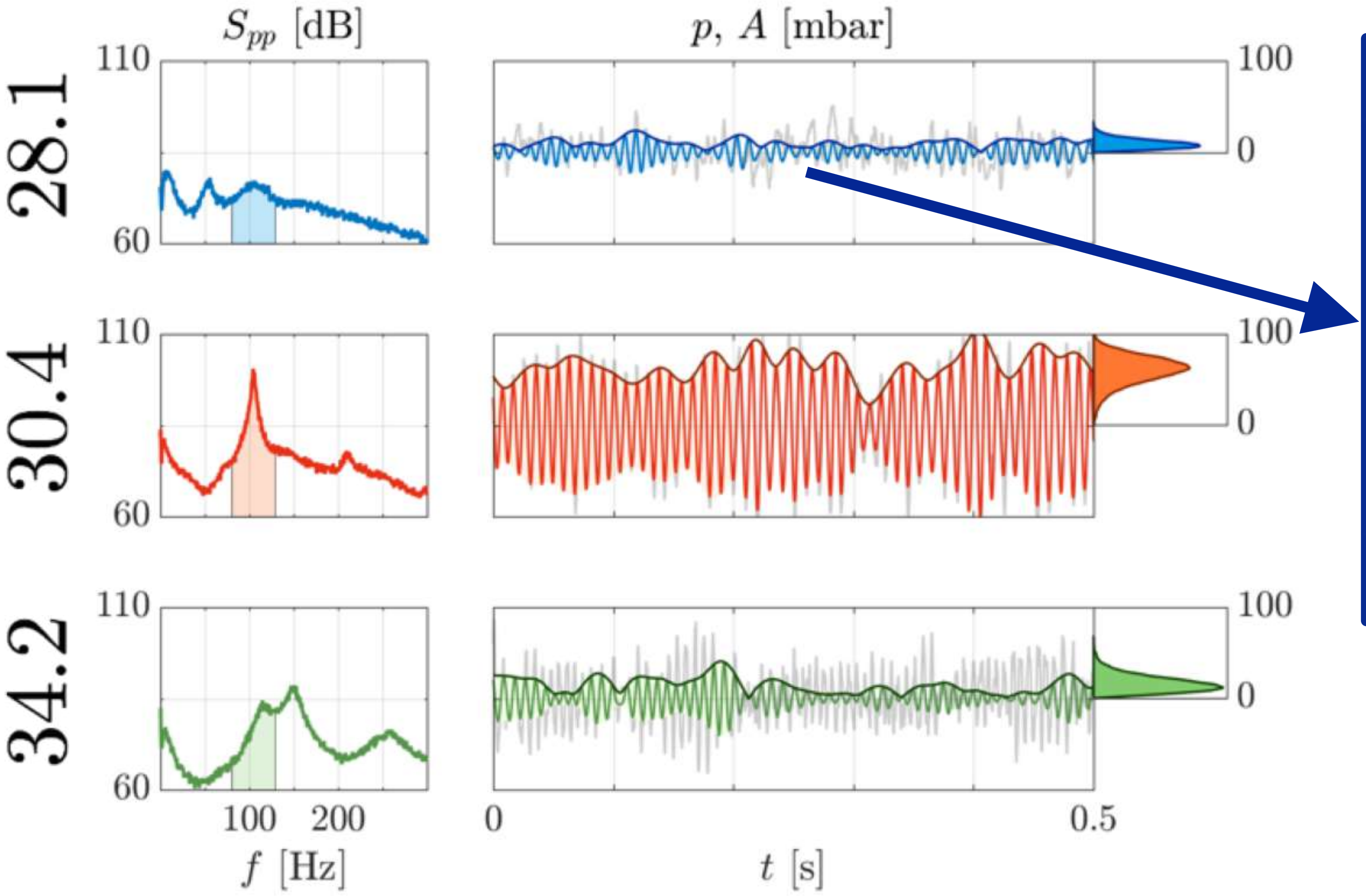


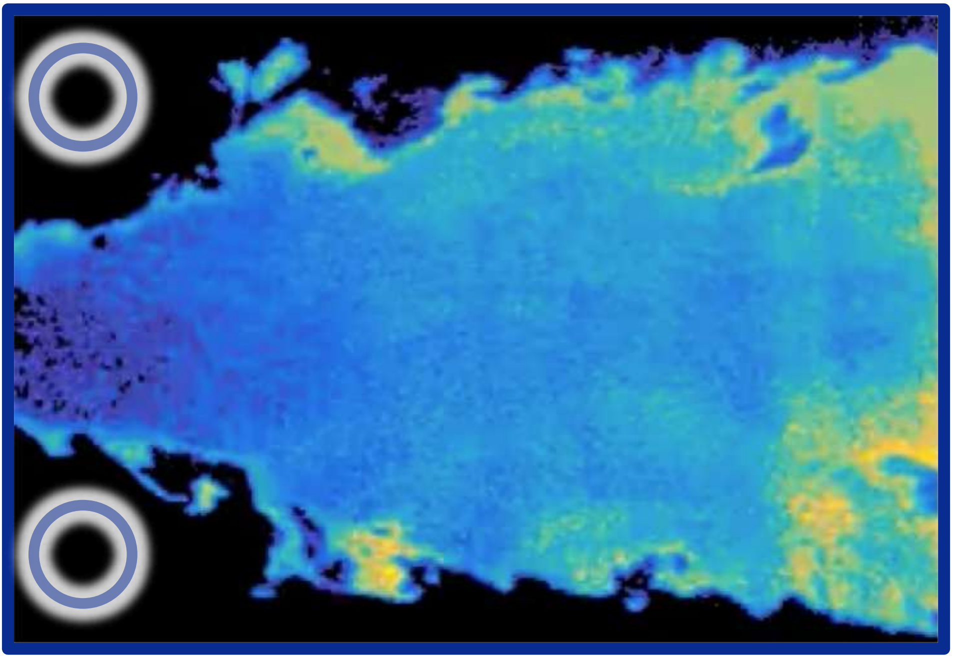
The current configuration features as well a double bifurcation





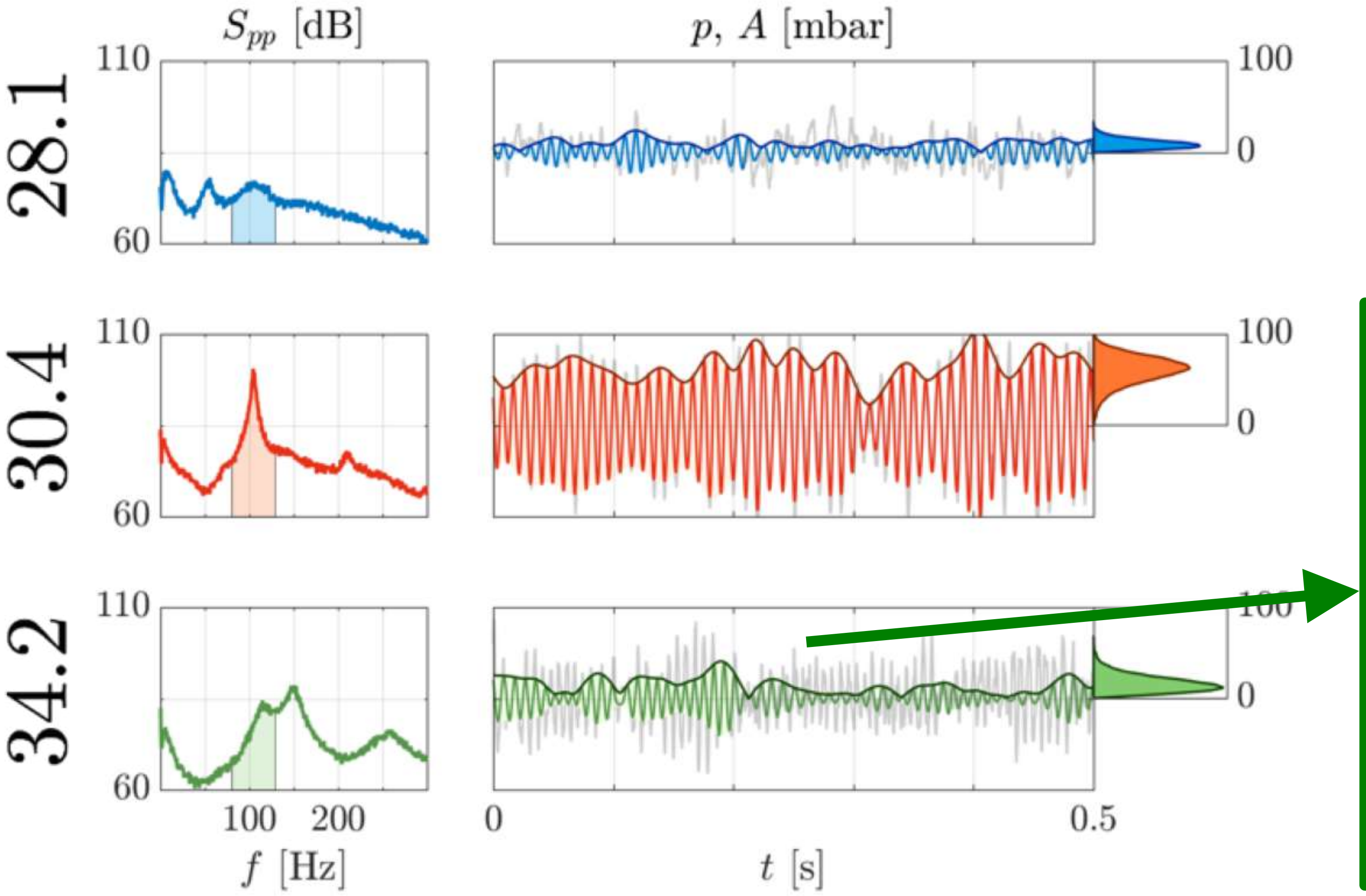
Flame topology change is the instability driver

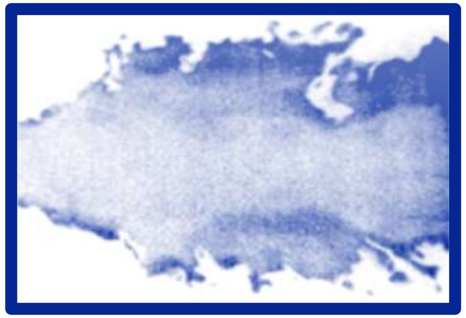




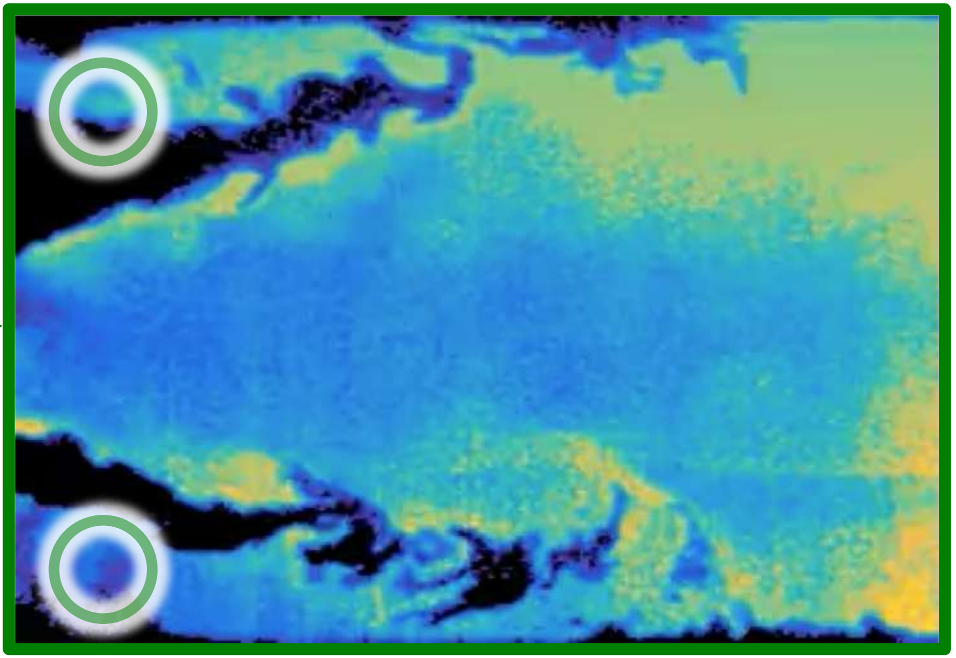
2000 FPS LIF

Flame topology change is the instability driver

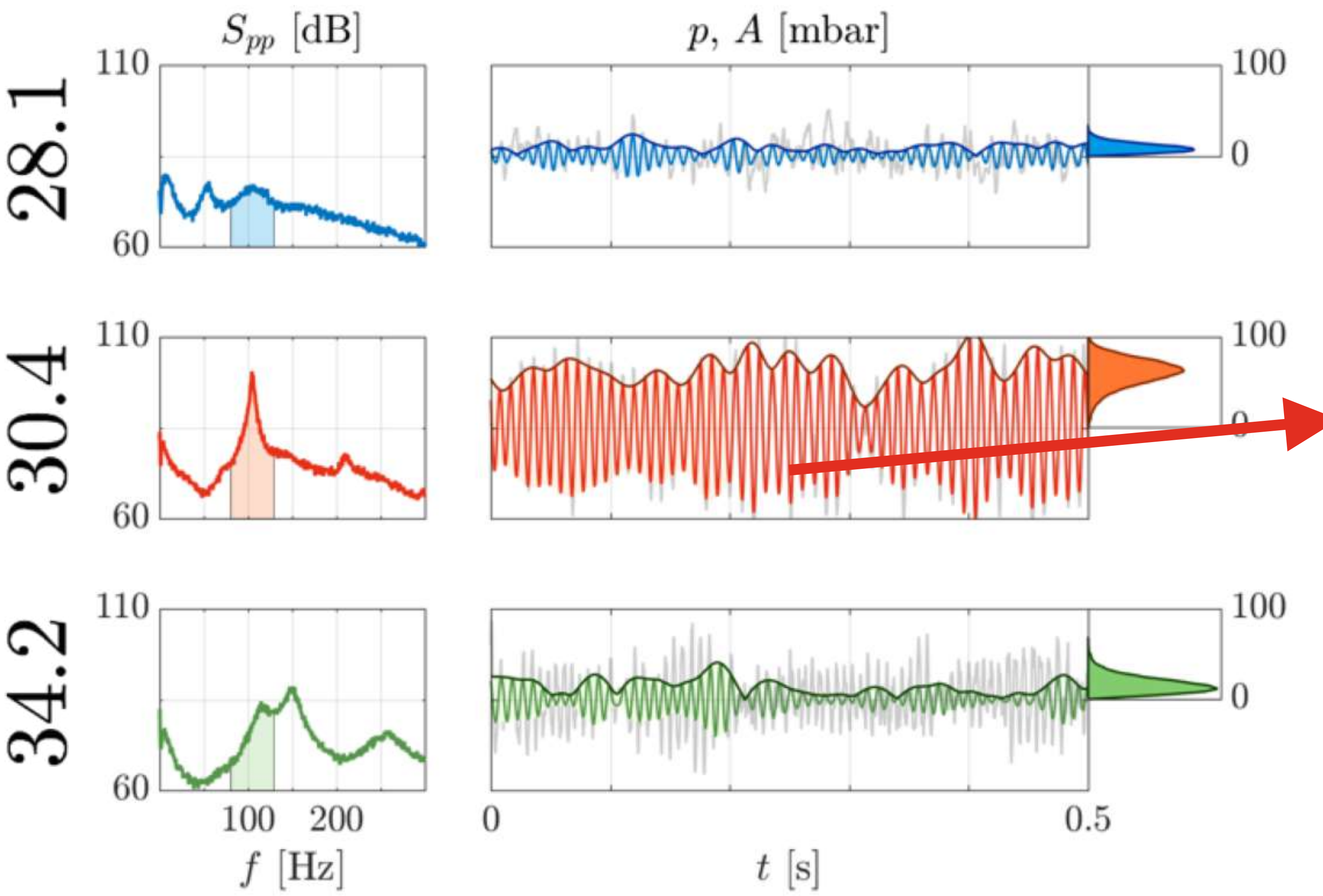


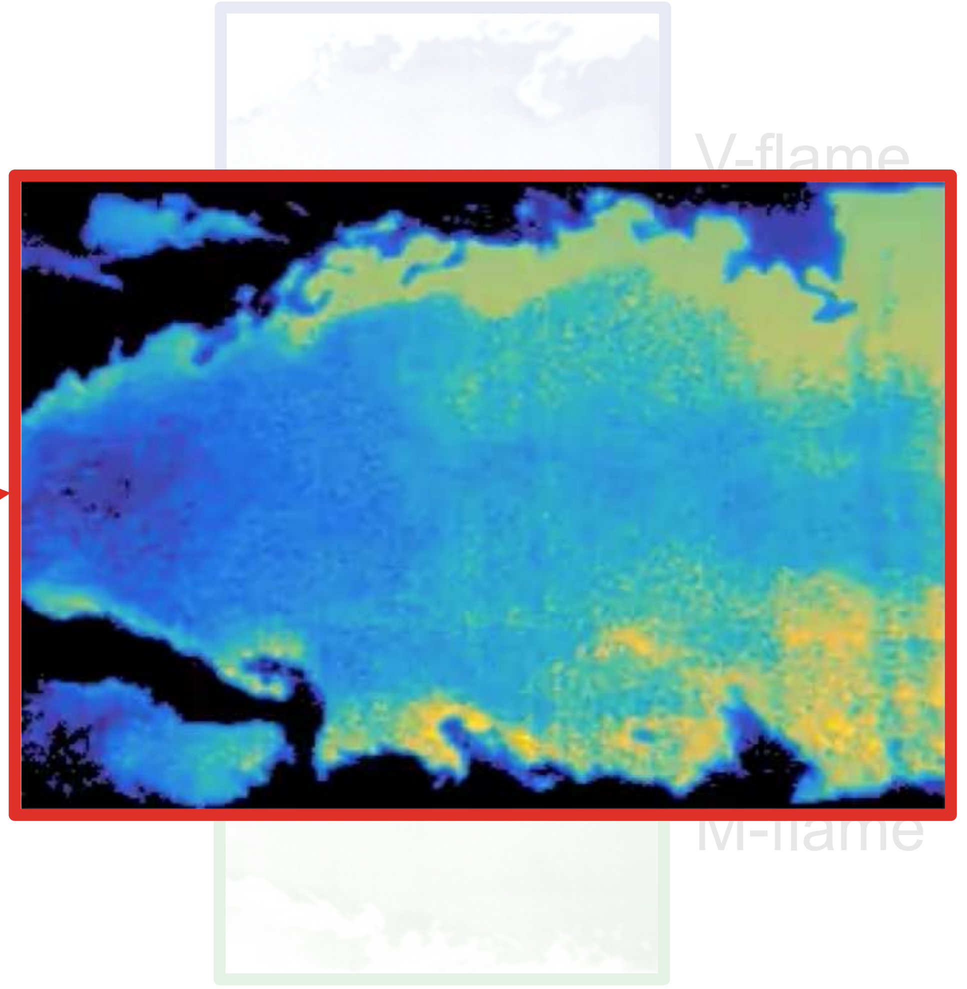


V-flame



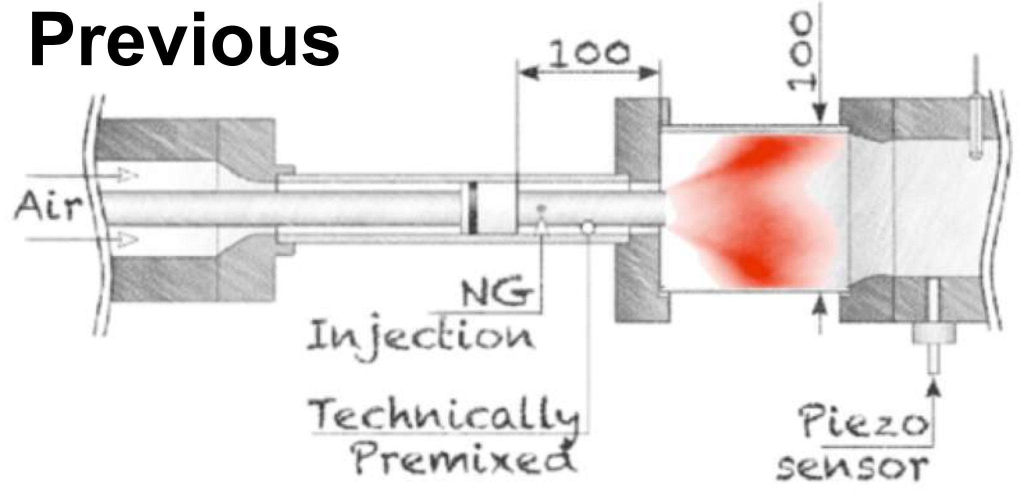
Flame topology change is the instability driver

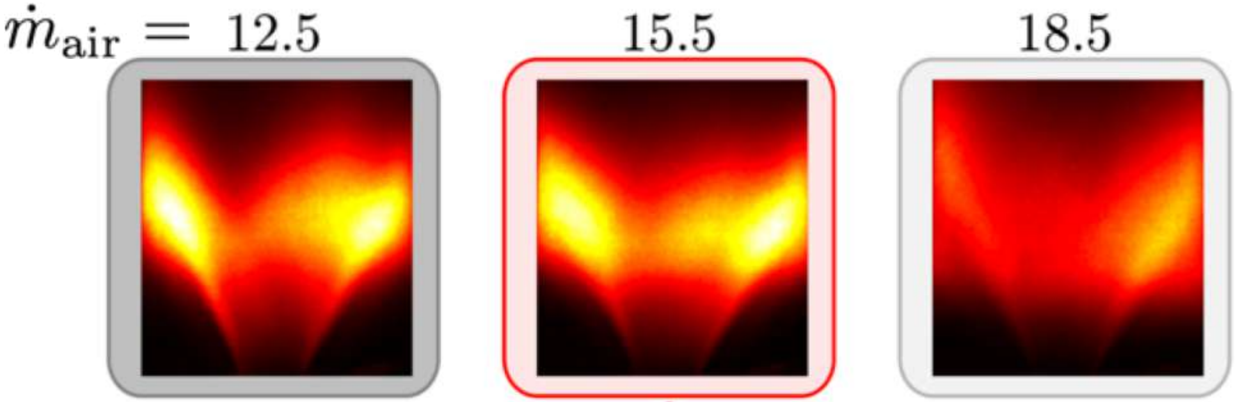




The instability driving mechanism has changed

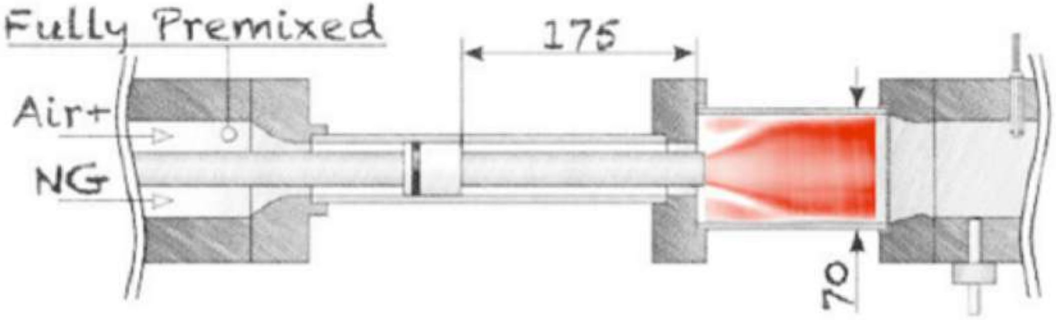
Previous

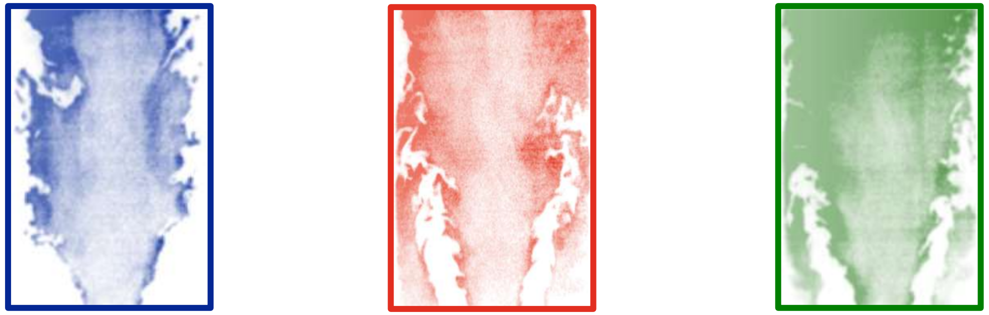




No flame topology change

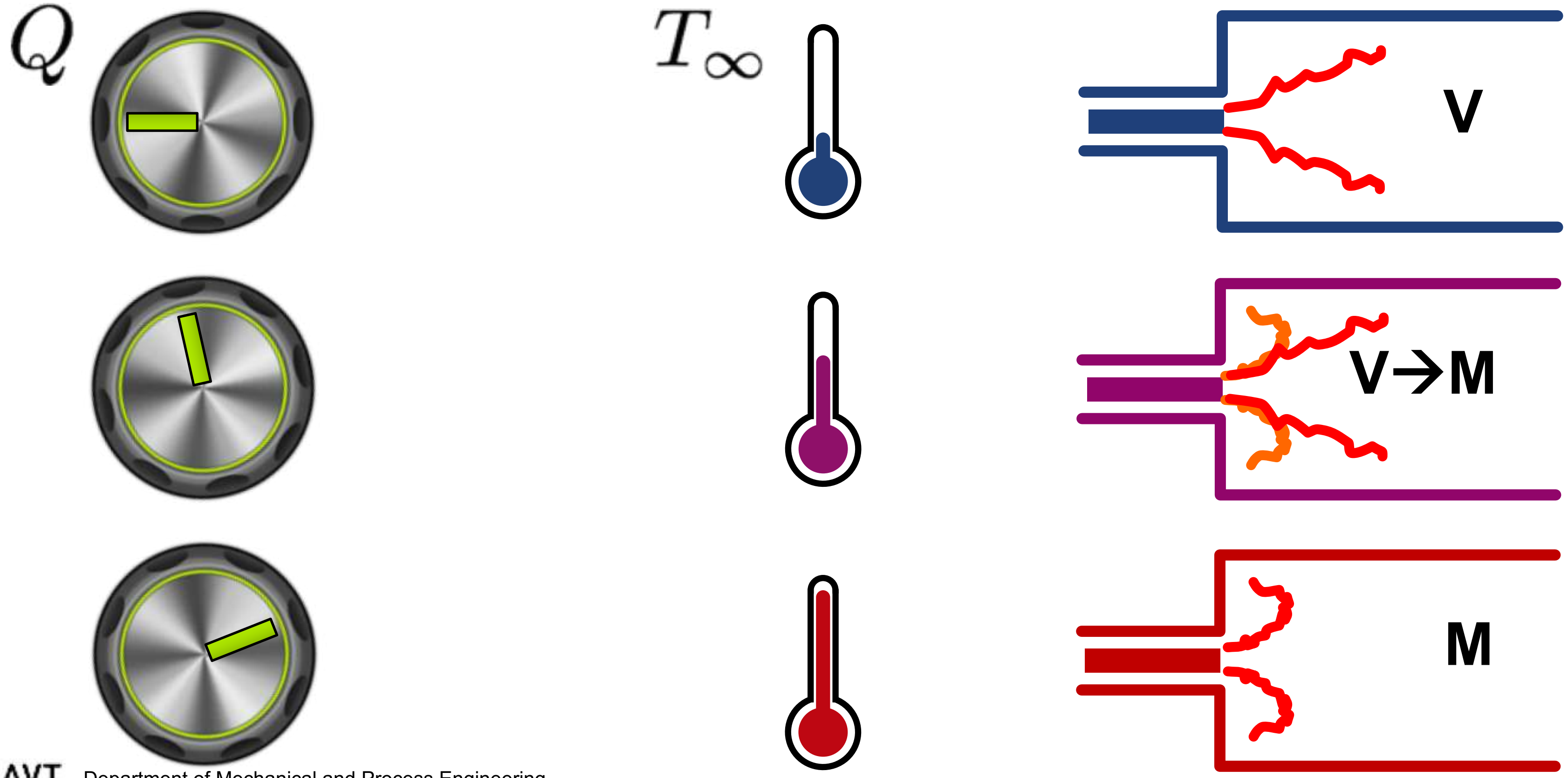
Current

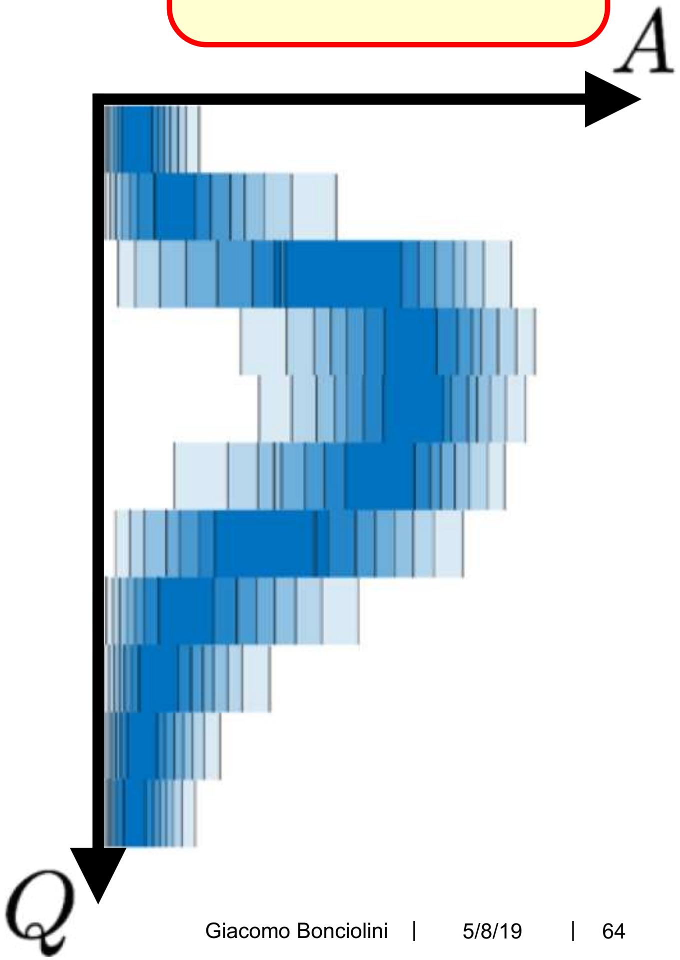




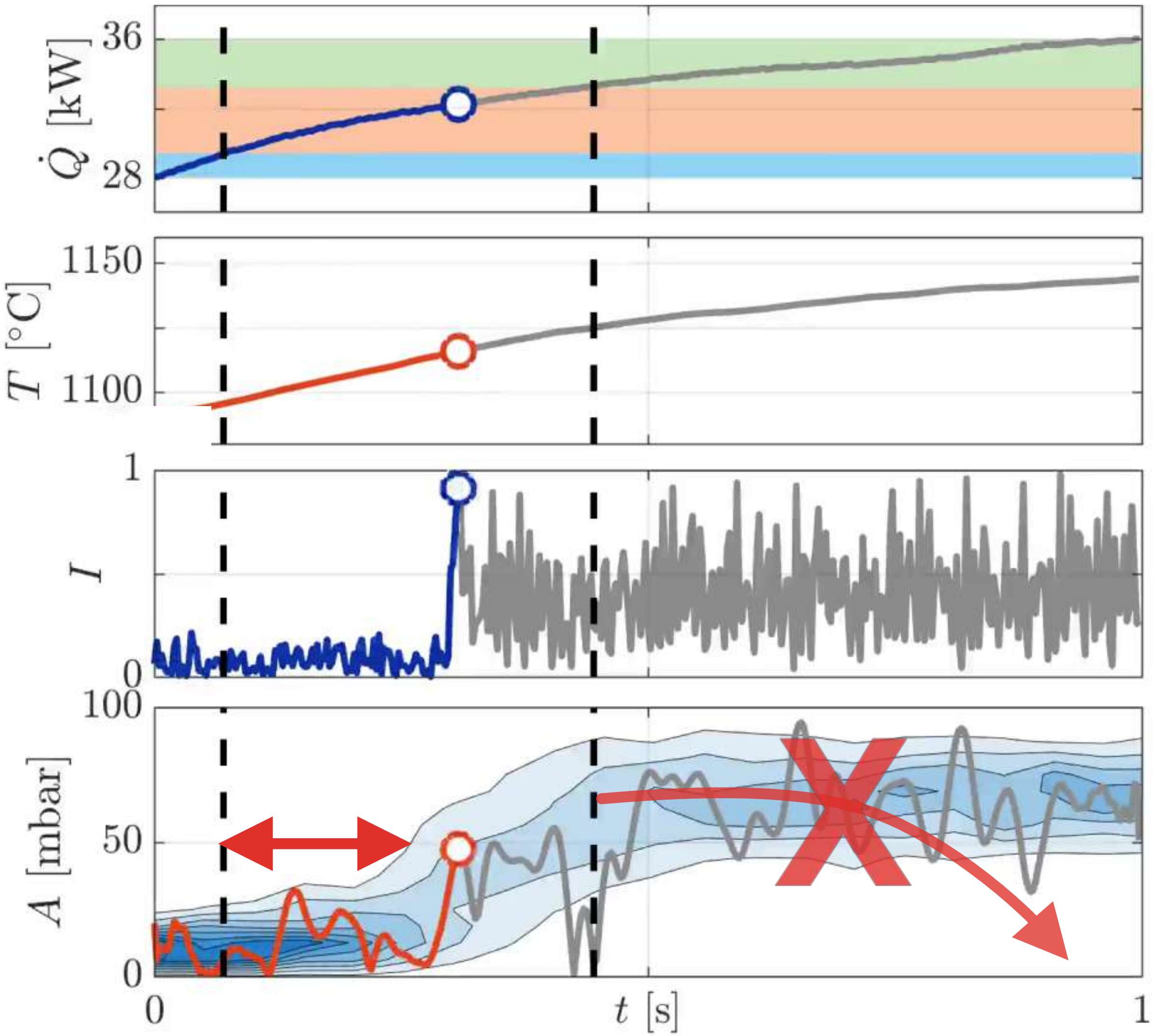
Wall temperature is the bifurcation parameter

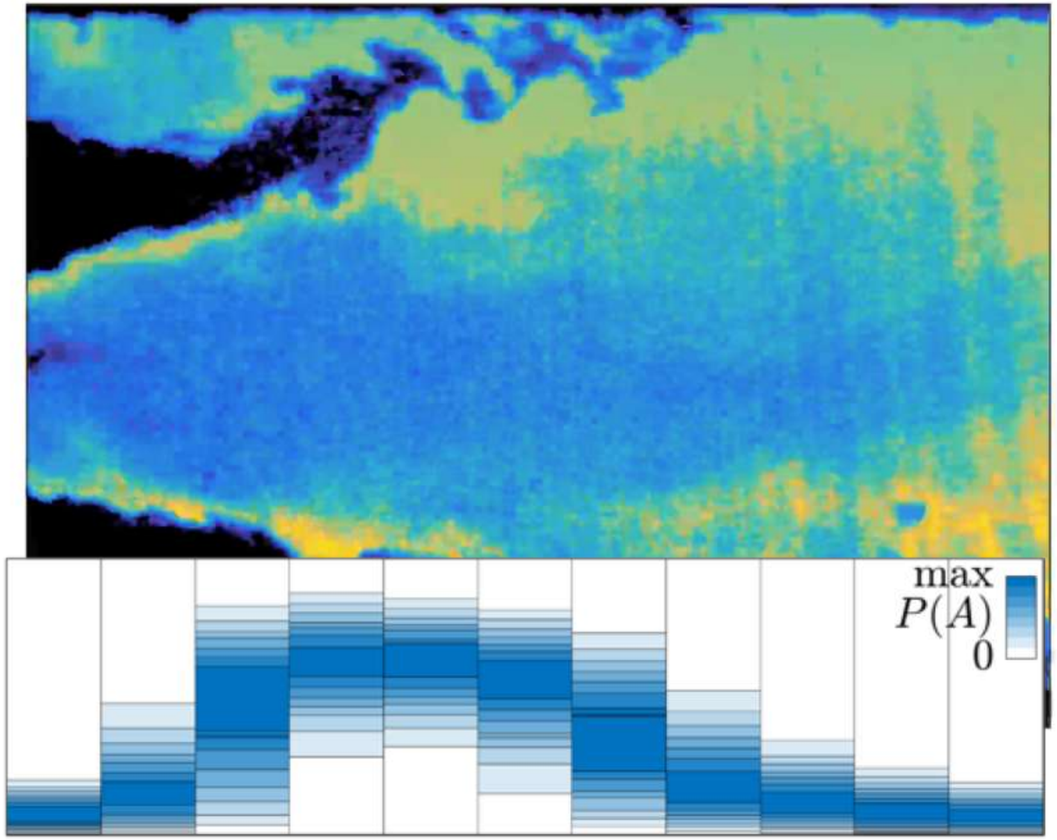
Thermal Power → Wall Temperature → Flame Shape → **Stability**



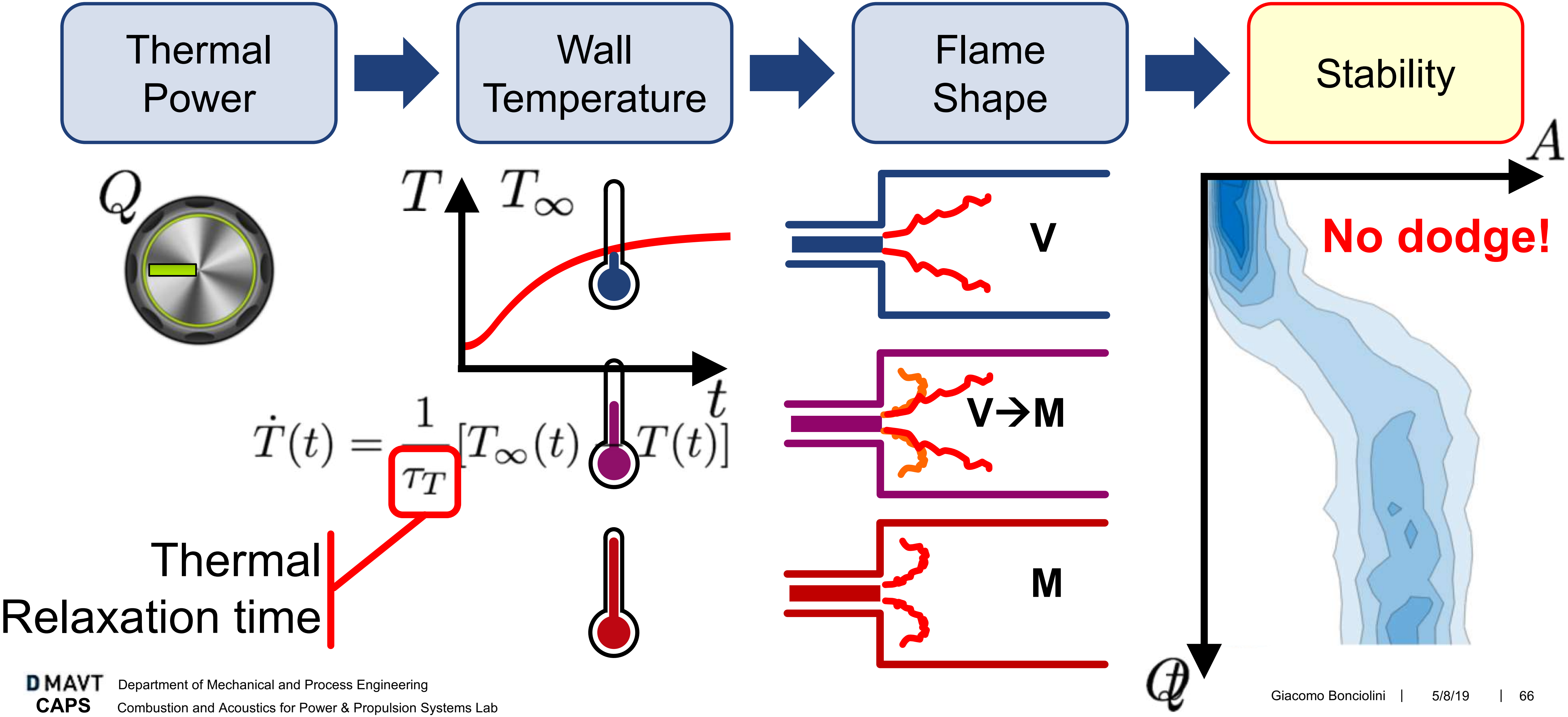


Thermal inertia defines the transient dynamics





Thermal inertia defines the transient dynamics



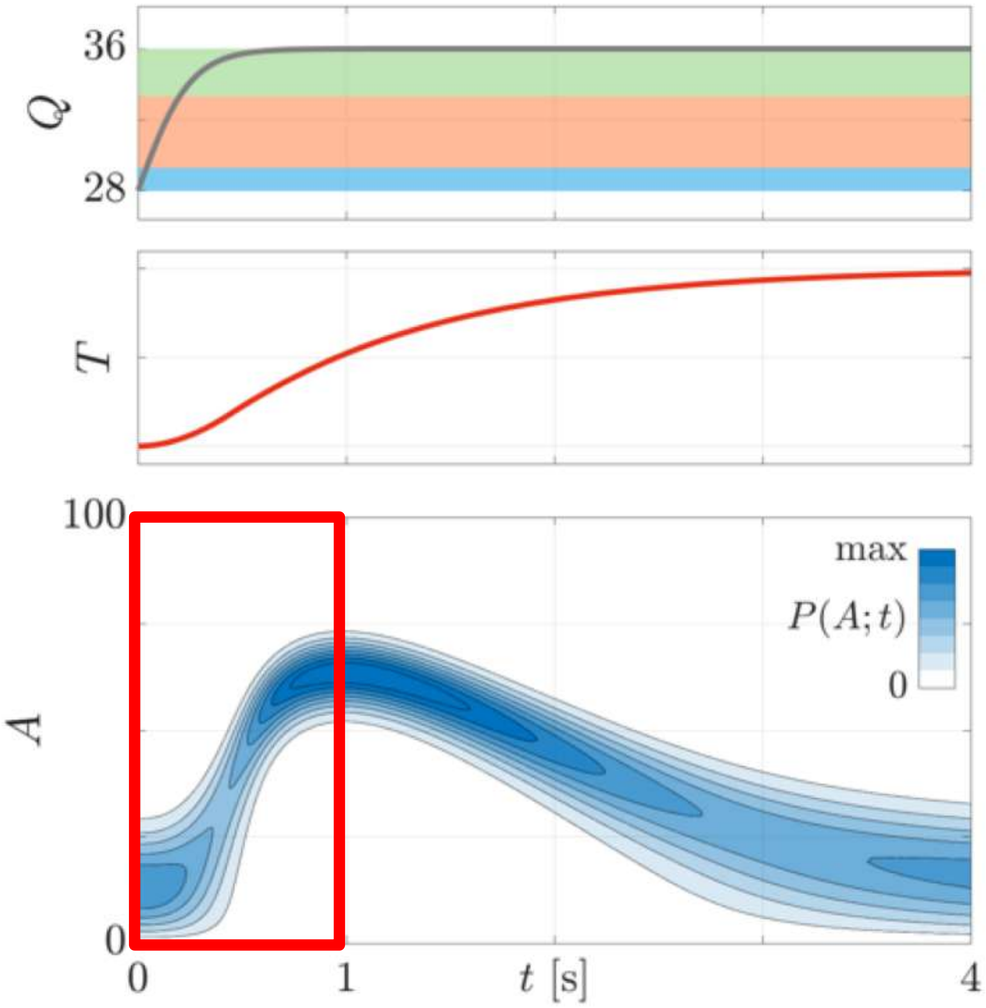
Thermal Power → Wall Temperature → Flame Shape → Stability

$$\dot{T}(t) = \frac{1}{\boxed{\tau_T}}[T_\infty(t) \quad T(t)]$$

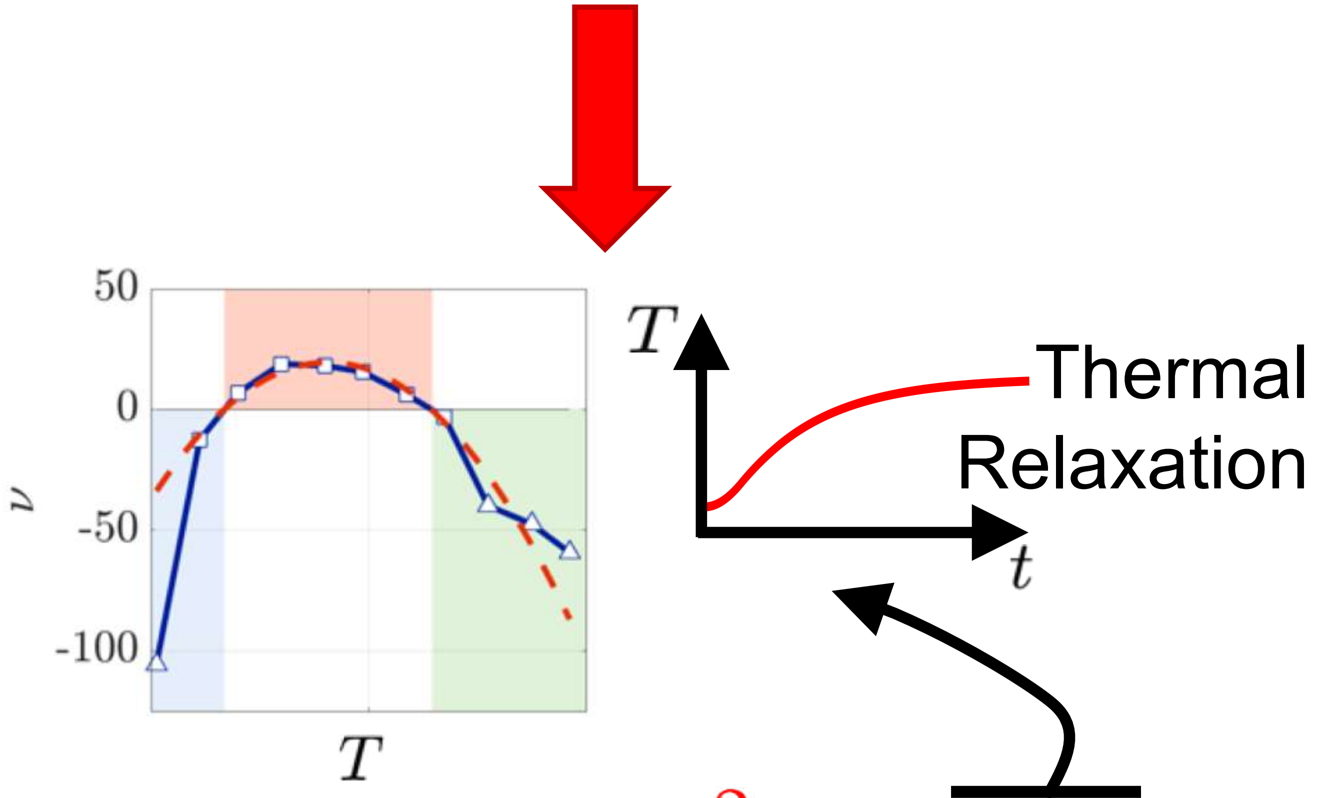
Thermal Relaxation time

No dodge!

The model can reproduce the observed dynamics



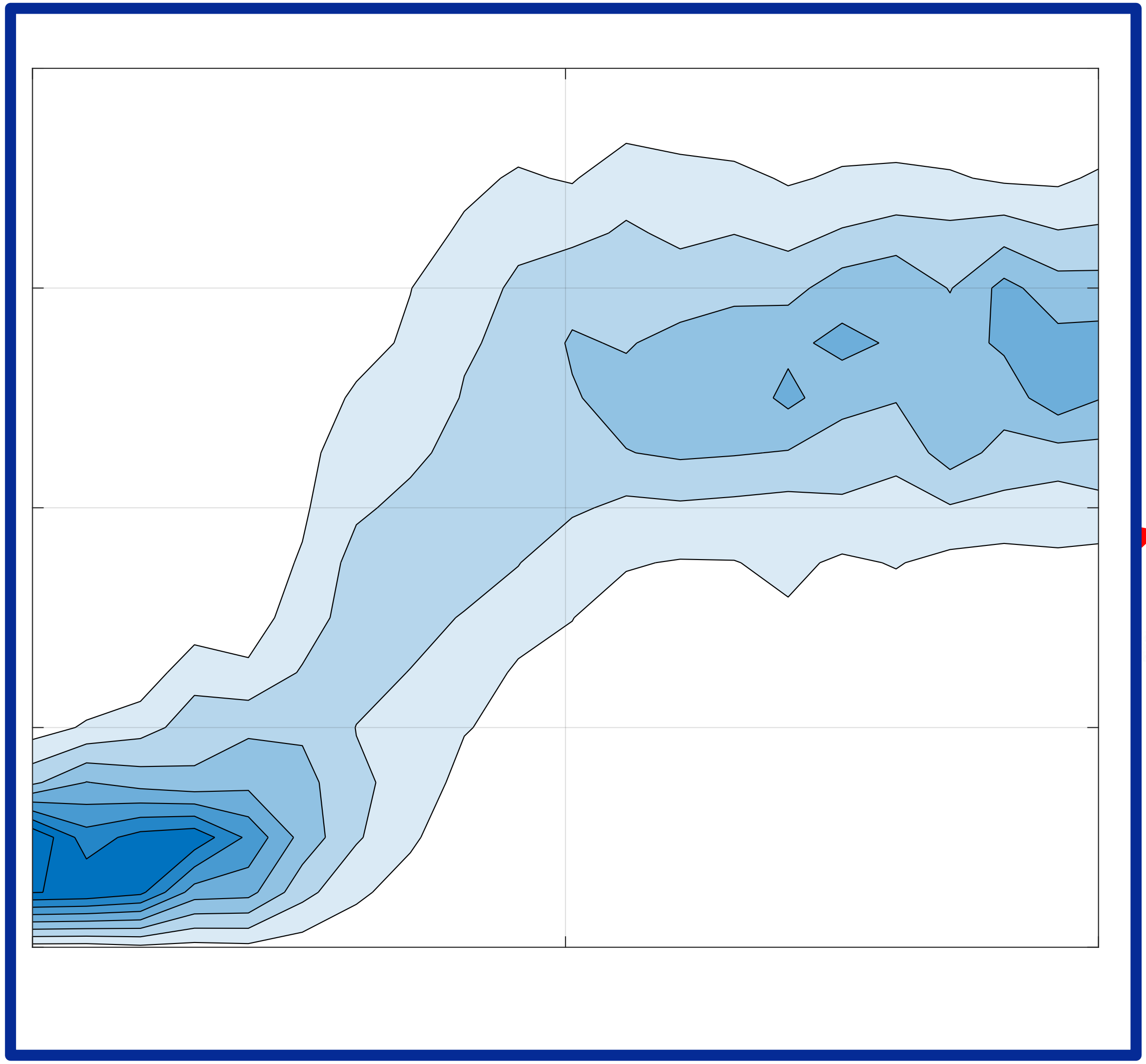
$$\ddot{p} + \omega_0^2 p = 2\nu(t)\dot{p} - \kappa p^2 \dot{p} + \xi(t)$$

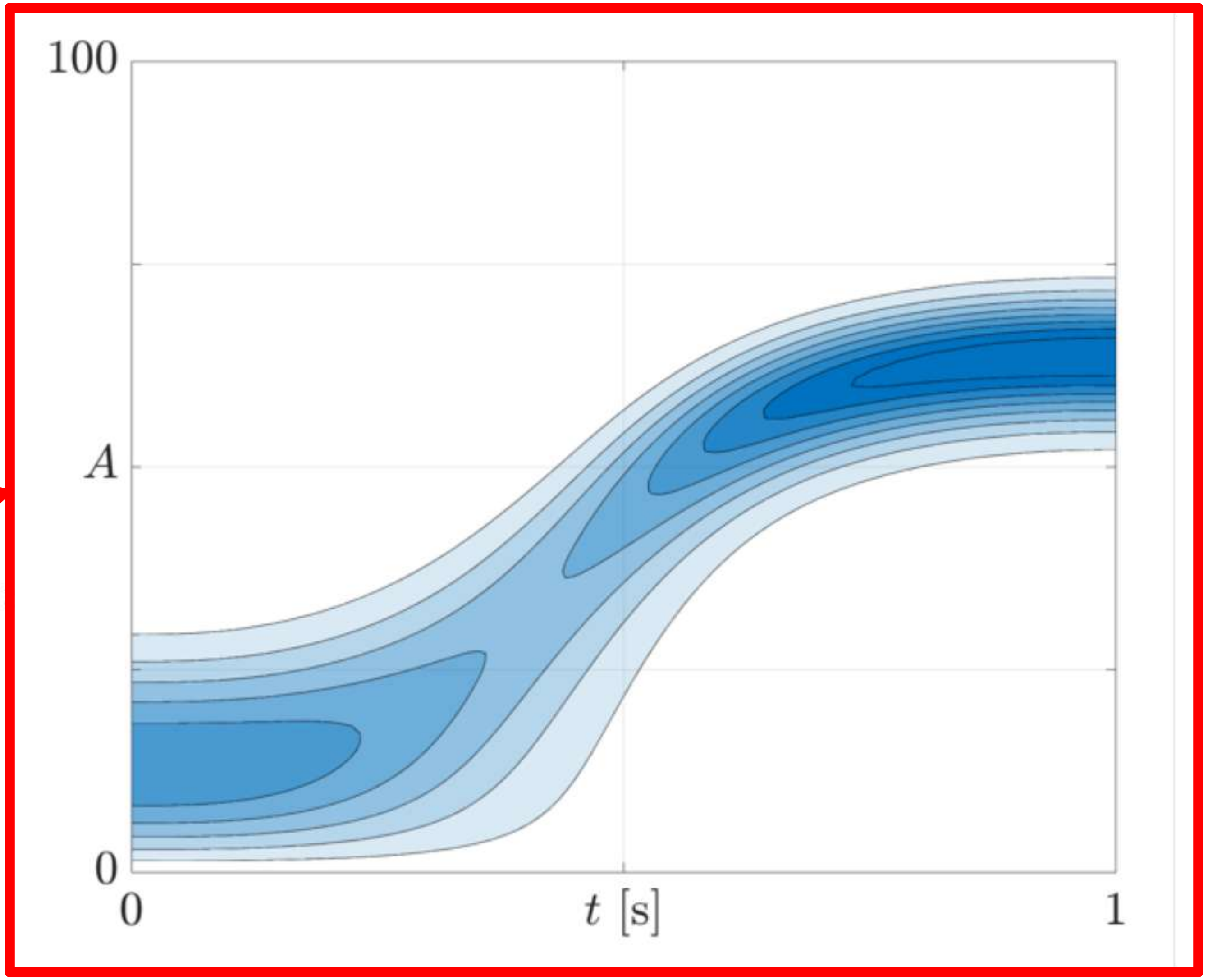


Thermal Relaxation

$$\nu(t) = a_\nu T(t)^2 + b_\nu T(t) + c_\nu$$

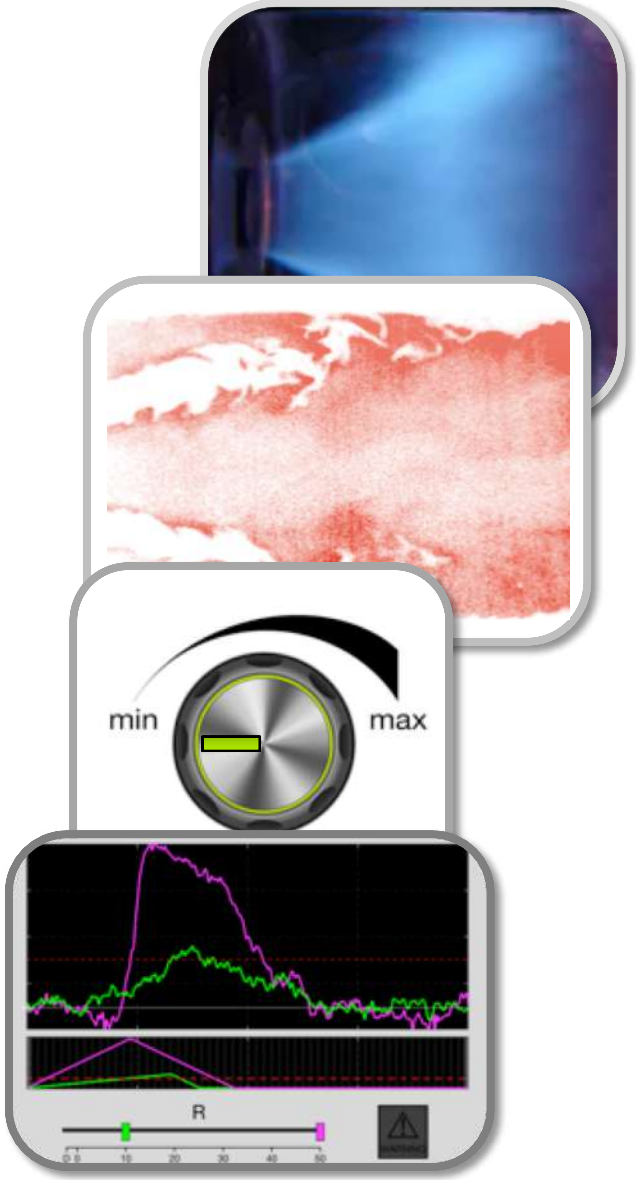
The model can reproduce the observed dynamics





Summary

- ➢ Thermoacoustic bifurcation: operating point ⇔ stability
- ➢ Many aspects contribute to the system dynamics
- ➢ Transient thermoacoustics shows peculiar phenomena
- ➢ Nonlinear oscillator model to understand the physics and perform additional analyses



List of Publications

6.

Delay - NL

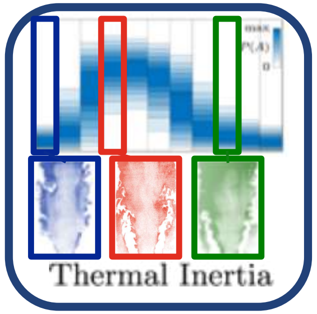
G. Bonciolini, C. Bourquard, N. Noiray, *“Effect of flame response delay and nonlinearity on the modelling of thermoacoustic instabilities”*, in preparation

5.

Bifurcation Dodge

G. Bonciolini, N. Noiray, *“Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation”*, Nonlinear Dynamics, (2019).

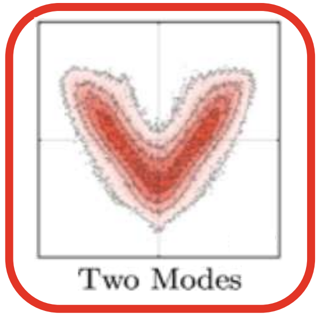
4.



Thermal Inertia

G. Bonciolini, D. Ebi, U. Doll, M. Weilenmann, N. Noiray, *“Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame”*, Proceedings of the Combustion Institute, (2019).

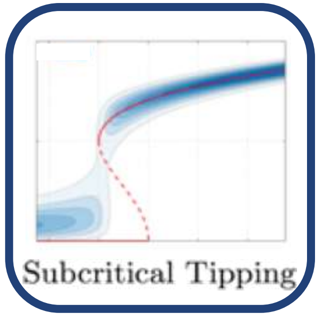
3.



Two Modes

G. Bonciolini, N. Noiray, *“Synchronization of Thermoacoustic Modes in Sequential Combustors”*, Journal of Engineering for Gas Turbines and Power, (2018)

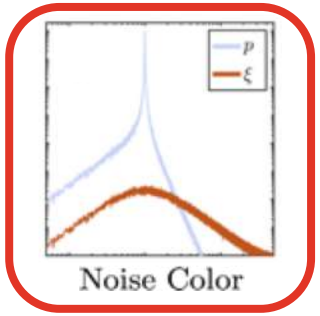
2.



Subcritical Tipping

G. Bonciolini, D. Ebi, E. Boujo, N. Noiray, *“Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation”*, Royal Society open science, (2018).

1.



Noise Color

G. Bonciolini, E. Boujo, N. Noiray, *“Output-only parameter identification of a colored-noise-driven Van-der-Pol oscillator: Thermoacoustic instabilities as an example”*, Physical Review E, (2017).

Supported by: FNSNF Swiss National Science Foundation

Thanks for your attention

Questions

Giacomo Bonciolini – CAPS Lab – ETH Zürich

giacomob@ethz.ch