Modelling of thermoacoustic dynamics in gas turbines applications

Giacomo Bonciolini

CAPS Laboratory, MAVT department ETH Zürich
T Anomaly [°C] vs 1950-81

Credit: NASA
Central Italy energy consumption by source in the last 24h

- **Gas**
- **Biomass**
- **Hydro**
- **Solar**
- **Wind**
- **Coal**

**Power [GW]**

- **0**
- **2**
- **4**
- **6**
- **8**

**Time**

- **00:00**
- **06:00**
- **12:00**
- **18:00**
GT combustion is studied in laboratory burners
GT combustion is studied in laboratory burners
GT combustion is studied in laboratory burners

Thermoacoustic Instability
Thermoacoustic coupling first discussed in 18\textsuperscript{th} and 19\textsuperscript{th} century

Doctor Higgins’ tube

The pyrophone

Lord Rayleigh
Thermoacoustic coupling stems from heat-sound interaction

- Turbulence
- Flame perturbation
- Flame Surface Variation
- Heat R.R. fluctuation
Thermoacoustic coupling stems from heat-sound interaction

Turbulence

Flame perturbation

Heat R.R. fluctuation

Source

Acoustic Oscillations

Pressure
Thermoacoustic coupling stems from heat-sound interaction

- Turbulence
- Flame perturbation
- Heat R.R. fluctuation
- Thermoacoustic Feedback
- Acoustic Oscillations

Pressure → Heat
Thermoacoustic coupling concerns today’s practical systems

Rocket engines

Gas turbines

Aero engines

Structural damages
Thermoacoustics can be modelled with different levels of detail

\[
p(x, t) = \sum_{i=1}^{\infty} \psi_i(x) \eta_i(t)
\]

\[
i = 1 \quad i = 2
\]
Thermoacoustics can be modelled with different levels of detail

\[ p(\mathbf{x}, t) = \sum_{i=1}^{\infty} \psi_i(\mathbf{x}) \eta_i(t) \rightarrow \ddot{\eta}_i + \omega_i^2 \eta_i = 2\nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j, \ldots)_{j=1,\ldots,\infty} + \xi_i \]
Thermoacoustics can be modelled with different levels of detail

\[ p(x, t) = \sum_{i=1}^{\infty} \psi_i(x) \eta_i(t) \]

\[ \ddot{\eta}_i + \omega_i^2 \eta_i = 2\nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j, \ldots)_{j=1}^{\infty} + \xi_i \]

Oscillation
Thermoacoustics can be modelled with different levels of detail

\[ p(\mathbf{x}, t) = \sum_{i=1}^{\infty} \psi_i(\mathbf{x}) \eta_i(t) \]

\[ \ddot{\eta}_i + \omega^2_i \eta_i = 2\nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j, \ldots)_{j=1,\ldots,\infty} + \xi_i \]

Linear decay/growth

\[ \nu < 0 \]

\[ \nu > 0 \]
Thermoacoustics can be modelled with different levels of detail

\[ p(x, t) = \sum_{i=1}^{\infty} \psi_i(x) \eta_i(t) \]

\[ \ddot{\eta}_i + \omega_i^2 \eta_i = 2 \nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j, \ldots)_{j=1...\infty} + \xi_i \]

Flame Non-Linear Response + Modes coupling
Thermoacoustics can be modelled with different levels of detail

\[ p(x, t) = \sum_{i=1}^{\infty} \psi_i(x) \eta_i(t) \]

\[ \ddot{\eta}_i + \omega_i^2 \eta_i = 2\nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j, \ldots)_{j=1}^{\infty} + \xi_i \]

Effect of turbulence: Random Amplitude
Changing oscillator’s elements to reproduce different dynamics

\[ p(x, t) = \sum_{i=1}^{N} \psi_i(x) \eta_i(t) \]

\[ \ddot{\eta}_i + \omega_i^2 \eta_i = 2 \nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j) + \xi_i \]

Experiment – 2 Modes

Model

1 Mode

Choice #1: Number of modes

Changing oscillator’s elements to reproduce different dynamics

\[ \ddot{\eta} + \omega_0^2 \eta = 2\nu \dot{\eta} - g(\eta, \dot{\eta}) + \xi \]

Noise color ↔ Spectral content


Choice #2: Noise color
Changing oscillator’s elements to reproduce different dynamics

\[ \ddot{\eta} + \omega_0^2 \eta = 2\nu \dot{\eta} - g(\eta, \dot{\eta}) + \xi \]

**Perturbations** - Time delay \( \tau \)

**Choice #3: Flame Response Delay**

Bonciolini, Bourquard, Noiray, “Effect of flame response delay and nonlinearity on the modelling of thermoacoustic instabilities”, in preparation

**Effective LGR**

\( \nu_e \)
Changing oscillator’s elements to reproduce different dynamics

\[ \ddot{\eta} + \omega_0^2 \eta = 2\nu \dot{\eta} - g(\eta, \dot{\eta}) + \xi \]

Choice #4: Nonlinearity
Model Elements

My PhD

Transients

Delay - NL

Noise Color

Two Modes

Subcritical Tipping

Bifurcation Dodge

Thermal Inertia
Thermoacoustics depends on operating conditions

More air

\[ \rho_{\text{avg}} [g/s] \]

\[ P_{\text{rms}} \]

\[ f \ [Hz] \]

\[ S_{pp} \]

\[ p \ [A] \]

\[ P(A) \]

\[ A \ [25] \]
Thermoacoustics depends on operating conditions

- Bifurcation: dynamics change with one parameter
- Transient dynamics through the bifurcation?
Transient Thermoacoustics is a relevant problem for Gas Turbines
The transient thermoacoustics trilogy

Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation
Royal Society Open Science

Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation
Nonlinear Dynamics

Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame
Proceedings of the Combustion Institute
The dynamics changes for different fuel/air ratios

Stable

Bi-stable

Unstable
The dynamics changes for different fuel/air ratios

Stable

Bi-stable

Unstable

Subcritical bifurcation

\[ \phi \]

\[ P(p) \]

\[ P(A) \]
Transient dynamics is explored by ramping the fuel/air ratio.

This cycle is repeated 100 times.
Transient dynamics is explored by ramping the fuel/air ratio

Fold point

Quasi-steady ramps: Hysteresis

Hopf point
Transient dynamics is explored by ramping the fuel/air ratio up & down continuously.
Transient dynamics is explored by ramping the fuel/air ratio

Fuel/air ratio ramped up & down continuously

High-A inertial effects

Bifurcation delay
Tipping delay depends on the ramp rate

Faster ramp → more delay
Tipping delay is observed experimentally

No bistability → Hysteresis

Bifurcation delay
The model can reproduce the thermoacoustic bifurcation

$$\ddot{p} + \omega_0^2 p = [2\nu + \kappa p^2 - \gamma p^4] \dot{p} + \xi$$

Envelope

$$\dot{A} = \nu A + \frac{\kappa}{8} A^3 - \frac{\gamma}{32} A^5 + \frac{\Gamma}{4\omega_0^2 A} + \zeta$$

Fokker-Planck eq.

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial A} [\mathcal{F}(A, t) P] + \frac{\Gamma}{4\omega_0^2} \frac{\partial^2 P}{\partial A^2}$$
The model can reproduce the transient dynamics

Hysteresis

Normalized time

Higher $R \rightarrow$ More delay
Tipping delay is a risk

Control threshold
Tipping delay is a risk
Tipping delay is a risk

\[ E_{th} = 1.88 \]
\[ E_{th} = 0.23 \]
Tipping delay is a risk

5000 realizations of the process
Tipping delay is a risk

- Higher Ramp Rate → More Delay → Higher triggered Limit cycle Amplitude
- More energy released
- Slower = Safer
The transient thermoacoustics trilogy

Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation
Royal Society Open Science

Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation
Nonlinear Dynamics

Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame
Proceedings of the Combustion Institute
Two consecutive and mirrored supercritical bifurcations

Can we dodge the bifurcation by ramping fast?
Transient dynamics is explored with multiple ramp experiments
Transient dynamics is explored with multiple ramp experiments
Transient dynamics is explored with multiple ramp experiments

Stationary reference

\[ \dot{m}_{\text{air}} [\text{g/s}] \]

\[ 12.5 \quad \Phi \quad 20.0 \]

\[ p, A [\text{mbar}] \]

\[ \frac{t}{T_r} \]
Transient statistics depends on the ramp time

Stationary reference

$\dot{m}_{\text{air}} \quad T_r = 1.19\text{s}$

12.5 20.0
Transient statistics depends on the ramp time

Stationary reference

Bifurcation dodge!
The model can reproduce the observed transient dynamics

Van der Pol Model

\[ \ddot{p} + \omega_0^2 p = 2\nu(t)\dot{p} - \kappa p^2 \dot{p} + \]

Identification

\[ \nu(t) = n_1 \cos \left[ n_2 \dot{m}_{\text{air}}(t) \right] + n_3 \]

\[ \frac{\partial P}{\partial t} = -\frac{\partial}{\partial A} [\mathcal{F}(A,t)P] + \frac{\Gamma}{4\omega_0^2} \frac{\partial^2 P}{\partial A^2} \]
The model can be used for preliminary risk estimation

Stationary mapping

- Oscillation frequency: $f_0 = \frac{\omega_0}{2\pi}$
- Growth Rate range: $\nu \in [0, 2\%]\omega_0$
- Max Amp: $\kappa \approx \frac{8\nu}{A_{\text{max}}^2}$
The model can be used for preliminary risk estimation.

Explore the system dynamics in a parametric 2D space.
The model can predict the dodge probability
The model can predict the dodge probability

\[ T_r = 1.19 \text{s} \]

**Dodge Threshold** ↔ **Absorbing BC**
The model can predict the dodge probability

\[ Pr_d = Pr_{nc}(T_r) \]

\[ T_r = 1.19s \]

\[ T_r = 0.16s \]

Faster = Safer

No dodge

Full dodge
A fast growth is harder to dodge

\[ e^{\nu_1 t} \]

\[ \nu_1 < \nu_2 \]

\[ e^{\nu_2 t} \]
The transient thermoacoustics trilogy

Experiments and modelling of rate-dependent transition delay in a stochastic subcritical bifurcation
Royal Society Open Science

Bifurcation Dodge: Avoidance of a Thermoacoustic Instability under Transient Operation
Nonlinear Dynamics

Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame
Proceedings of the Combustion Institute
The configuration is modified
The current configuration features as well a double bifurcation instability.
Flame topology change is the instability driver

\[ S_{pp} \text{ [dB]} \]

\[ p, A \text{ [mbar]} \]

\[ f \text{ [Hz]} \]

\[ t \text{ [s]} \]

2000 FPS LIF
Flame topology change is the instability driver
Flame topology change is the instability driver
The instability driving mechanism has changed

Current

Previous

\[ \dot{m}_{\text{air}} = 12.5 \]

No flame topology change
Wall temperature is the bifurcation parameter

Thermal Power → Wall Temperature → Flame Shape → Stability

\[ Q \rightarrow T_\infty \rightarrow V \rightarrow M \]

\[ A \rightarrow Q \]
Thermal inertia defines the transient dynamics
Thermal inertia defines the transient dynamics

Thermal Power → Wall Temperature → Flame Shape → Stability

\[ \dot{T}(t) = \frac{1}{\tau_T} [T_\infty(t) - T(t)] \]

Thermal Relaxation time

No dodge!
The model can reproduce the observed dynamics

\[ \ddot{p} + \omega_0^2 p = 2\nu(t) \dot{p} - \kappa p^2 \dot{p} + \xi(t) \]

\[ \nu(t) = a_\nu T(t)^2 + b_\nu T(t) + c_\nu \]
The model can reproduce the observed dynamics
Summary

- Thermoacoustic bifurcation: operating point ↔ stability
- Many aspects contribute to the system dynamics
- Transient thermoacoustics shows peculiar phenomena
- Nonlinear oscillator model to understand the physics and perform additional analyses
List of Publications


Thanks for your attention

Questions

Giacomo Bonciolini – CAPS Lab – ETH Zürich

giacomob@ethz.ch