Important Note

The following are redacted (incomplete) slides. This is because some of the images in the original version were taken from the course “Engineering Acoustics 2” by Prof. Noiray at ETH, and this material will not be made public. In case you are interested in more details on the presentation, please contact me under ptiemo@ethz.ch

Modeling Combustion Instability in Large Gas Turbines

A Frontiers in Energy Research lecture by Tiemo Pedergnana

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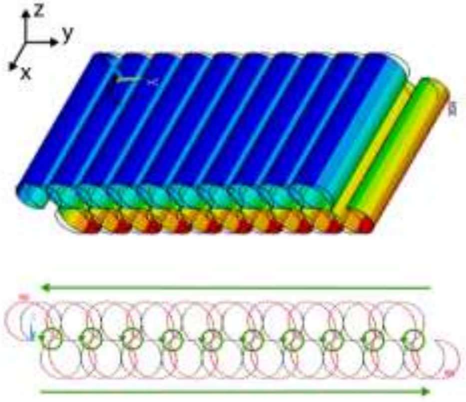
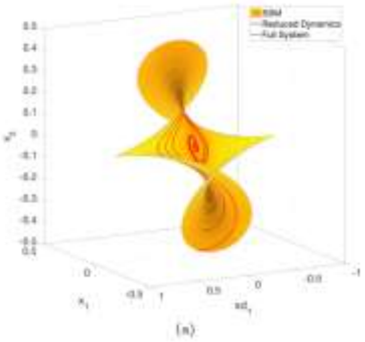
Current Research

- Aero-and thermoacoustic instabilities
- Hydrodynamic formulation

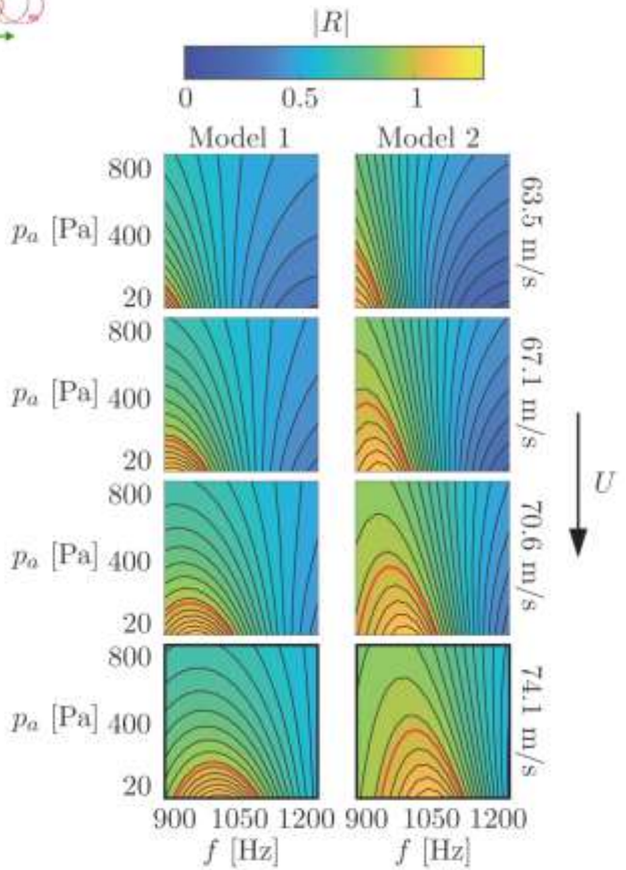
Previous Research

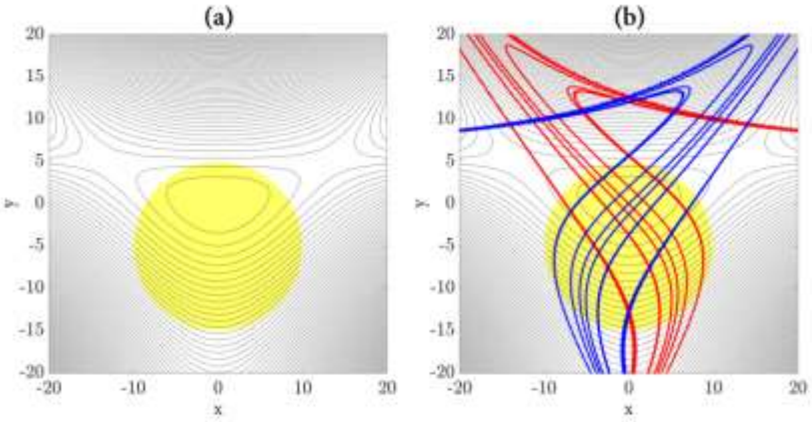
- Exact model order reduction
- Anisotropic plate vibrations
- Exact Navier-Stokes solutions, vortex criteria and objectivity





$$\mathbf{v}(\mathbf{x},t)=\begin{pmatrix}\mathbf{u}(\mathbf{x},t)\\ w(\mathbf{x},t)\end{pmatrix}=\begin{pmatrix}\mathbf{h}(t)\\ w_0\end{pmatrix}+\frac{1}{2}\omega(t)\begin{pmatrix}-y\\ x\\ 0\end{pmatrix}+\begin{pmatrix}\sum_{k=1}^{n}\begin{pmatrix}a_k(t) & b_k(t)\\ b_k(t) & -a_k(t)\end{pmatrix}\begin{pmatrix}\mathrm{Re}f_k\\ \mathrm{Im}f_k\end{pmatrix}\\ 0\end{pmatrix}$$





Overview

- Gas turbines (GT)
- Combustion instability (CI)
- Low-order modeling of CI
- Present work

Energy Challenge

- Why GT?
- CO_2 and pollutant emission reduction
- Improvement of GT technology mandatory

Energy Challenge

- Increase of renewables
- Strong negative impact on grid stability
- To compensate: combined cycle power plants based on GT (faster ramp-up than coal, nuclear)

Combined Cycle Power Plant

1: Electric generators

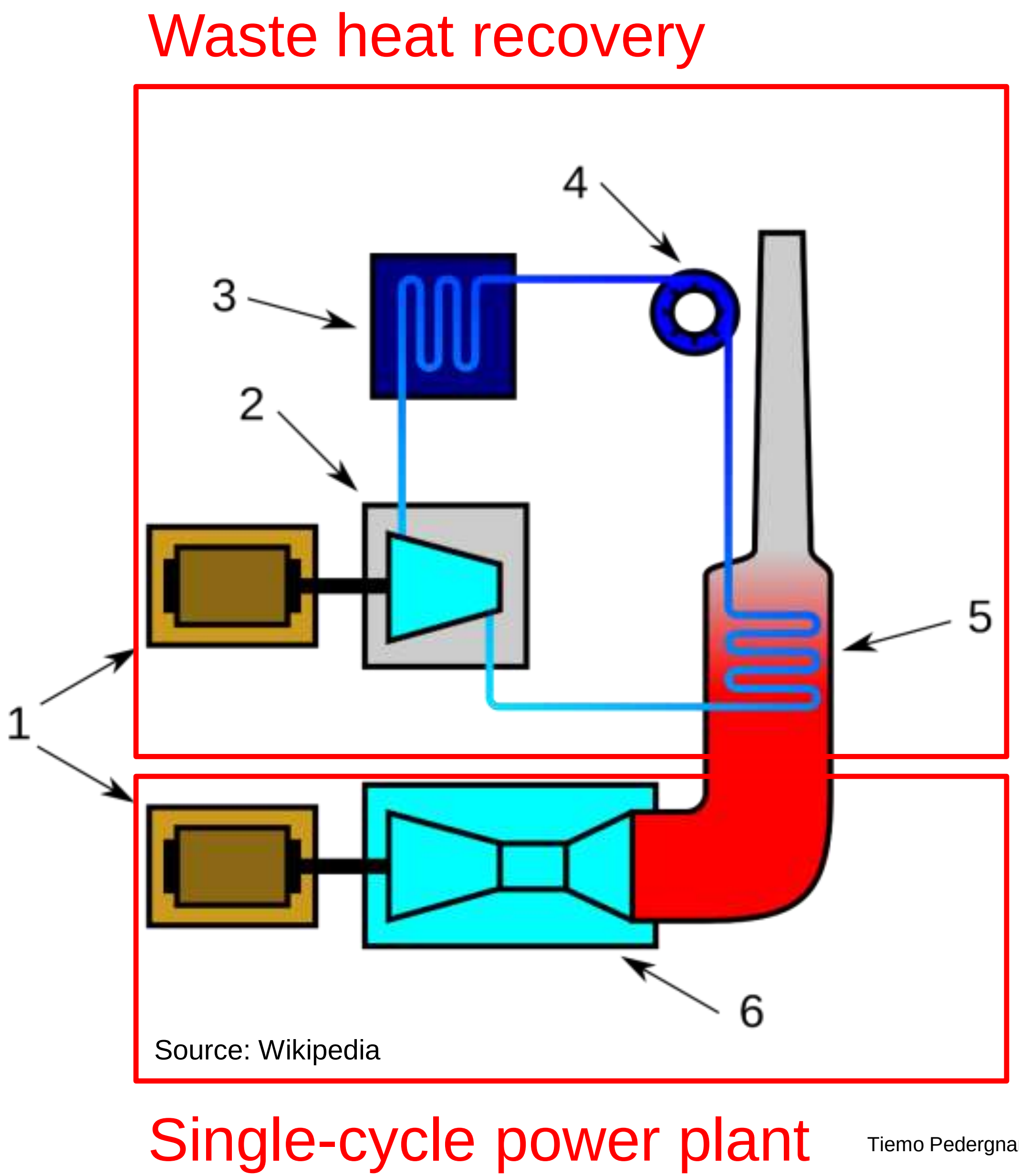
2: Steam turbine

3: Condenser

4: Pump

5: Boiler/heat exchanger

6: Gas turbine



Source: Wikipedia

Gas Turbines

- World record: *Most efficient combined-cycle power plant* using GE's HA7 GT
- Other manufacturers: Siemens, Ansaldo Energia Switzerland

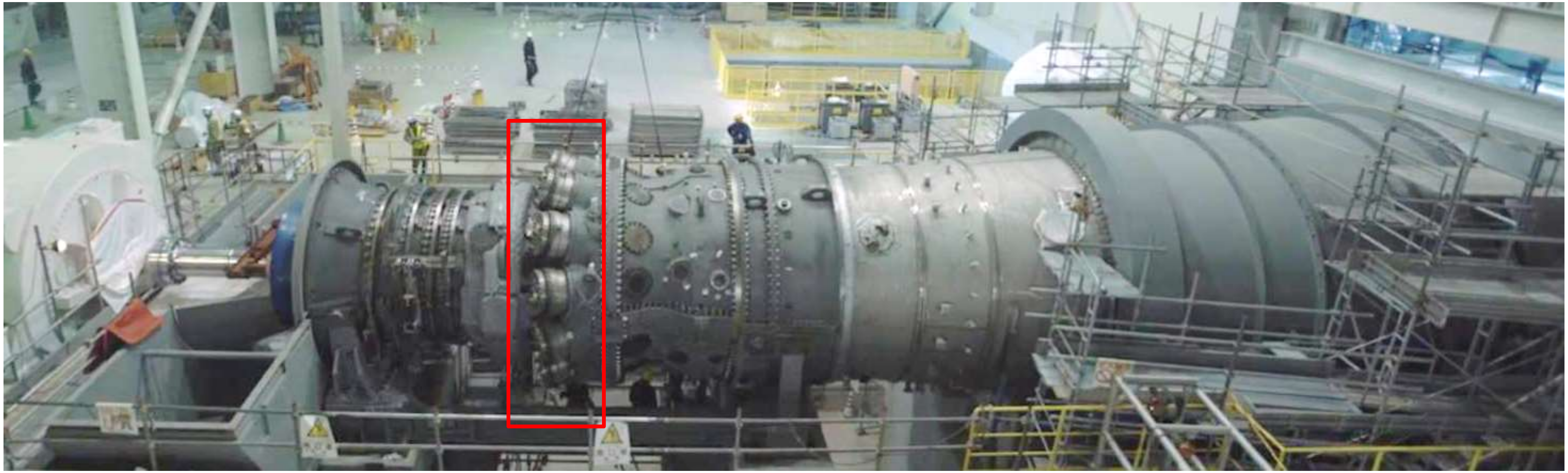
Gas Turbines

- Many requirements on GT manufacturers

- Efficiency, reliability, reduced NO_x emissions, flexibility (fuel and operation)

Gas Turbines

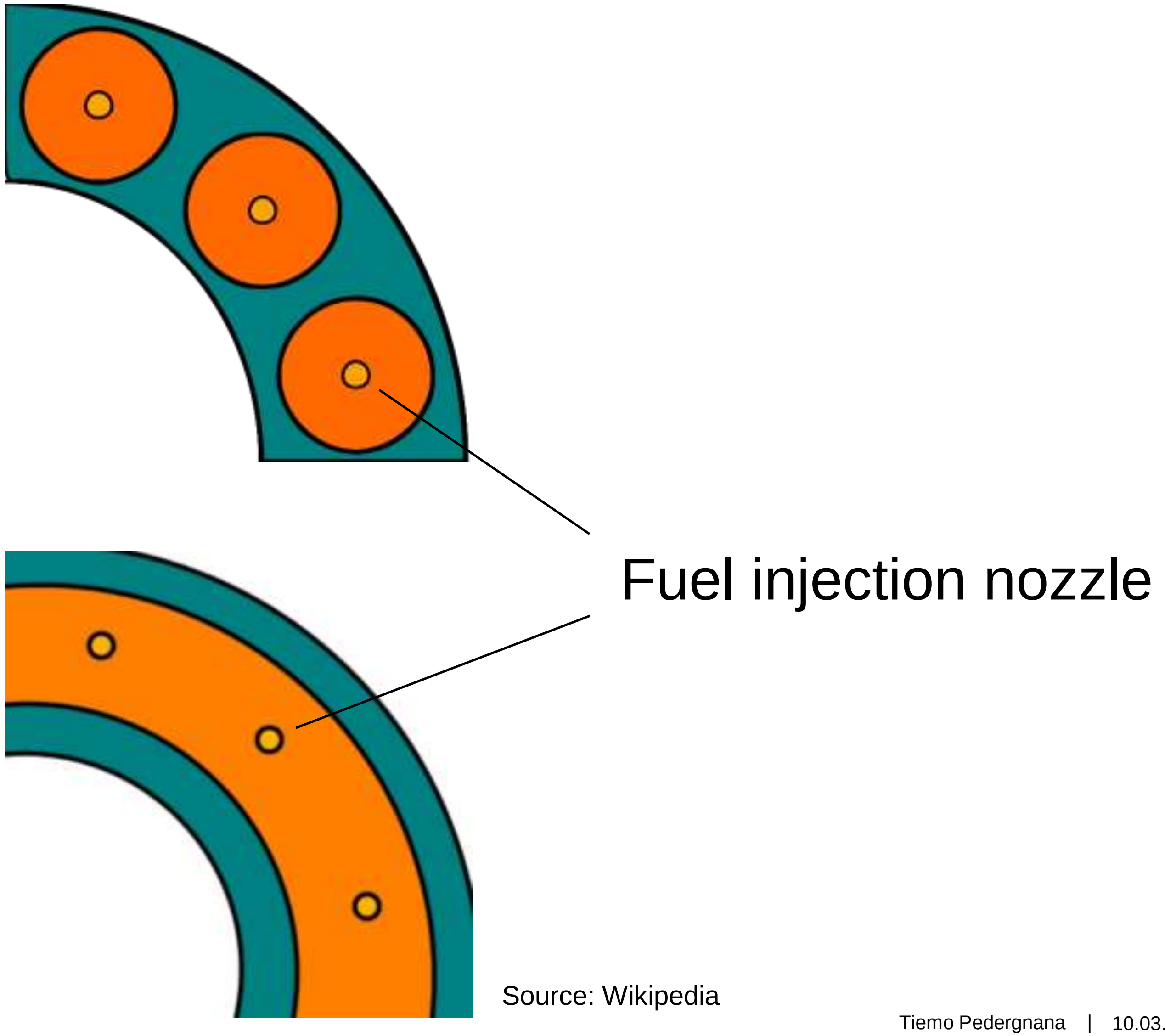
- Can-annular architecture favorable in large GT (testing & repair)



Source: GE

Gas Turbines

- Can-annular combustor (axis-on, through exhaust):
- Annular combustor:



Source: Wikipedia

Combustion Instability

- Thermoacoustic (TA) instabilities relevant for combustion systems in power, aeronautic and aerospace applications

Combustion Instability

- Over the last 20 years: GT technology development driven by the reduction of pollutant emissions → “Dry Low NO_x emissions”concepts (DLE)

Source: Wikipedia

Combustion Instability

- Technology changes for emission reduction, increased efficiency and performance lead to generally lead to TA instabilities
- Non-monotonous dependence on parameters
- Feared and hidden difficulty for most combustion tech.

Combustion Instability

- Classic picture of instability mechanism:

Combustion Instability

Low-Order Modeling of Combustion Instability

- Bonciolini et al.
  (Comb. Flame 2021):
- Low-order modelling (LOM) is the highest degree of simplification of the physics
- Valuable for academic study of new combustor concepts, TA oscillations

Low-Order Modeling of Combustion Instability

- Valuable insight into physics gained from LOM

- LOM enables real-time parameter identification from experiments

- Growth rate determines whether CI occurs, and how fast it grows/decays

So Far

- GT-based combined-cycle power plants preferable over coal, nuclear to bridge supply gaps caused by increased use of renewables
- Can-annular combustor architecture favorable in large GT
- Plaguing and recurring issue in GT development: Combustion instability
- Low-order models provide valuable physical insight into the TA instability mechanism, enable real-time parameter identification

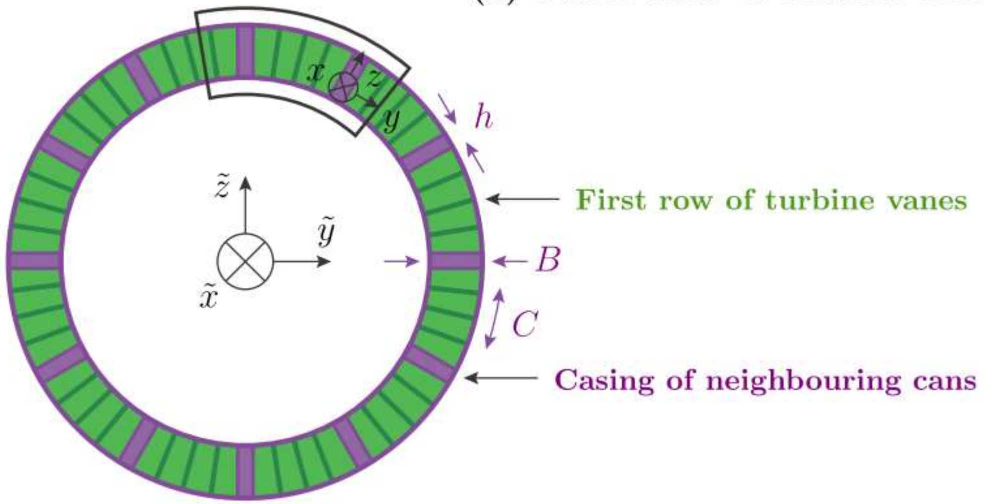
Present Work

- Initiate physics-based academic study of combustion instability in can-annular GT (industry has already started)
- Develop low-order model to unravel some of the complex underlying physics
- Determine generic dependence of system stability on physical parameters

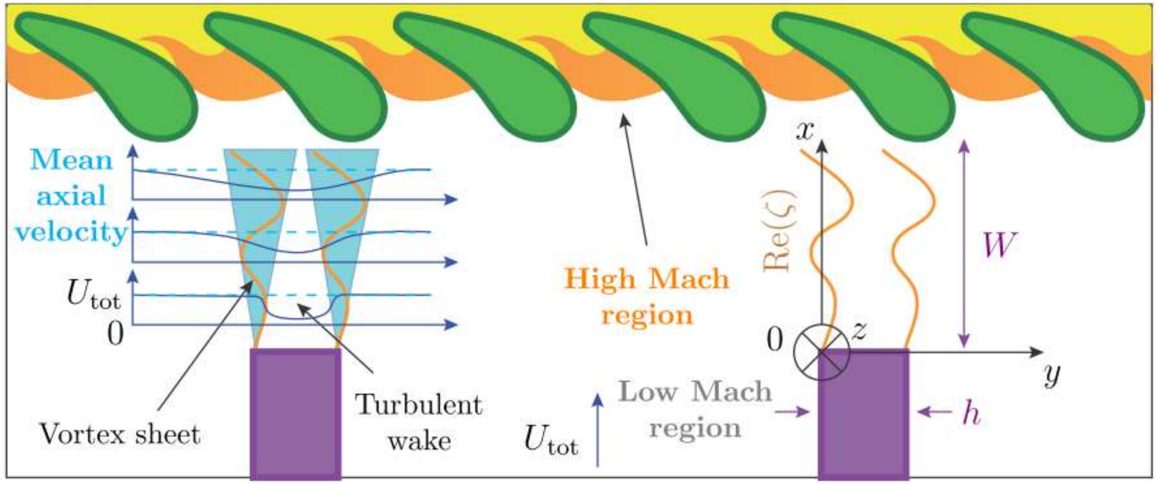
Present Work

- Research based on preprint (JSV)
- “Coupling-Induced Instability in a Ring of Thermoacoustic Oscillators”
- Focus on linear stability, modeling can-to-can coupling at turbine inlet

(a) Axial view of turbine inlet



(b) Radial view of can outlet and turbine inlet



Scope of Present Work

- Linear stability implies small acoustic perturbations
- Qualitative description of the onset of TA oscillations
- No description of high-amplitude behavior

Small acoustic perturbations

Linear stability

Source: PNGkey

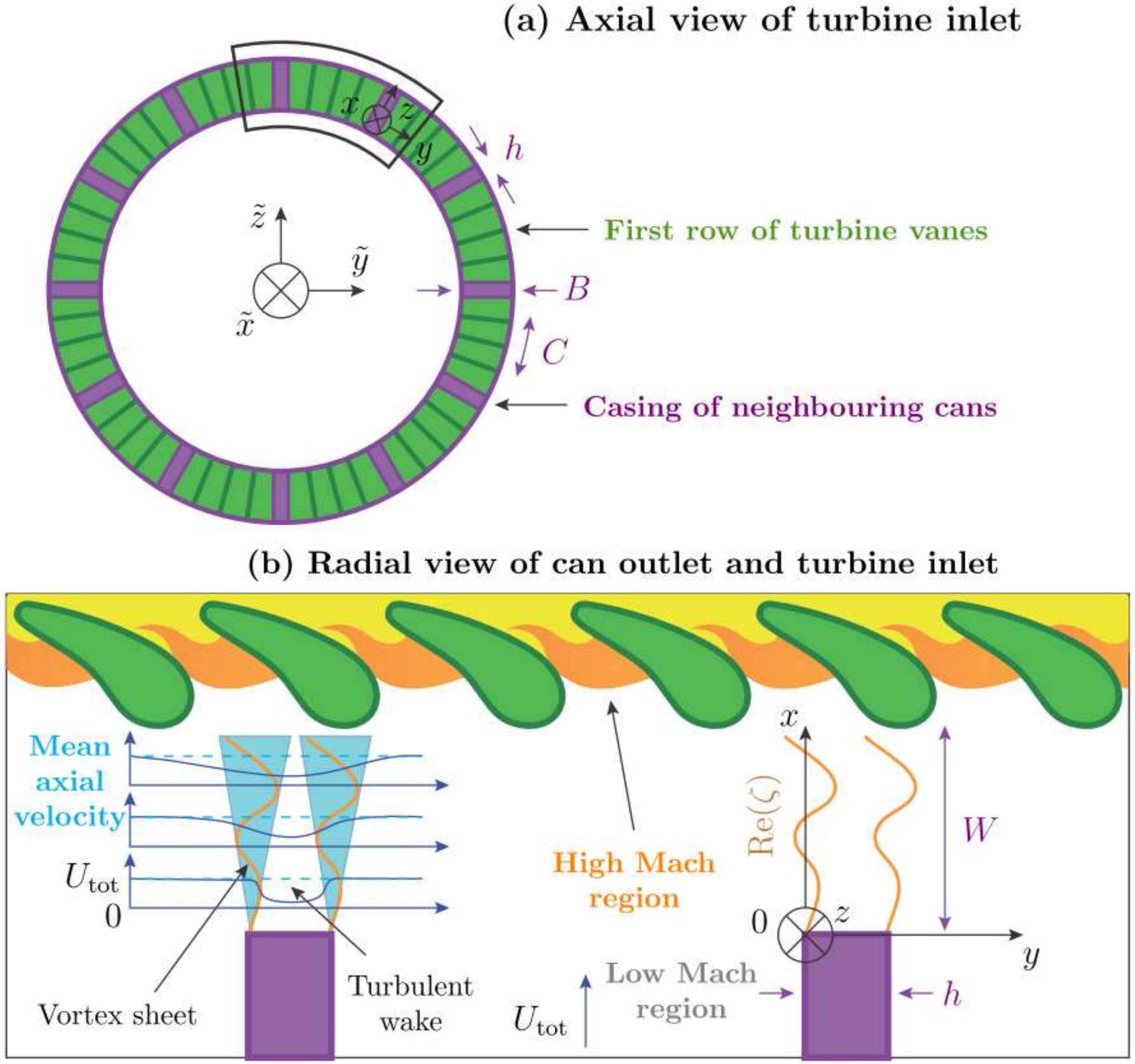
High-amplitude behavior

Acoustic Coupling at Turbine Inlet

- Turbulent wake forms at inlet
- Acoustic-Hydrodynamic interaction between sound waves and wake
- Energy exchange possible

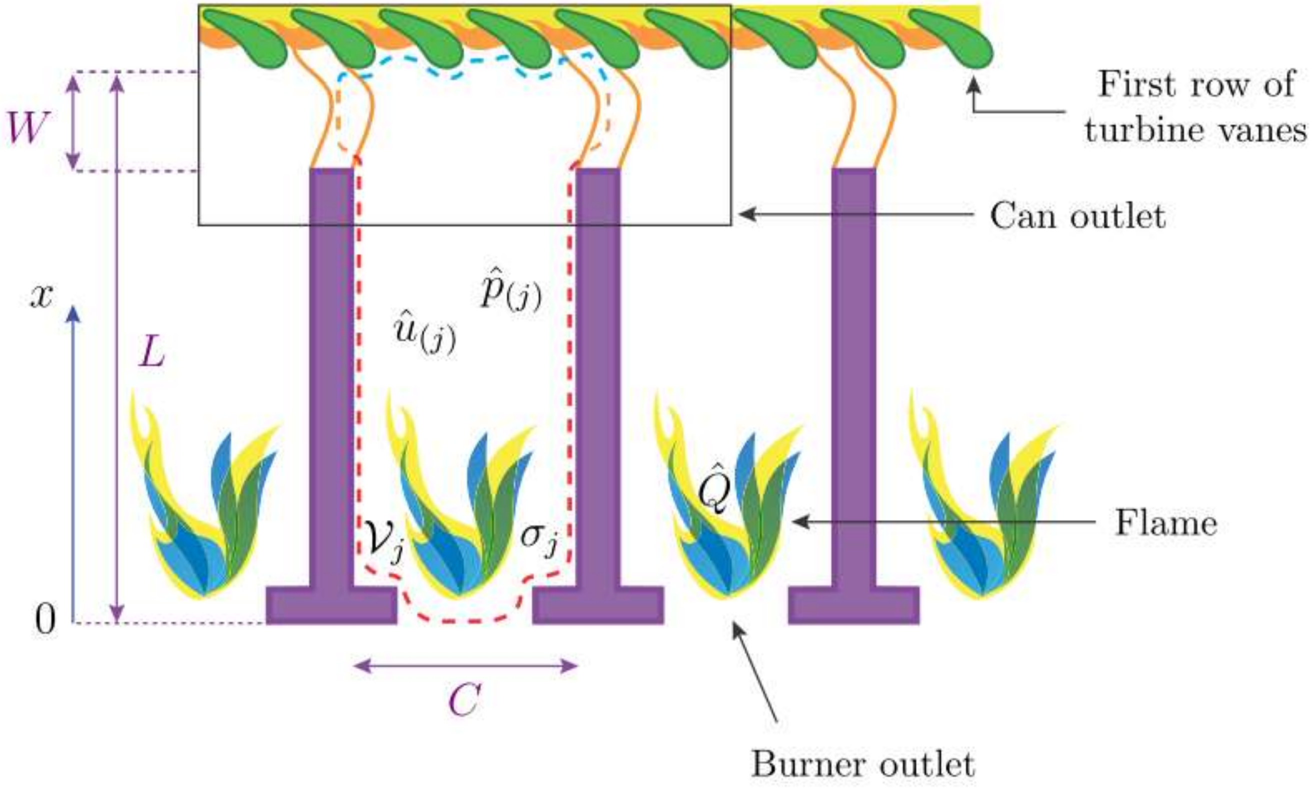
(a) Axial view of turbine inlet

(b) Radial view of can outlet and turbine inlet



Thermoacoustic model

- Sketch of TA model of j^{th} can
- Flames feed energy into the ac. field in the can outlet region
- Interaction between ac. pressure in neighboring cans at turbine inlet



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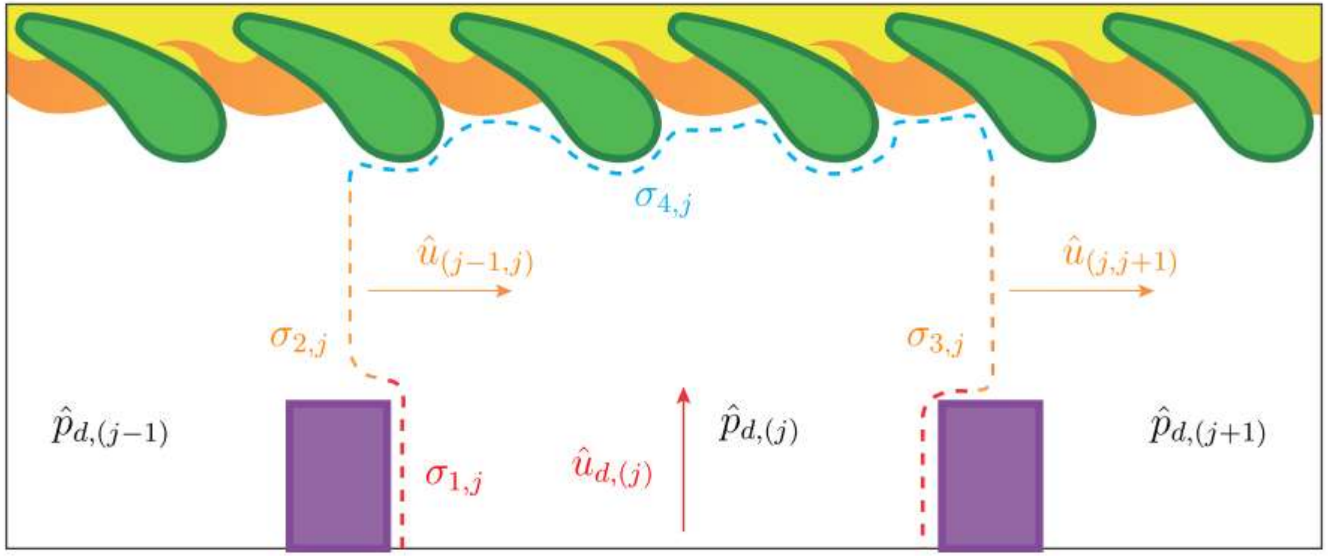
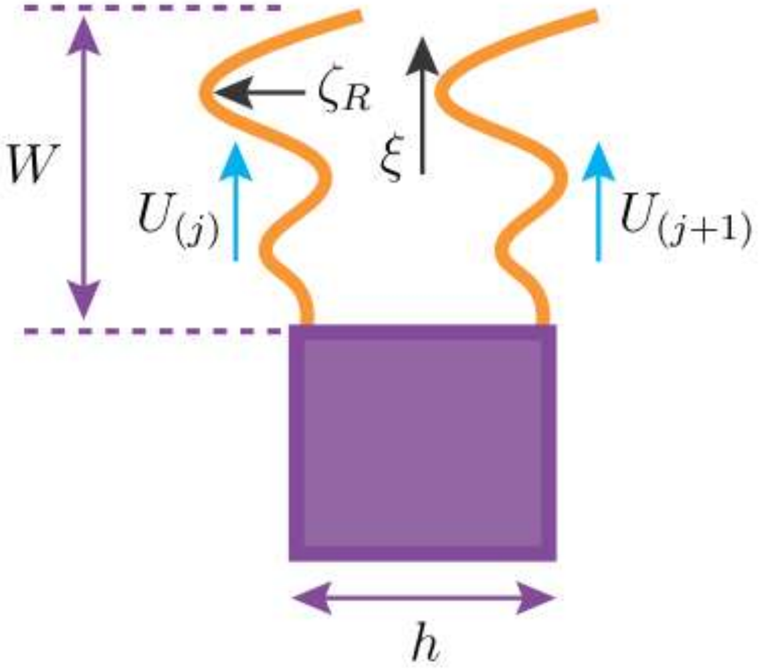
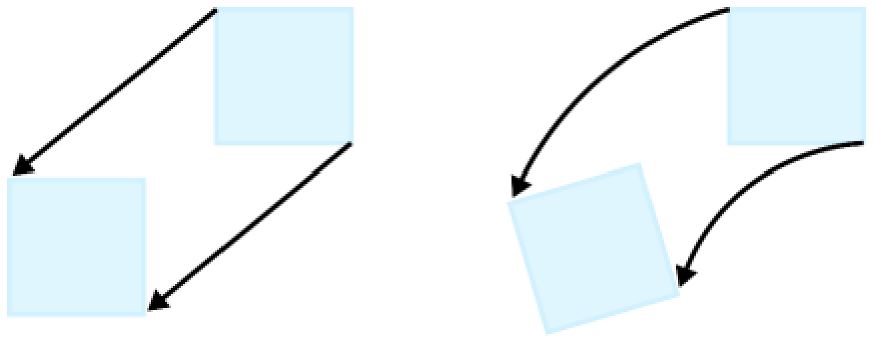


Hydrodynamic Formulation of Acoustic Coupling

- Classic model due to Howe (1997)
- Turbulent wake idealized by two vortex sheets separated by small distance h
- Vortex sheet displacements represents vorticity fluctuations in the aperture
- Vorticity fluctuations + mean flow = acoustic energy (Howe, 1980)

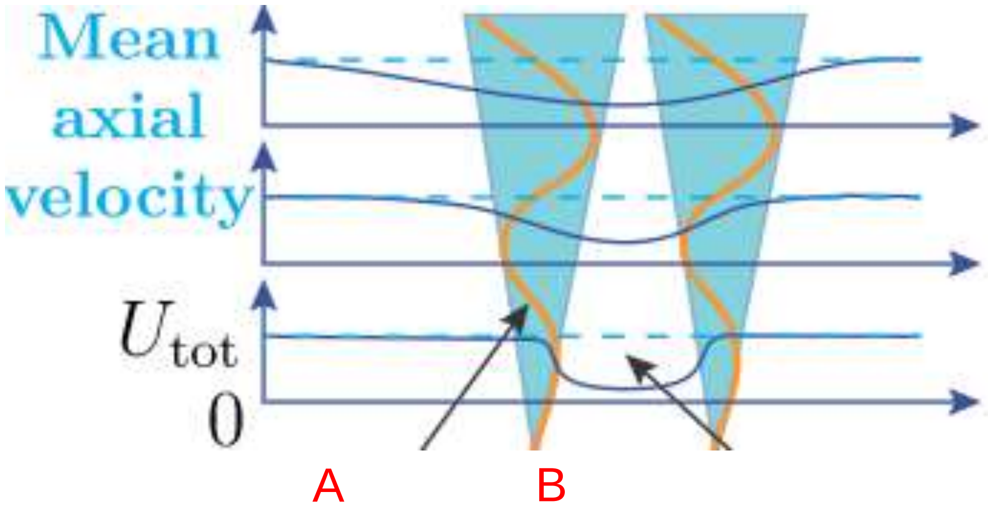


Excursion: Vorticity



Source: Wikipedia

- Fluid separated into many small parcels
- Vorticity proportional to rotation of two fluid parcels w.r.t. each other
- Different axial velocities lead to different rotations of fluid particles (no vortex)
- Parcel located at A will not rotate, but a parcel at B will, because its left side has higher axial velocity than its right side

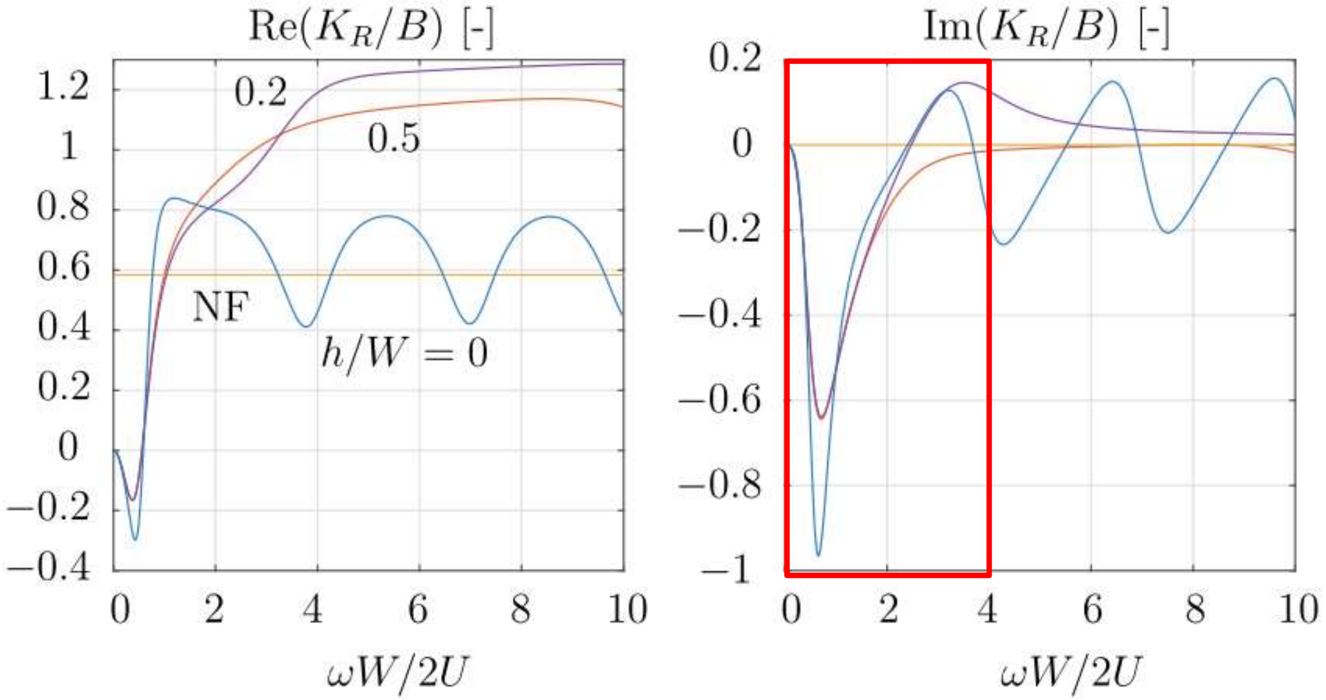


Hydrodynamic Formulation of Acoustic Coupling

- Acoustic coupling described by Rayleigh conductivity K_R, from solution of integral equation
- Positive imaginary part implies amplification of sound field by mean flow, negative imag. part implies damping
- Amplification suppressed at large enough can separation h

$$\int_{-1}^{1} \zeta'(\mu)\{\ln|\xi-\mu| + L_+(\xi,\mu)\}d\mu - \pi\Omega_c^2\left(\frac{h}{W}\right)\int_{-1}^{1}\zeta'(\mu)G(\xi,\mu)d\mu + (\lambda_+ + \lambda_-\xi)e^{\mathrm{i}\Omega_c\xi} = 1,$$

$$K_R(\omega_c) = -\pi B \int_{-1}^{1} \zeta'(\mu,\omega_c)d\mu.$$

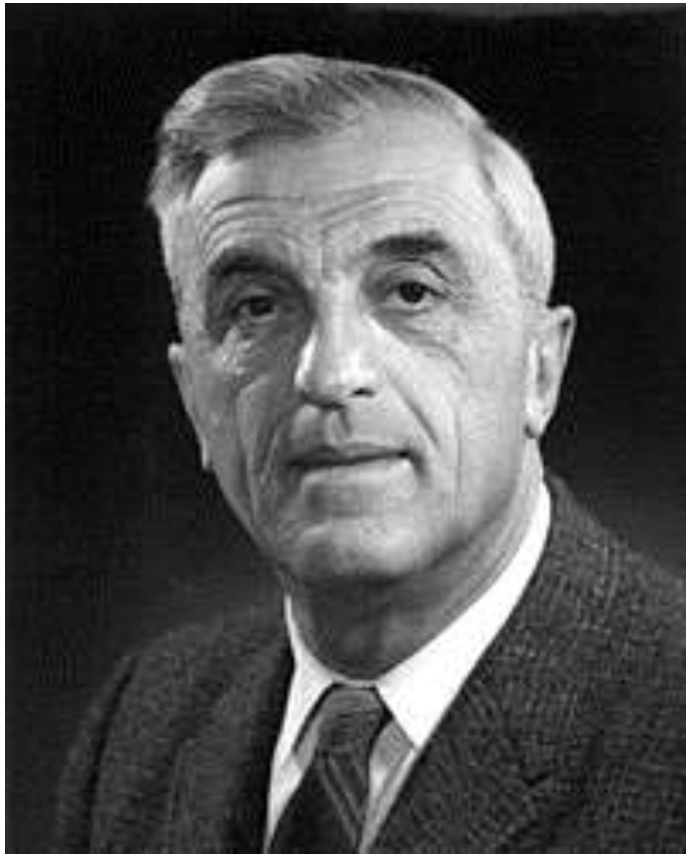


Bloch Wave Ansatz

- Felix Bloch (1929): Electrons in metals (crystals) feel periodic potential due to atoms in lattice

- Bloch wave ansatz enables simplified modelling by exploiting periodicity

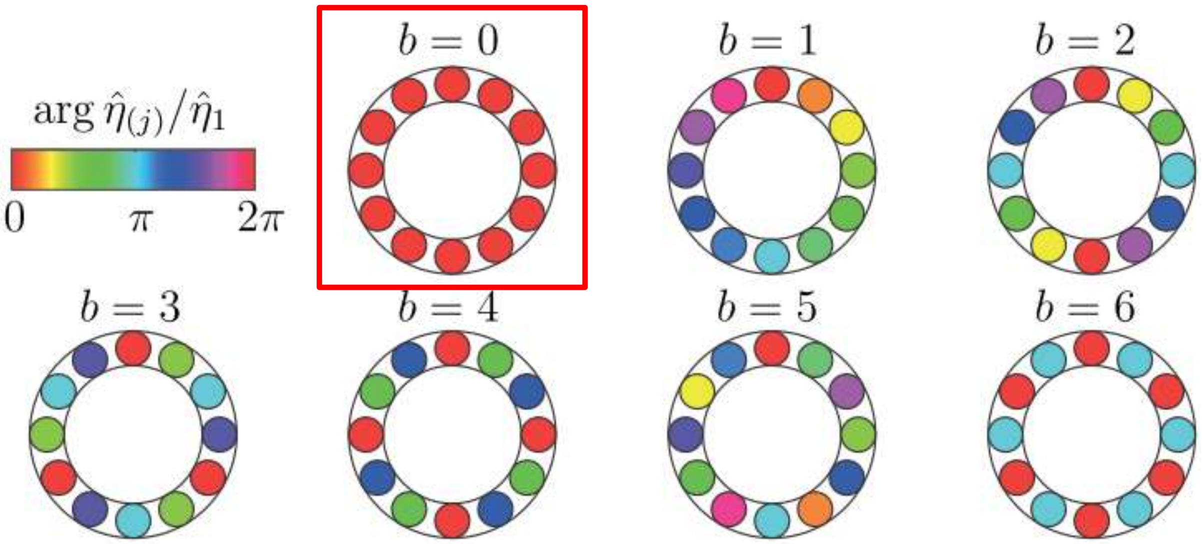
- Mensah et al. (2016): Adapted ansatz to TA oscillations in periodic/symmetric systems

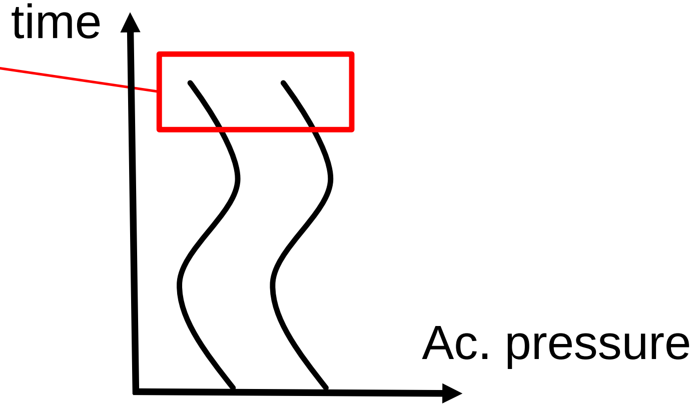


Bloch Wave Ansatz

- In present work, Bloch wave ansatz identifies mode patterns according to relative phase of the pressure signals
- If two cans are in phase, the aperture connecting them feels no pressure difference
- The more out of phase two cans, the larger the perceived pressure difference

«Bloch modes»





b: Bloch wavenumber

Bloch Wave Ansatz: What is really happening?

- Coupled identical damped harmonic oscillators

$$\ddot{x}_1 + \alpha_A \dot{x}_1 + \omega_A^2 x_1 = \kappa_A (x_2 - x_1)$$
$$\ddot{x}_2 + \alpha_A \dot{x}_2 + \omega_A^2 x_2 = \kappa_A (x_1 - x_2)$$

- Spectrum: ansatz $x_k = A_k e^{\lambda t}$ leads to

$$\begin{pmatrix} \lambda^2 + \alpha_A \lambda + \omega_A^2 + \kappa_A & -\kappa_A \\ -\kappa_A & \lambda^2 + \alpha_A \lambda + \omega_A^2 + \kappa_A \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Solution exists if determinant of matrix = 0 ➡ 4th order system

Bloch Wave Ansatz: What is really happening?

- Idea: reduce system using a symmetry ansatz, for example

$$\ddot{x}_1 + \alpha_A \dot{x}_1 + \omega_A^2 x_1 = \kappa_A (x_2 - x_1)$$
$$\ddot{x}_2 + \alpha_A \dot{x}_2 + \omega_A^2 x_2 = \kappa_A (x_1 - x_2)$$

- Define $k = k \mod 2$: minimal example of ring of oscillators
- Assume: x is an eigenfunction of the translation operator: $Tx_k = x_{k+1} = \Lambda x_k$
- Then, combining $\begin{matrix} k = k \mod 2 \\ x_{k+1} = \Lambda x_k \end{matrix}$ leads to $\Lambda = e^{ikb\pi}, b = 0, \pm 1$. Hence $\begin{matrix} x_1 = e^{ib\pi} x_2 \\ x_2 = e^{-ib\pi} x_1 \end{matrix}$

Bloch Wave Ansatz: What is really happening?

- By linearity and symmetry of system, can show that Bloch wave ansatz identifies 4 (all) solutions of the EV problem

$$\ddot{x}_1 + \alpha_A \dot{x}_1 + \omega_A^2 x_1 = \kappa_A (x_2 - x_1)$$
$$\ddot{x}_2 + \alpha_A \dot{x}_2 + \omega_A^2 x_2 = \kappa_A (x_1 - x_2)$$

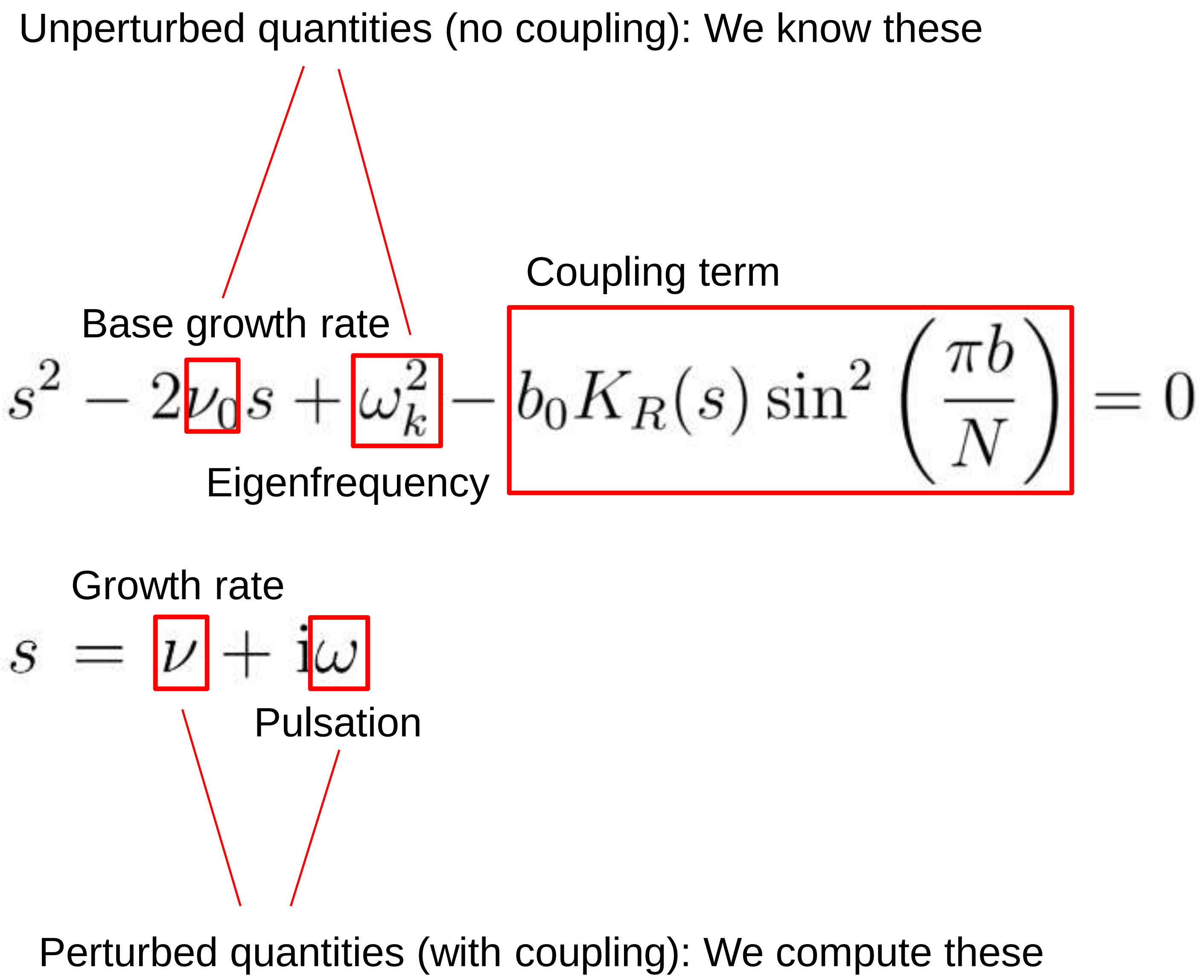
- Substitute $A_2 = e^{-ib\pi} A_1$ into $\begin{pmatrix} \lambda^2 + \alpha_A \lambda + \omega_A^2 + \kappa_A & -\kappa_A \\ -\kappa_A & \lambda^2 + \alpha_A \lambda + \omega_A^2 + \kappa_A \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to get

$$\lambda_\pm(b) = -\alpha_A / 2 \pm i \sqrt{\omega_A^2 + \kappa_A (1 - e^{ib\pi}) - (\alpha_A / 2)^2}$$
$$b = 0, \pm 1$$

- This method is used to reduce our model of N equations for the frequency spectrum of the system to a single equation

Results

- Damping of unstable modes
- Parameter study
- Coupling-induced instability
- Physical interpretation



Unperturbed quantities (no coupling): We know these

Base growth rate

Coupling term

$$s^2 - 2\boxed{\nu_0}s + \boxed{\omega_k^2} - \boxed{b_0 K_R(s) \sin^2\left(\frac{\pi b}{N}\right)} = 0$$

Eigenfrequency

Growth rate

$$s = \boxed{\nu} + \mathrm{i}\boxed{\omega}$$

Pulsation

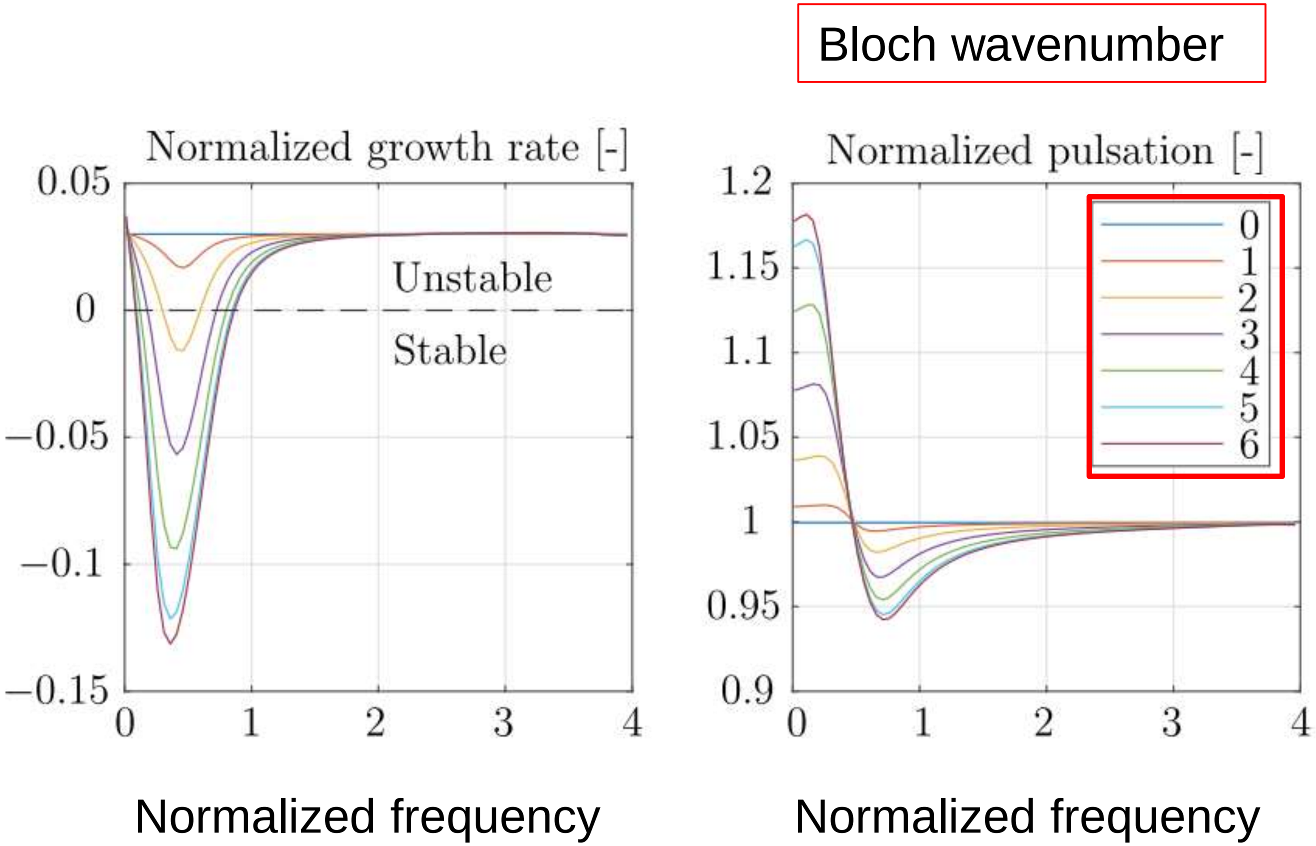
Perturbed quantities (with coupling): We compute these

Damping of Unstable Modes

- This condition is nominally unstable due to the acoustic-flame interaction

- Dashed line: stability border

- Acoustic coupling acts damping on higher-order Bloch modes in certain frequency range

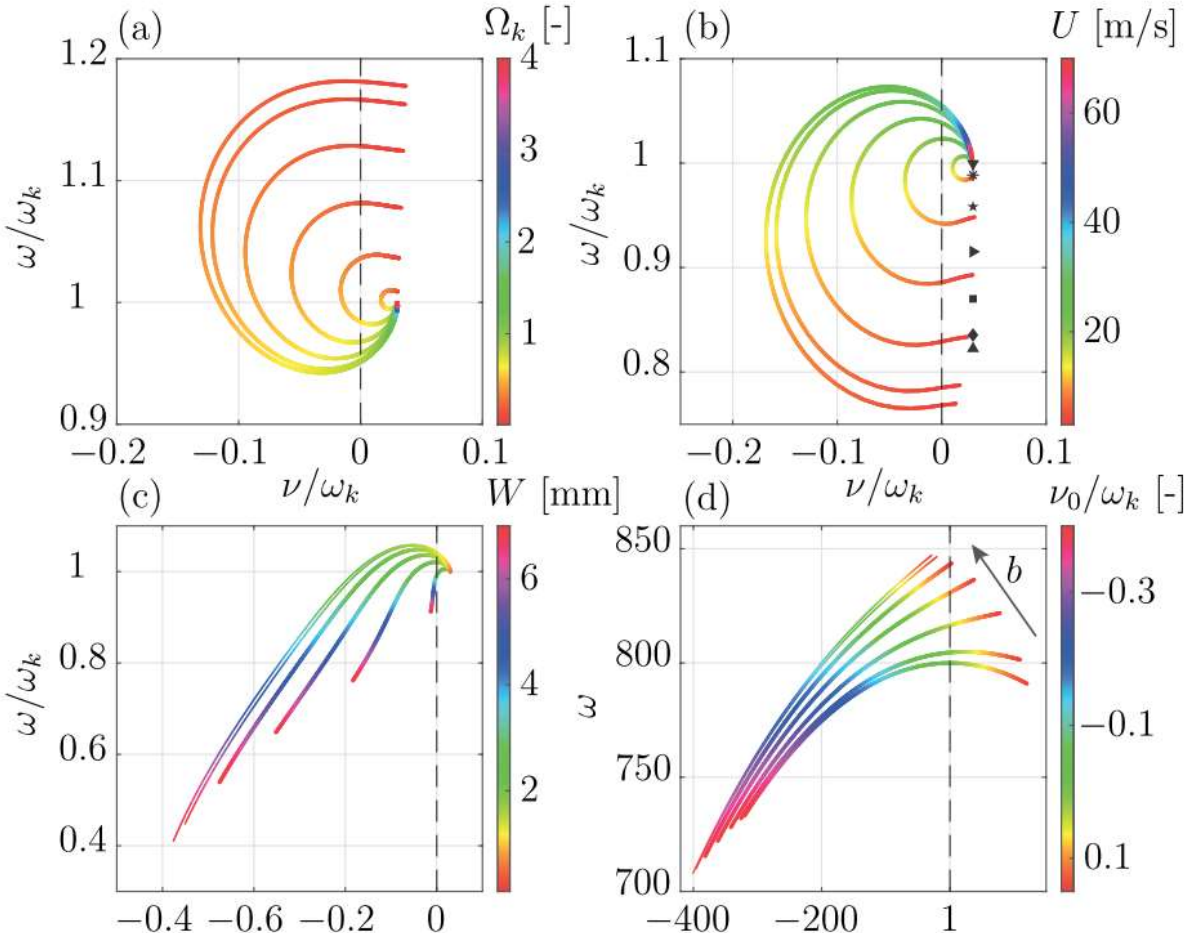


- In this condition, the cans act “normal”, the apertures act like acoustic dampers

Damping of Unstable Modes

Parameter Study

- (a) Non-monotonous dependence on frequency
- (b) Non-monotonous dependence on bulk flow speed (at which combustion products are injected)
- (c), (d) Monotonous dependence on aperture width W and base growth rate

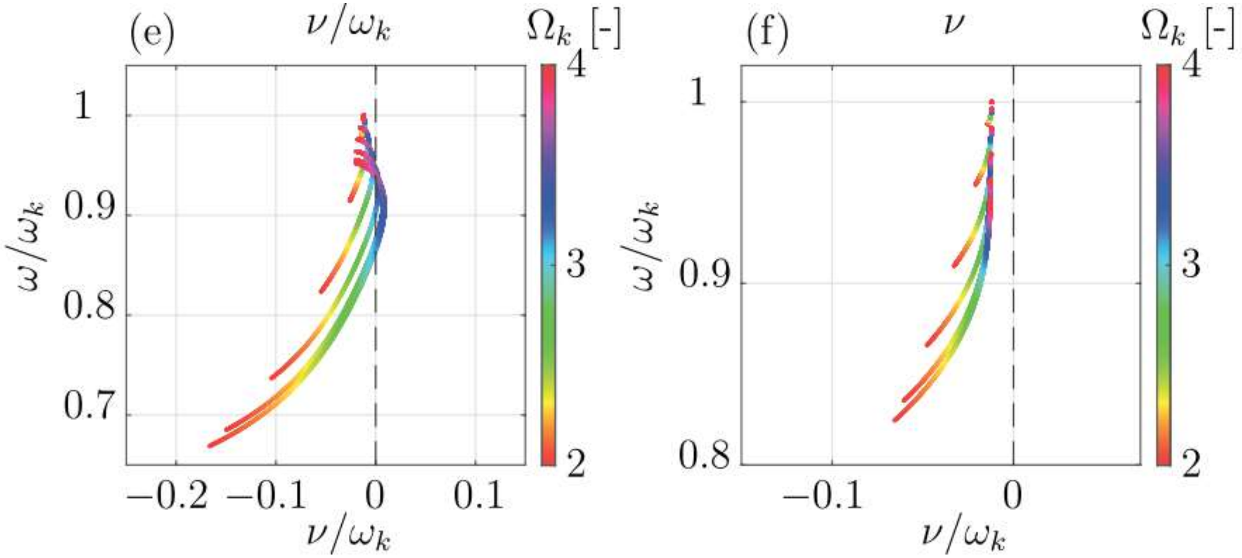


Norm. growth rate

Growth rate [1/s]

Coupling-Induced Instability

- This condition is nominally stable due to the acoustic-flame interaction
- (e) Coupling-induced instability for small can spacing h
- (f) This instability is suppressed if the can spacing h is increased
- In this condition, the cans act like whistles



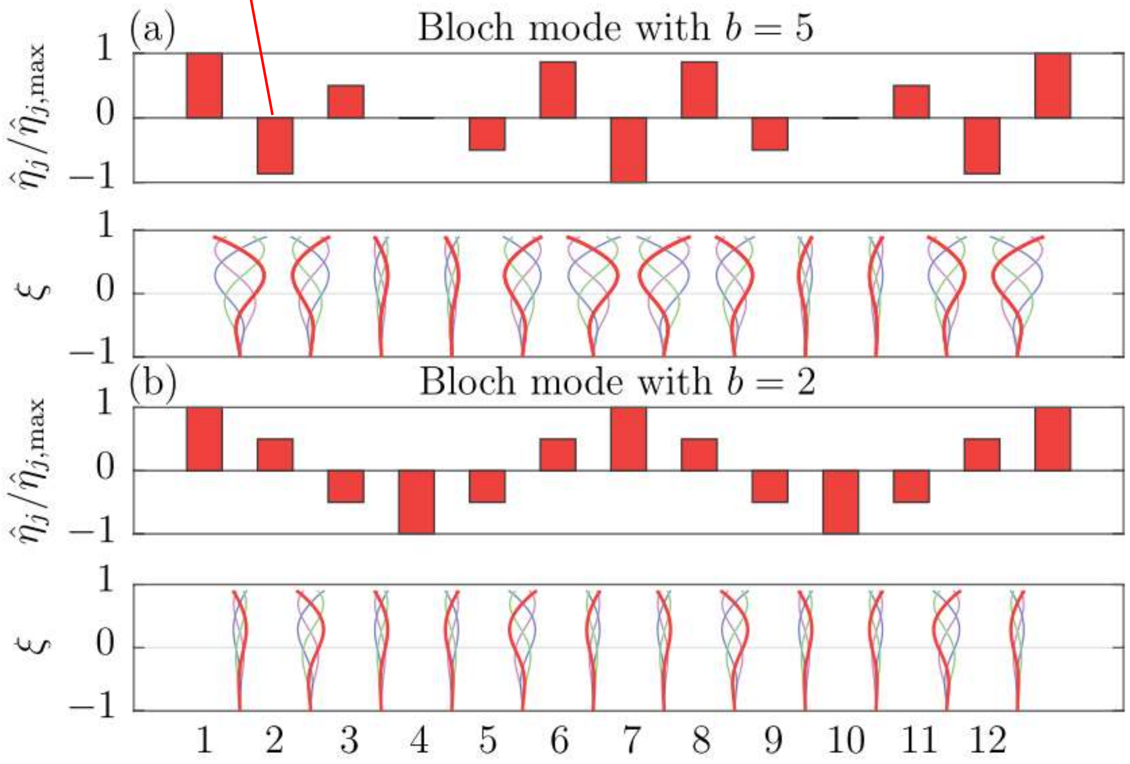
Norm. growth rate

Norm. growth rate

Physical Interpretation

- (a) unstable condition, (b) stable condition (same if coupling is removed)
- 2nd and 4th row: Vortex sheet displacement between cans over 1 acoustic cycle
- Larger pressure differences (higher b) increase potential for damping or amplification of TA oscillations (CI)

Normalized acoustic pressure



- Even in nominally stable conditions, coupling can lead to instability

Summary

- Relevance of GT technology with regard to the energy challenge
- Importance of CI (TA oscillations) for GT development
- Value of LOM for academic research into novel combustor concepts and control strategies for CI
- Novel low-order model of TA stability in can-annular combustor architecture

Acknowledgements

- Images taken from lecture “Engineering Acoustics 2” given by Prof. Noiray et al.
- Funding: Swiss National Science Foundation, Grant agreement 184617

Thank you for your attention!