Measuring the gravitational acceleration

Goal: Determination of the gravitational acceleration *g* by a free-fall experiment.

Equipment:
- Smartphone or meter stick or calibrated tape for height measurement
- Spherically shaped object (e.g. made of steel or stone)

Method:

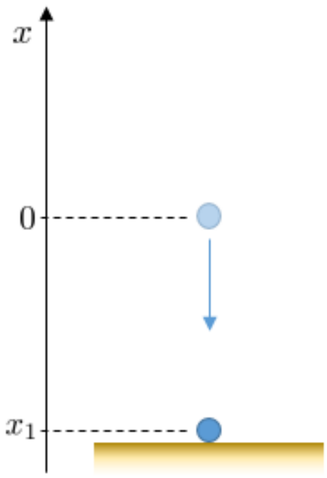

Fig.1 Reference system

In a free-fall experiment with a constant acceleration $-g$ the equation of motion is given by

$$\frac{d^2x(t)}{dt^2} = -g \tag{1}$$

where $x(t)$ represent the vertical position at time t according to a vertical axis oriented in upwards direction (see Fig.1). By integrating the equation of motion twice, one can obtain the displacement as

$$\int_0^t \left[\int_0^t \frac{d^2x}{dt^2}dt\right] = \int_0^t \left[\int_0^t -gdt\right] \longrightarrow x(t) = -g\frac{t^2}{2} + v_0 t + x_0 \tag{2}$$

where $v_0 = v(0)$ and $x_0 = x(0)$ are the velocity and position at time zero, respectively. Choosing the reference system as shown in Fig.1, we can set $x_0 = 0$. Also, for a free-fall experiment we can assume $v_0 = 0$. Therefore, from equation (2) one obtains the gravitational acceleration as

$$g = -\frac{2x_1}{t_1^2} \tag{3}$$

where t_1 is the time needed for the mass to arrive to the position $x_1 = x(t_1)$ whose value is negative, given the reference system in Fig. 1.

Procedure:

1. Perform a low-height drop experiment to record the time of the free fall. Measure a *2m* distance from the floor and make a mark at your wall. From the same point, drop the sphere and record the time required to touch the ground. Repeat the experiment until you collect time results from ten different measurements.
2. Next, perform the same experiment at a higher position which is not exposed to high wind (e.g. a stairway). Make sure that nobody is close to the free fall area during the drop experiments. You can measure the height of the stairway through your smartphone apps (e.g. *Smart Distance*, *EasyMeasure*). Conduct the free-fall experiment ten times.

Analysis and discussion

1. Make a table to present the different values of $x(t)$, t, *and* g for both experiments. Which out of the two experiments gives more accurate values in comparison to $g = 9{,}81 m/s^2$?
2. Compare the variation of the measured free-fall time t and acceleration due to gravity g for both the low- and high-altitude drop experiment. Which experiment is more precise (lower variance)?
3. What effects could be responsible for the differences between the obtained values?
4. What could be a better set up to determine the gravity constant?