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Oxidation dynamics of soot or carbon black accounting for its core-shell structure and pore network

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ABSTRACT

Internal oxidation is most important for removal of soot in diesel particulate filters. Furthermore, it enhances the porosity of carbon blacks (CBs), increases their specific surface area, *SSA*, determining their performance in batteries, inks and tires, as well as their environmental and health impact. Current models for oxidation of soot or CB neglect its fractal-like pore network during internal oxidation and underestimate its *SSA* by 60 % on average! Here, a lattice Monte Carlo (LMC) model elucidates both external and internal oxidation dynamics of soot (or CB) accounting for its fractal-like pore structure, for the first time to the best of our knowledge. Random internal oxidation takes place at the bulk soot (or CB), expanding its pore network and increasing the pore fractal dimension, *Dfp*, up to 3, in good agreement with data. This increases the *SSA* up to a factor of ten at 75 % conversion. In the presence of a less reactive shell though, internal oxidation stops after the reactive core is consumed. Thus the *SSA* levels off at large conversions (*>*50 %). Hence, accounting for the realistic core-shell, fractal-like pore structure during simultaneous external and internal oxidation nicely explains several measurements of soot and CB *Dfp*, *SSA* and particle diameter. So the LMC-derived oxidation dynamics of soot or CB presented here can be used to assist the design of highly porous CB grades from first principles, as well as the mitigation of soot emissions by enhancing their oxidation.

1. Introduction

Oxidation facilitates the elimination of soot emissions to minimize their environmental and health footprint. For example, soot nanoparticles are partially oxidized in regenerative traps and particulate filters at the engine exhaust during diesel [[1](#page-6-0)] and jet fuel [[2](#page-6-0)] combustion. This increases their porosity and specific surface area, *SSA* [[1](#page-6-0)], and thus their toxicity [\[3\]](#page-6-0) and propensity to act as cloud condensation nuclei [[4](#page-6-0)]. Furthermore, internal oxidation enhances the porosity of carbon blacks and determines their performance in various applications (e.g. tires, inks, paints). Low temperature oxidation has been used to optimize the porosity of carbon blacks [[5](#page-6-0)] and candle soot [\[6\]](#page-7-0) and improve their performance in electrochemical storage. So, a quantitative understanding of the carbonaceous nanoparticle (CNP) porosity dynamics during oxidation is essential for the process design of highly porous carbon black grades [[7](#page-7-0)], as well as for the mitigation of the impact of soot on public health and climate [[8](#page-7-0)].

The internal porosity of CNPs depends on the combustion conditions during the CNP formation and growth [[9](#page-7-0)]. For example, nascent soot formed at low temperatures and short residence times is rather amorphous and contains mostly single and randomly arranged PAHs [[10\]](#page-7-0). In contrast, mature soot [[10\]](#page-7-0) and carbon black [[11\]](#page-7-0) particles formed at high temperatures and long residence times consist of a graphitic, less reactive porous shell of stacked and concentrically-oriented PAHs [\[12](#page-7-0)] that encloses an amorphous, reactive core [\[13](#page-7-0)]. The average PAH length of nascent and mature soot or carbon black varies with fuel type (e.g. benzene vs. ethanol [\[14](#page-7-0)]). Both nascent and mature CNPs contain micropores having width that ranges from 0.2 $[15]$ $[15]$ to 0.9 nm $[16]$ $[16]$ between each PAH stack. Small angle X-ray scattering $[17]$ $[17]$, Ar, CO₂ $[18]$ $[18]$ and N₂ adsorption [[19\]](#page-7-0) revealed that the pores of soot and carbon blacks form fractal-like networks having a fractal dimension, D_{fp} , of about 2–2.5. In the absence of oxidation, the total pore surface area of CNPs is negligible compared to their external particle surface area [[20\]](#page-7-0). So, the specific surface area, *SSA*, of unoxidized CNPs ranges from 50 m^2/g for diesel

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Table 1

Initial $D_{fp,o}$ of pore networks created here by DLA for unoxidized CNPs, along with those obtained in literature using DLA [[44,45\]](#page-7-0) and experimental data from acetylene black [[18\]](#page-7-0), Printex 95 [\[19](#page-7-0)], NIST and NEU soot [\[17](#page-7-0)].

Fig. 1. Representative carbonaceous nanoparticles (CNPs) containing a single straight pore (a), or a fractal-like pore network without (b) and with (c) a surrounding shell (red-shaded). The dark blue cubic cells represent the pores. The light blue cells in (b) and (c) represent reactive solid sites, while the red-shaded ones in (c) represent non-reactive solid sites of the particle shell.

soot [[20\]](#page-7-0) to 300 m²/g for fine carbon black grades [\[21](#page-7-0)].

The evolution of CNP porosity during oxidation at various temperatures, *T*, has been elucidated experimentally for NEU, NIST [[17\]](#page-7-0), diesel soot [[20\]](#page-7-0) and carbon blacks (e.g. Regal 600 [\[22](#page-7-0)] and Printex 95 [\[19](#page-7-0)]). That way, it was shown that O_2 reacts at the external surface of carbon black particles at 800 ◦C, reducing their diameter without affecting their internal structure [\[19](#page-7-0)]. So, external oxidation enhances the CNP *SSA* only up to 50 % at a 75 % conversion [\[19](#page-7-0)]. At low temperatures (e.g. 500–600 °C), O_2 diffuses and reacts within the bulk CNP [[23\]](#page-7-0). Internal oxidation compacts the pore network of soot [[17\]](#page-7-0) and carbon black [[19\]](#page-7-0), increasing its D_f up to 2.9. This results in hollow carbon black [[19\]](#page-7-0), diesel [\[20](#page-7-0)] and biodiesel [[24\]](#page-7-0) soot particles and increases their *SSA* up to a factor of four at 50–75 % conversion [[20\]](#page-7-0). In this regard, the detailed experimental characterization of the CNP porosity obtained at various *T* can enable the derivation of robust models for the external and internal oxidation of CNPs.

External oxidation of CNPs is typically simulated using moment [\[25](#page-7-0)], fixed [\[26](#page-7-0)] and moving monodisperse [\[27\]](#page-7-0) or polydisperse [\[23](#page-7-0)] sectional models by reducing uniformly the primary particle diameter, *dp*, following closely the analytical shrinking particle model [\[28](#page-7-0)]. Fixed [[29\]](#page-7-0) and moving [\[23](#page-7-0)] sectional models have been used also to derive the internal oxidation dynamics of CNPs containing cylindrical pores. These pores grow uniformly by internal oxidation at low temperatures [\[23](#page-7-0)]. Despite this simplification, a moving sectional model for external and internal oxidation [[23\]](#page-7-0) explained mobility size distribution data from nascent [\[30](#page-7-0)] and mature soot [[31\]](#page-7-0) oxidized at high and low *T*. However, the *SSA* derived by this model is up to 60 % smaller than that measured at 60 % conversion [\[22](#page-7-0)]. So, accounting for the realistic fractal-like pore network of CNPs seems to be essential to describe their porosity and *SSA* dynamics.

In this regard, lattice Monte Carlo (LMC) models [[32\]](#page-7-0) have been used to elucidate the uniform [\[33](#page-7-0)] or random [\[34](#page-7-0)] internal oxidation of single [[35\]](#page-7-0) and multi-component [\[36](#page-7-0)] coal particles, as well as the reduction of iron oxide particles [\[37](#page-7-0)] accounting for their fractal-like pore network [[38\]](#page-7-0). Here, a LMC model is developed to simulate the concurrent external and internal oxidation of CNPs, for the first time to the best of our knowledge. Uniform and random internal oxidation of CNPs

containing a single cylindrical pore are firstly derived by LMC and compared to the shrinking particle model for external oxidation [\[28](#page-7-0)]. Then, the simultaneous external and internal oxidation of CNPs containing fractal-like pore networks is elucidated at high and low temperatures, respectively. The LMC-derived evolution of D_{fn} and *SSA* is validated over a wide range of experimental data from NEU, NIST [\[17](#page-7-0)], diesel soot [\[20](#page-7-0)] and commercial carbon blacks [[19,22\]](#page-7-0). The impact of the CNP core-shell structure on the oxidation dynamics is also quantified.

2. Methods

2.1. Internal structure of CNPs

Spherical CNPs with diameter, $d_p = 18$ nm are discretized into a cubic lattice of 113[']100 cells. The CNP d_p used here is on par with the carbon black [\[19\]](#page-7-0) and soot [\[31](#page-7-0)] d_p used often in oxidation experiments. Each cell *i* can be part of the CNP solid or pore networks having a solid mass fraction, $y_i = 1$ or 0, respectively. The cell size is set to 0.3 nm that is on par with the smallest pore sizes of 0.2 and 0.36 nm used in sectional models [\[23](#page-7-0)] and molecular dynamics [[39\]](#page-7-0), respectively. The oxidation dynamics of CNPs having a single straight pore of 18 nm length and 0.6 nm width are simulated. The initial fractal dimension, $D_{fp,0} = 1$ derived here for this pore using the Sandbox algorithm [[40\]](#page-7-0) is in excellent agreement with the theoretical $D_{fp} = 1$ for linear structures [[41\]](#page-7-0).

CNPs containing fractal-like pore networks are also created here using a diffusion limited aggregation (DLA) algorithm [[42\]](#page-7-0). DLA results in percolated pore networks, similar to those of large coal [32–[36\]](#page-7-0) and iron oxide [\[37](#page-7-0)] particles obtained from percolation theory [[43\]](#page-7-0). In brief, a single pore cell is placed at the CNP center. Then, new pore cells are generated next to existing ones based on diffusion-limited random walks [[42\]](#page-7-0), resulting in a CNP porosity of 2 % [[23\]](#page-7-0) and matching that way the SSA_o of 240.3 m²/g measured for unoxidized carbon black [\[19](#page-7-0)]. The D_{fp} , σ = 2.21 \pm 0.08 of the fractal-like pore networks derived that way (Table 1) is in excellent agreement with that obtained by Eggersdorfer & Pratsinis [[44\]](#page-7-0) for DLA-derived structures with similar size, but slightly smaller than that of Tolman & Meakin [\[45](#page-7-0)] obtained for larger fractal-like networks. This can be attributed to finite size effects [\[46](#page-7-0)]. The *Dfp,o* derived here is well within the range of the *Dfp,o* measured for unoxidized carbon blacks $(2-2.42)$ $[18,19]$ $[18,19]$ and soot $(2.19-2.37)$ $[17]$ $[17]$. That way realistic initial CNP pore structures are used here before oxidation starts. The $D_{fp,o}$ derived here is also on par with the $D_{fp,o} = 2.53$ obtained for large coal [\[32](#page-7-0)–36] and iron oxide [[37\]](#page-7-0) particles from percolation theory [[43\]](#page-7-0).

CNPs having a core-shell structure are also generated to investigate

Fig. 2. LMC-derived evolution of m (a), *SSA* (b), χ (c: left ordinate) and d_p (c: right ordinate) as a function of *t* during simultaneous external and uniform (solid lines, orange insets) or random (broken lines, blue insets) internal oxidation of CNPs containing a single straight pore at 550 ◦C.

the limited shell reactivity of mature soot $[47]$ $[47]$ and carbon black $[48]$ $[48]$ nanoparticles. The CNP shell reacts much more slowly with O_2 due to its large degree of graphitization $[12]$ $[12]$. The diameter of the reactive core, d_c , is varied from $0.5d_p$ to $0.75d_p$. This range is consistent with the $d_c =$ 0.4*dp* - 0.8*dp* obtained from microscopy images of mature soot from diesel engines [[12\]](#page-7-0) and diffusion flames [\[49](#page-7-0)]. CNPs with $d_c/d_p = 0$ are also simulated here to account for graphitic CNPs (e.g. Fischer-Tropsch soot [[24\]](#page-7-0)) that undergo mostly external oxidation even at low *T*. [Fig. 1](#page-1-0) summarizes the pore structures used in this work.

2.2. Lattice Monte Carlo simulation of CNP oxidation

Lattice Monte Carlo (LMC) is a computational algorithm that relies on random sampling to simulate stochastic processes (e.g. diffusion and reaction of gases in porous materials) on a cubic lattice [[32\]](#page-7-0). Here, LMC is used to elucidate the oxidation dynamics of CNPs at 550 or 800 ◦C and 10 vol % of O2 [[19\]](#page-7-0). The evolution of CNP mass, *m*, as a function of time, *t,* during external and internal oxidation is [[23\]](#page-7-0):

$$
\frac{dm}{dt} = -\omega \left(A_{ext} + A_{int} \right) \tag{1}
$$

where $\omega = 9.5 \cdot 10^{-8}$ and 1.2 $\cdot 10^{-5}$ kg/(m²s) are the oxidation rates of Nagle & Strickland-Constable (NSC) at 550 and 800 ◦C, respectively [[50\]](#page-7-0), while *Aext* and *Aint* are the external and internal areas, respectively. So, ω is the same for both external and internal oxidation, consistent with "overall" rates derived for external and internal oxidation of diesel [[51\]](#page-7-0) and flame [[52\]](#page-7-0) soot.

At $T = 800$ °C, internal oxidation is negligible [[19](#page-7-0)] and integrating Eq. (1) yields:

$$
\Delta m_{ext} = - \omega A_{ext} \Delta t \tag{2}
$$

where *Δmext* is the mass removed by external oxidation after time Δ*t*. The CNP *m* at *t* is:

$$
m(t) = \rho \frac{\pi}{6} d_p(t)^3
$$
\n(3)

where $\rho = 1.8$ g/cm³ is the CNP bulk density [\[53](#page-7-0)]. The d_p at $t + \Delta t$ can be obtained by combining Eqs. (2) and (3):

$$
d_p(t + \Delta t)^3 = d_p(t)^3 - \frac{6\omega A_{ex}\Delta t}{\pi \rho}
$$
\n(4)

So, solid cells with center of mass larger than $d_p(t + \Delta t)$ are converted to CO2 by external oxidation and their *yi* is set to zero.

At $T = 550$ °C, the Thiele effectiveness factor is about 1 [[23\]](#page-7-0) and O₂ molecules can diffuse throughout the small CNPs investigated here [\[15](#page-7-0)]. The total mass, Δm_{inb} , removed by internal oxidation at Δt is obtained by integrating Eq. (1):

$$
\Delta m_{int} = - \omega A_{int} \Delta t \tag{5}
$$

Based on the uniform internal oxidation model [[33\]](#page-7-0), all internal solid cells adjacent to pores are oxidized uniformly resulting in:

$$
\Delta m_{c,i} = -\omega A_{c,i} \Delta t \tag{6}
$$

where $\Delta m_{c,i} = m_{c,i}(t + \Delta t) - m_{c,i}(t)$ is the mass of cell *i* removed by internal oxidation and $A_{c,i}$ is its external area. At small t , uniform internal oxidation is incomplete and solid cells are only partly converted to $CO₂$. So, *yi* is given by:

$$
y_i(t) = \frac{m_{c,i}(t)}{m_{c,o}}
$$
 (7)

where $m_{c,o}$ is the initial cell mass before any internal oxidation takes place. The cell y_i at $t + \Delta t$ is obtained by combining Eqs. (6) and (7):

Fig. 3. LMC-derived D_{fp} (a) and normalized *SSA*/*SSA*_o (b) as a function of χ during concurrent external and uniform (solid lines) or random (broken lines) internal oxidation of CNPs containing a single straight pore, along with the shrinking particle model (dotted line [\[28](#page-7-0)]) and experimental data from Printex 95 (triangles [[19\]](#page-7-0)) and Regal 600 (squares [\[22](#page-7-0)]) carbon blacks, as well as NIST (inverse triangles [\[17](#page-7-0)]), NEU (circles [[17\]](#page-7-0)) and diesel (diamonds [[20\]](#page-7-0)) soot.and have D_p > 1 [17-[19\]](#page-7-0).

Fig. 4. Cross-sections of CNPs containing fractal-like pore networks with initial $D_p = 2.21 \pm 0.08$ [\(Table 1\)](#page-1-0) derived by LMC during solely external (a–d) and concurrent external and random internal (e–h) oxidation at 800 (a–d) or 550 ◦C (e–h) and *χ* = 0, 25, 50 and 75 %.

Fig. 5. LMC-derived *SSA*/*SSAo* as a function of *χ* during solely external (a) and concurrent external & random internal oxidation (b) at 800 (a: dot-broken line) or 550 ℃ (b: broken line) of CNPs containing fractal-like pore networks, along with the shrinking particle model (dotted lines [\[28](#page-7-0)]) and experimental data from Fig. 3.

$$
y_i(t + \Delta t) = y_i(t) - \frac{\omega A_{c,i} \Delta t}{m_{c,o}} \tag{8}
$$

When internal oxidation is completed and $y_i = 0$, the solid cell is converted to a pore $[33]$ $[33]$. The CO₂ generated by internal oxidation is removed rapidly from the bulk CNP [\[33](#page-7-0)]. This is a common assumption in LMC models of coal combustion [[33,34](#page-7-0)].

A random internal oxidation model [[34\]](#page-7-0) is also explored here to account for the random walk of O_2 in the bulk CNP [\[15](#page-7-0)]. So, internal solid cells adjacent to pores are selected using a random number generator [[54\]](#page-7-0) and oxidized completely [\[34](#page-7-0)]. The total number of cells, *nc*, oxidized that way is given by:

$$
n_c = \frac{\Delta m_{int}}{m_{c,o}}\tag{9}
$$

where *Δmint* is given by Eq. [\(5\)](#page-2-0).

3. Results & discussion

3.1. Oxidation dynamics of CNPs containing a single straight pore

[Fig. 2](#page-2-0) shows the CNP mass, *m* (a), specific surface area, *SSA* (b), conversion, χ (c: left ordinate), and diameter, d_p (c: right ordinate), as a function of time, *t*, during simultaneous external and uniform (solid lines, orange insets) or random (broken lines, blue insets) internal oxidation of CNPs containing a single straight pore derived by lattice Monte Carlo (LMC) at 550 ◦C. The shaded area represents the statistical variation of 50 LMC simulations for external and random internal oxidation. Uniform internal oxidation results in uniform oxidation of all solid sites adjacent to pores [[33\]](#page-7-0). Random oxidation results in random (stochastic) removal of solid sites adjacent to pores [[34\]](#page-7-0). During concurrent external and uniform internal oxidation, the pore width increases uniformly as a function of *t*. This results in a linear increase of the CNP *SSA* and reduction of its *m*. This is consistent with moving sectional modeling of concurrent external and uniform internal oxidation of soot containing cylindrical pores [[23\]](#page-7-0). In contrast, random internal oxidation results in a fractal-like pore network, as nicely shown by the insets in [Fig. 2](#page-2-0)b. This drastically increases the porosity of CNP, enhancing its *SSA* and reducing its m ([Fig. 2a](#page-2-0)). The evolution of d_p ([Fig. 2c](#page-2-0)) is determined only by external oxidation and is not affected by internal oxidation.

[Fig. 3](#page-3-0) shows the LMC-derived pore fractal dimension, D_{fp} (a), and normalized *SSA* by the initial SSA_0 (SSA/SSA_0) (b) as a function of χ during simultaneous external and uniform (solid lines, orange insets) or random (broken lines, blue inset) internal oxidation of CNPs containing initially a single straight pore, along with the shrinking particle model (dotted line [\[28](#page-7-0)]) and experimental data from Printex 95 (triangles [[19\]](#page-7-0)) and Regal 600 (squares [[22\]](#page-7-0)) carbon blacks, as well as NIST (inverse triangles [[17\]](#page-7-0)), NEU (circles [[17\]](#page-7-0)) and diesel (diamonds [\[20](#page-7-0)]) soot. The shaded areas represent the statistical variation of the LMC simulations. The soot (inverse triangles, circles [[17\]](#page-7-0), diamonds [\[20](#page-7-0)]) and carbon black (triangles [\[19\]](#page-7-0), squares [\[22](#page-7-0)]) data have been obtained

at 500 and 550 \degree C, respectively. Their *SSA*₀ has been measured by N₂ adsorption [\[19](#page-7-0)] or microscopy [\[20,22\]](#page-7-0), while their *SSA* evolution has been obtained by N_2 adsorption. Uniform internal oxidation does not alter the pore morphology. So, $D_{fp} = 1$ at all χ [\(Fig. 3](#page-3-0)a), while *SSA* doubles by $\chi = 80$ % [\(Fig. 3](#page-3-0)b). Thus, accounting for concurrent external and uniform internal oxidation of a single straight pore results in up to 50 % larger *SSA* than that obtained by the shrinking particle model for external oxidation only [\[28](#page-7-0)] (dotted line, [Fig. 3b](#page-3-0)). However, this uniform oxidation model vastly underestimates (up to 60 %) the measured *SSA* of CNPs. Random internal oxidation results in fractal-like pore networks with large D_{fp} of about 3 at $\chi = 80$ %. This increases rapidly the *SSA*, explaining some of the large *SSA* measured at *χ >* 50 %. However, this model for simultaneous external and random internal oxidation of CNPs with a single straight pore underestimates up to 50 % the *SSA* measured at χ < 50 %. This can be attributed to the compact pore networks that make up unoxidized CNPs.

3.2. Oxidation dynamics of CNPs containing fractal-like pore networks

[Fig. 4](#page-3-0) shows cross-sections of CNPs containing fractal-like pore networks with initial $D_{fp} = 2.21 \pm 0.08$ ([Table 1\)](#page-1-0) derived by LMC during solely external (a-d) and concurrent external and random internal (e-h) oxidation at 800 (a-d) or 550 °C (e-h) and conversion $\chi = 0$, 25, 50 and 75 %. At 800 ◦C (a-d), oxidation takes place largely at the external surface of CNPs [\[19](#page-7-0)], reducing their diameter without much affecting their internal porosity. The internal surface area of CNPs is larger than their external one. So, at 550 ◦C (e-h), internal oxidation dominates increasing the CNP porosity with increasing χ , while limited external oxidation reduces only slightly the size of the CNP.

[Fig. 5](#page-3-0) shows the LMC-derived evolution of *SSA*/*SSAo* as a function of *χ* during solely external (a) and simultaneous external and random internal oxidation (b) at 800 (a: dot-broken line) or 550 ◦C (b: broken line) of CNPs containing fractal-like pore networks, along with the shrinking particle model (dotted lines [[28\]](#page-7-0)) and experimental data from [Fig. 3](#page-3-0). The shaded area represents the statistical variation of the LMC simulations obtained from 50 fractal-like pore networks. In the absence of internal oxidation (a), the LMC-derived *SSA* increases by up to 50 % by

Fig. 6. Cross-sections of core-shell CNPs with $d_c = 0.5$ (a–d) or 0.75 d_p (e–h) derived by LMC for concurrent external and random internal oxidation at 550 °C and χ 0, 25, 50 and 75 %, along with microscopy images of Printex 95 [\[19](#page-7-0)] oxidized at the same conditions (i–l).

external oxidation at 800 ◦C, in excellent agreement with the shrinking particle model and the data obtained at the same conditions. This further validates the present LMC simulations.

Concurrent external and random internal oxidation of CNPs containing fractal-like pore networks at 550 ◦C (b) increases the *SSA* by up to an order of magnitude at *χ* = 70 %. The *SSA* evolution derived by LMC at these conditions is in good agreement with data at χ < 50 %, but overestimates the measured *SSA* by 130 % on average at $\chi = 50{\text -}80$ %. This indicates that random internal oxidation levels off at such large *χ* for the mature soot $[17,20]$ $[17,20]$ and carbon black $[19,22]$ $[19,22]$ nanoparticles shown in [Fig. 5,](#page-3-0) proving that their further oxidation is somehow hindered. Internal oxidation results in 20 % larger *SSA* (on average) compared to that obtained by concurrent internal and external oxidation at χ < 50 % and 10 % smaller *SSA* (on average) at χ \geq 50 % ([Fig. 5b](#page-3-0)). Most importantly, internal oxidation overestimates the measured *SSA* by 85 % (on average). So, the slow external burning rates proposed based on microscopy images of diesel [\[51,55](#page-7-0)] and flame [[52\]](#page-7-0) soot cannot explain the measured *SSA* dynamics of diesel soot and carbon black at low *T*.

3.3. Impact of core-shell structure on the CNP internal oxidation

[Fig. 6](#page-4-0) shows the cross-sections of core-shell CNPs with a reactive core diameter, $d_c = 0.5d_p$ (a-d) or 0.75 d_p (e-h) derived by LMC for simultaneous external and random internal oxidation at 550 ◦C and conversion χ = 0, 25, 50 and 75 %, along with microscopy images of Printex 95 [\[19](#page-7-0)] oxidized at the same conditions (i-l). The shell of CNPs contains large stacks of PAHs [\[49](#page-7-0)] that limit its oxidation [[48\]](#page-7-0). So internal oxidation of CNPs with $d_c/d_p = 0.5$ (a-d) or 0.75 (e-h) results in hollow structures at *χ* = 0.75 %, in good agreement with those observed for Printex 95 at similar oxidation conditions [\[19](#page-7-0)].

Fig. 7 shows the LMC-derived m (a), *SSA* (b) and d_p (c) at 550 °C as a function of *t* during simultaneous external and random internal oxidation of CNPs having $d_c/d_p = 0$ (dot-broken lines), 0.5 (dotted lines), 0.75 (solid lines) and 1 (broken lines). Again, the shaded areas represent the statistical variation of the LMC simulations obtained from 50 fractal-like pore networks as in [Fig. 5b](#page-3-0). The CNPs with $d_c/d_p = 0$ are oxidized slowly, as their core is not reactive and oxidation takes place only at their external surface decreasing their diameter, as seen in [Fig. 2](#page-2-0)c. So, *m* and *dp* decrease, while *SSA* increases linearly with time.

The CNPs that contain a reactive core $(d_c/d_p > 0)$ are oxidized faster as both internal and external oxidation take place simultaneously, consistent with moving sectional modelling [[23\]](#page-7-0) and microscopy mea-surements [\[48](#page-7-0)]. In the presence of a less reactive shell though $(d_c/d_p =$ 0.5 or 0.75), internal oxidation practically stops when the reactive core has been fully consumed. This levels off the *m* and *SSA* reduction. The *dp* reduction is not affected by the core-shell structure of CNPs, as it is determined by external oxidation only [\[23](#page-7-0)].

[Fig. 8](#page-6-0) shows the LMC-derived evolution of D_{fp} (a) and SSA/SSA_0 (b) as a function of *χ* during simultaneous external and random internal oxidation of CNPs having $d_c/d_p = 0$ (dot-broken lines), 0.5 (dotted lines), 0.75 (solid lines) and 1 (broken lines), along with experimental data from carbon blacks [[19,22](#page-7-0)] and soot [\[17,20](#page-7-0)]. In the absence of a reactive core $(d_c/d_p = 0)$, oxidation takes place only at the external CNP surface without altering its *Dfp* and *SSA*/*SSAo*. This underestimates the measured CNP *SSA* up to 60 %. On the other hand, neglecting the less reactive shell at the CNP surface $(d_c/d_p = 1)$ results in internal oxidation throughout the particle volume. This results in a fast increase of D_{fp} and SSA/SSA_{o} and overestimates SSA/SSA_0 up to a factor of three at $\chi = 75$ %, as discussed already in [Fig. 6](#page-4-0).

In the presence of a less reactive shell $(d_c/d_p = 0.5 \text{ or } 0.75)$, internal oxidation gradually stops when the reactive core has been fully consumed. So, both D_f and SSA/SSA _o level off at $\chi > 50$ %. The LMCderived evolution of CNP D_{fp} and SSA/SSA_o using $d_c/d_p = 0.5 - 0.75$ nicely explains all data presented here. This indicates that the core-shell structure of the CNPs shown in [Fig. 8](#page-6-0) (symbols) determines the

Fig. 7. LMC-derived m (a), *SSA* (b) and d_p (c) as a function of t during concurrent external and random internal oxidation of CNPs having $d_c/d_p = 0$ (dotbroken lines), 0.5 (dotted lines), 0.75 (solid lines) and 1 (broken lines).

dynamics of their porosity during internal oxidation.

[Fig. 9](#page-6-0) shows the normalized d_p by the initial $d_{p,o}$, $d_p/d_{p,o}$, as a function of *χ* derived here by LMC for external and internal oxidation of CNPs with $d_c/d_p = 1$ (broken line), 0.75 (solid line), 0.5 (dotted line) and 0 (dot-broken line) at 550 ◦C, along with Fischer-Tropsch (triangles [[24\]](#page-7-0)) and diesel soot data obtained at 500 (diamonds [[20\]](#page-7-0)) or 450 $°C$ (squares, circles [\[51](#page-7-0)]) and exhaust gas recirculation (EGR) of

Fig. 8. LMC-derived D_{fp} (a) and *SSA/SSA₀* (b) as a function of *χ* during simultaneous external and random internal oxidation of CNPs having $d_c/d_p = 0$ (dot-broken lines), 0.5 (dotted lines), 0.75 (solid lines) and 1 (broken lines), along with experimental data from [Fig. 3.](#page-3-0)

Fig. 9. Normalized d_p by the initial $d_{p,o}$ ($d_p/d_{p,o}$) as a function of χ derived here by LMC for oxidation of CNPs with *dc*/*dp* = 1 (broken line), 0.75 (solid line), 0.5 (dotted line) and 0 (dot-broken line) along with Fischer-Tropsch (triangles [[24\]](#page-7-0)) or diesel soot data obtained at 500 (diamonds [[20\]](#page-7-0)) or 450 °C (squares, circles [[51\]](#page-7-0)) and EGR = 0 (diamonds [\[20](#page-7-0)], squares [[51](#page-7-0)]) or 20 % (circles [51]).

0 (diamonds [[20\]](#page-7-0), squares [\[51](#page-7-0)]) or 20 % (circles [[51\]](#page-7-0)). At *dc*/*dp* = 0, only external oxidation takes place and reduces d_p up to about 40 % at $\chi = 80$ %. The *dp* data obtained from graphitic Fischer-Tropsch soot (triangles [[24\]](#page-7-0)) lie mostly between the d_p dynamics derived by LMC for $d_c/d_p =$ 0 and 0.5. This confirms that Fischer-Tropsch soot is graphitic and undergoes mostly external oxidation. Internal oxidation is dominant at $d_c/d_p \geq 0.5$ and removes mass from the bulk CNP. So, external oxidation is limited and reduces d_p by 5–20 % at $d_c/d_p = 1 - 0.5$ and $\chi = 80$ %. In this regard, the d_p data measured for diesel soot oxidized at 450–500 $^{\circ}$ C and EGR = $0-20$ % (squares, circles [\[51](#page-7-0)] & diamonds [[20\]](#page-7-0)) are in agreement with the d_p derived here by LMC for CNPs with $d_c/d_p =$ 0.5–0.75 (solid & dotted lines). This confirms that diesel soot particles undergo both internal and external oxidation at these conditions.

4. Conclusions

Lattice Monte Carlo (LMC) simulations for concurrent external and internal oxidation were used here to elucidate the dynamics of carbonaceous nanoparticle (CNP) porosity, for the first time to the best of our knowledge. So, the evolution of the CNP specific surface area, *SSA*, during random internal oxidation is presented here for fractal-like pore networks. Internal oxidation compacts the pore network, increasing the

pore fractal dimension, *Dfp*, up to 3 and enhancing the *SSA* up to an order of magnitude at 75 % conversion. In the presence of a less reactive shell though, internal oxidation stops after the more reactive core is consumed. Thus the *SSA* levels off at large conversions (*>*50 %) of CNPs with core-shell pore networks, in excellent agreement with experimental data for NIST, NEU $[17]$ $[17]$, diesel soot $[20,51]$ $[20,51]$ and carbon black $[19,22]$ $[19,22]$ oxidation. In specific, most soot and carbon black data are bracketed between the LMC dynamics obtained for $d_c/d_p = 0.5$ –0.75 (Fig. 8a). So, accounting for the realistic internal pore structure of CNPs is essential to simulate accurately their oxidation dynamics. In this regard, the present LMC model can be used to guide the design of highly porous carbon black grades by low temperature oxidation [\[19](#page-7-0)] and optimize their performance in various applications from first principles. The porosity dynamics obtained by LMC can be used also to derive robust oxidation rates for soot nanoparticles [[23\]](#page-7-0) and eliminate soot emissions by judicious air injection at the combustor exhaust [2].

CRediT authorship contribution statement

Georgios A. Kelesidis: Conceptualization, Formal analysis, Methodology, Supervision, Validation, Writing – original draft, Writing – review & editing. **Patrizia Crepaldi:** Formal analysis, Methodology, Software, Validation, Writing – original draft, Visualization. **Sotiris E. Pratsinis:** Funding acquisition, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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