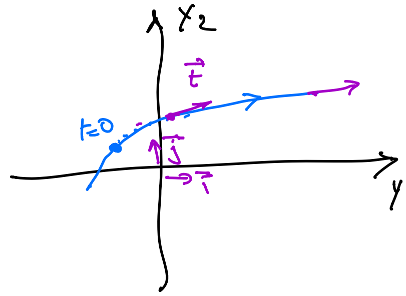
(I) vector field associated to $\dot{\underline{y}} = \underline{f}(\underline{y})$ – (II) nullclines

population dynamics : (III) Ex 1 – Ex 2 (IV)

(I) $\begin{cases} \dot{y}_1 = f_1(y_1, y_2) \\ \dot{y}_2 = f_2(y_1, y_2) \end{cases}$ $\quad \underline{y} \in \mathbb{R}^2$

solution $\rightarrow$ $\begin{cases} y_1 = \varphi(t) \\ y_2 = \psi(t) \end{cases}$ line in parametric form



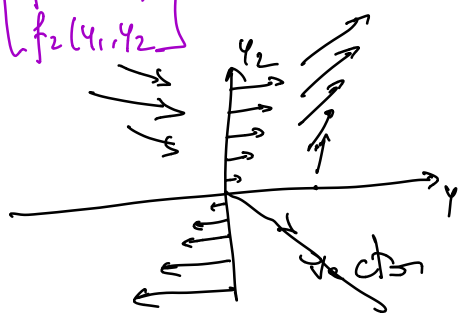
phase plane

$$\vec{t} = \frac{d\varphi}{dt}\vec{i} + \frac{d\psi}{dt}\vec{j}$$

$$\underline{t} = \begin{bmatrix} d\varphi/dt \\ d\psi/dt \end{bmatrix} = \begin{bmatrix} f_1(y_1, y_2) \\ f_2(y_1, y_2) \end{bmatrix}$$

$\underline{y} \in \mathbb{R}^n \qquad \underline{t} = \underline{f}(\underline{y})$

Ex $\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_1 \end{cases}$



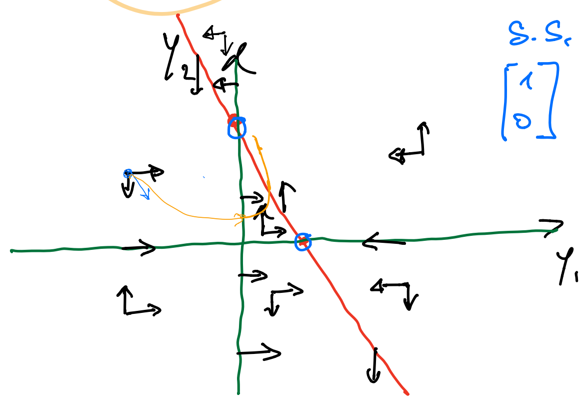
vector field

$$\underline{t} = \begin{bmatrix} y_2 \\ y_1 \end{bmatrix}$$

(II) null clines

N_1 : where the horizontal component of the vector field is 0

$\dot{y}_1 = f_1(y_1, y_2) = 0$



S.S. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

N_2 : where the vertical component is zero

$\dot{y}_2 = f_2(y_1, y_2) = 0$

$$\begin{cases} \dot{y}_1 = -2y_1 - y_2 + 2 \\ \dot{y}_2 = y_1 y_2 \end{cases}$$

s.s. $\underline{f}(\underline{y}_{ss}) = \underline{0}$

N_1 : $\dot{y}_1 = 0 = -2y_1 - y_2 + 2 = 0 \Rightarrow \boxed{y_2 = -2y_1 + 2}$

N_2 : $\dot{y}_2 = 0 = y_1 y_2 \Rightarrow \boxed{y_1 = 0 \text{ or } y_2 = 0}$

(IV) two species, x_1 and x_2 — ch. 2.16

logistic equation

$$\begin{cases} \dot{x}_1 = x_1(\alpha_1 - \beta_1 x_1 - \gamma_1 x_2) \\ \dot{x}_2 = x_2(\alpha_2 - \beta_2 x_2 - \gamma_2 x_1) \end{cases}$$

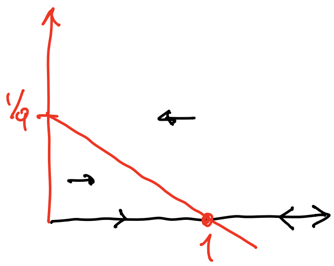
$$x = \frac{\beta_1}{\alpha_1} x_1 \qquad y = \frac{\beta_2}{\alpha_2} x_2 \qquad a = \frac{\alpha_2 \gamma_1}{\alpha_1 \beta_2} > 0 \qquad b = \frac{\alpha_1 \gamma_2}{\alpha_2 \beta_1} > 0$$

$$\boxed{\begin{aligned} \dot{x} &= \alpha_1 x (1 - x - a y) \\ \dot{y} &= \alpha_2 y (1 - y - b x) \end{aligned}}$$

generic situations $\Rightarrow$ $a \neq 1$, $b \neq 1$

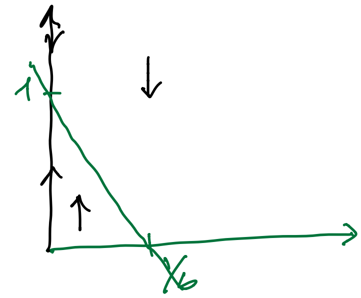
N_1

$\dot{x} = 0$

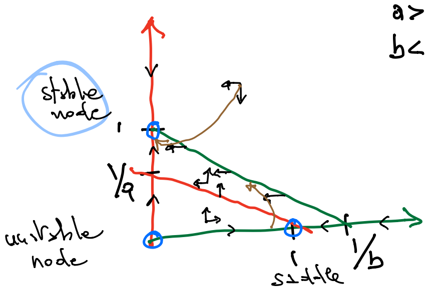


N_2

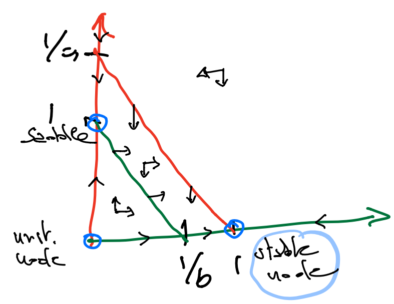
$\dot{y} = 0$



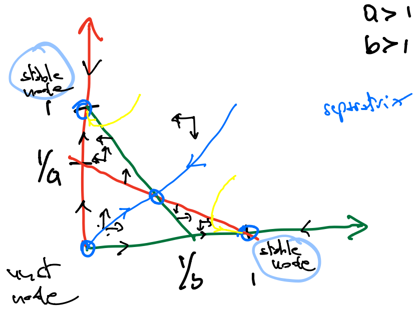
$a > 1$, $b < 1$



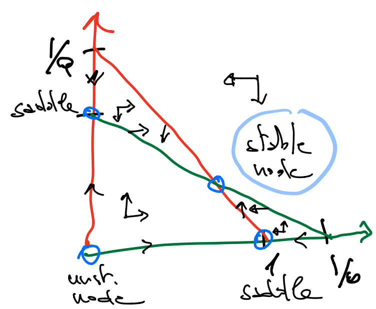
$a < 1$, $b > 1$



$a > 1$, $b > 1$



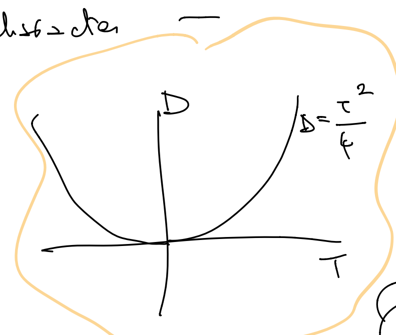
$a < 1$, $b < 1$



- s.s. multiplicity
- different stability character
- nullclines/vector field

s.s. $(0,0)$ $(0,1)$ $(1,0)$ $($ " " $)$

$$\underline{\underline{J}} = \begin{bmatrix} \alpha_1(1 - 2x - ay) & -\alpha_1 a x \\ -\alpha_2 b y & \alpha_2(1 - 2y - bx) \end{bmatrix}$$



(IV) Predation - Prey model   Lotka-Volterra

(P) (N)

$\dot{P} = -bP$ (when alone) $\dot{N} = aN$ (when alone)

$a, b, c, d > 0$

$$\boxed{\dot{P} = (dN - b)P \qquad \dot{N} = (a - cP)N}$$

$$y = \frac{c}{a}P \qquad x = \frac{d}{b}N$$

$$\Rightarrow \begin{cases} \dot{x} = ax(1-y) \\ \dot{y} = by(x-1) \end{cases}$$