## Constant pattern

$\frac{1}{\alpha}$ non-linear

$$\boxed{\frac{1}{Pe\,St}c''' - \left(\frac{1}{\lambda Pe} + \frac{1-\lambda}{St}\right)c'' + \frac{1-\lambda}{\lambda}c' - \nu f' = 0} \rightarrow c(\xi)$$

$\xi = x - \lambda\tau$

B.C. $\xi \to -\infty$ $\quad c \to c_L$, $\quad c', c'' \to 0$

$\xi \to +\infty$ $\quad c \to c_R$, $\quad c', c'' \to 0$

$f'(\xi) = \frac{df}{dc}\frac{dc}{d\xi} = f'(c)\,c'$

$\int_{+\infty}^{-\infty} \Rightarrow$ $\boxed{0 = \frac{1-\lambda}{\lambda}(c_L - c_R) - \nu(f_L - f_R)}$

Integration in $\xi$ : $\int_{+\infty}^{\xi} \boxed{\phantom{x}}\, d\xi = 0$

$$\boxed{\frac{1}{Pe\,St}c'' - \left(\frac{1}{\lambda Pe} + \frac{1-\lambda}{St}\right)c' + \frac{1-\lambda}{\lambda}(c - c_R) - \nu\underbrace{\left(f(c) - f(c_R)\right)}_{(f - f_R)} = 0}$$

$\to 0$ simplifying assumption

$$\boxed{\frac{dc}{d\xi} = \alpha\left[\frac{1-\lambda}{\lambda}(c - c_R) - \nu\left(f(c) - f_R\right)\right]}$$

non linear autonomous
1° order ODE
$\hookrightarrow c = c(\xi)$

① $\lambda$? $\to$ same as $\tilde{\sigma}$ from Eq. Theory. $\to$ no need for simplifying assumption.

② $c_R, c_L \sim$ steady states and their stability ?

B.C. at $\xi \to -\infty$

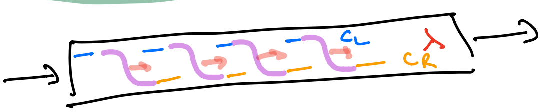
Ⓐ $\hookrightarrow$ $0 = \boxed{\alpha\left[\frac{1-\lambda}{\lambda}(c_L - c_R) - \nu\left(f_L - f_R\right)\right] = 0}$ $\quad \lambda = ?$

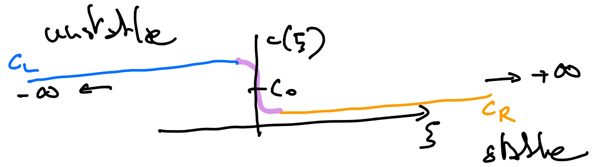
$$\boxed{\lambda = \left(1 + \nu\frac{\Delta f}{\Delta c}\right)^{-1}}$$

$x = \frac{z}{L} \qquad \tau = \frac{t v}{L}$

$\lambda = \frac{dx}{d\tau} = \frac{1}{\cancel{L}}\frac{dz}{dt}\frac{\cancel{L}}{v} \qquad \frac{1}{v}\frac{dz}{dt} = \left(1 + \nu\frac{\Delta f}{\Delta c}\right)^{-1}$ ①

$$\frac{dt}{dz} = \frac{1}{v}\left(1+\nu\frac{\Delta f}{\Delta c}\right) = \tilde{\sigma}(c_L, c_R)$$ as calculated from Eq. Theory




(2) steady states of $\frac{dc}{d\xi} = \alpha\left[\frac{1-\lambda}{\lambda}(c-c_R) - \nu\left(f(c)-f_R\right)\right]$

$$\boxed{\frac{dc}{d\xi} = \alpha\nu\left[\frac{\Delta f}{\Delta c}(c-c_R) - (f-f_R)\right]} = \alpha\nu\left[F(c)\right] \Rightarrow c(\xi)$$

$$\frac{dc}{d\xi}=0 \qquad \begin{array}{ll} (\mathrm{I}) & c = c_R \\ (\mathrm{II}) & c = c_L \end{array}$$

**Reminder**

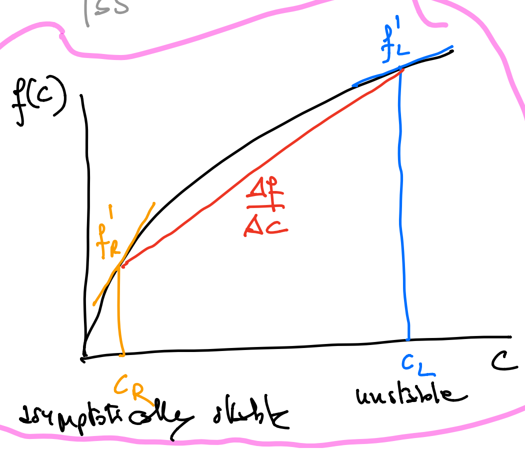
$\dot{y} = G(y)$ $\qquad y_{ss}:\ G(y_{ss})=0$

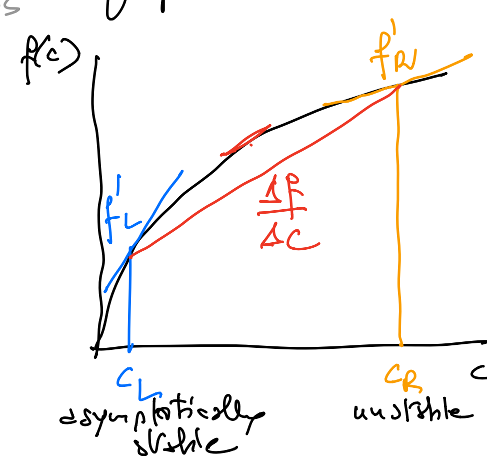
$\hookrightarrow$ condition for stability

$\dot{y} = G'(y_{ss})(y-y_{ss})$

$x = y-y_{ss}$ $\qquad \boxed{\dot{x} = G'(y_{ss})\,x}$ $\qquad x(t)=x_0\exp\left(G'_{ss}t\right)$

$$F'(c)\Big|_{ss} = \frac{dF}{dc} = \frac{\Delta f}{\Delta c} - \frac{df}{dc}\Big|_{ss} \quad \begin{cases} \text{if } <0 & \text{stable ss.} \\ \text{if } >0 & \text{unstable ss.} \end{cases}$$




(2)

$$f(c) = \frac{Hc}{1+Kc} \qquad \frac{dc}{d\xi} = \alpha\nu\left[\frac{\Delta f}{\Delta c}(c-c_R) - (f-f_R)\right]$$

$$\boxed{\frac{1}{F(c)}\,dc = \alpha\nu\, d\xi} \Rightarrow \boxed{\frac{1+Kc}{(c-c_R)(c-c_L)}\,dc = \alpha\nu K \frac{\Delta f}{\Delta c}\,d\xi}$$

$$\int_{c_0}^{c} \frac{dc}{F(c)} = \int_0^{\xi} \alpha\nu\, d\xi = \alpha\nu\,\xi =$$

$$\boxed{(1+Kc_L)\ln\left(\frac{c-c_L}{c_0-c_L}\right) - (1+Kc_R)\ln\left(\frac{c-c_R}{c_0-c_R}\right) = \alpha\nu K\,\Delta f\,\xi}$$

$$c_0: \quad \frac{d^2c}{d\xi^2} = 0 = F'(c_0) \Rightarrow \frac{\Delta f}{\Delta c} = f'(c_0)$$

$$1 + Kc_0 = \sqrt{(1+Kc_L)(1+Kc_R)}$$

Conservation law $\quad M(w)\,,\ F(w)$

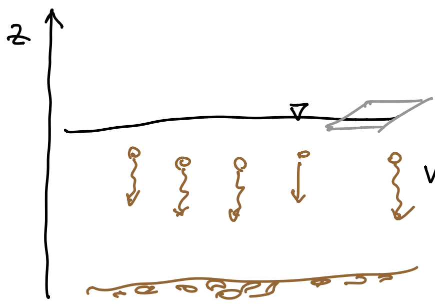
↳ differential form: $\quad \frac{\partial M}{\partial t} + \frac{\partial F}{\partial z} = 0$

↳ $\sigma(w) = \dfrac{M'(w)}{F'(w)}$

↳ finite form → $\tilde{\sigma} = \dfrac{\Delta M}{\Delta F}$

Map 4th

# Sedimentation




$V_0$ : terminal velocity

when isolated,
gravity vs. drag

all of the same size → When there are many particles there are hindrance effects, depending on concentration!

↳ traffic (Prigogine)

$$\boxed{\begin{array}{l} M(u) = m = m_{max}\, u \qquad 0 \le u \le 1 \\ F(u) = -m_{max} V_0\, \phi(u) \end{array}}$$

$\left[\frac{\#}{m}\right]$ (w/ cross section of $1\,m^2$)

dimensionless concentration

$$\begin{cases} (I) & \phi(0) = 0 \\ (II) & \phi(1) = 0 \end{cases}$$


$$\phi(u) = u - u^2$$

$$\phi'(u) = 1 - 2u$$

$$\phi(u) = u(1-u)$$

$$\frac{dz}{dt} = \lambda = \frac{1}{\sigma} = \frac{F'}{M'} = -V_0\,\phi'(u) \overset{V_0=1}{=} -\phi'(u) = 2u-1 \Rightarrow \text{slope of characteristics in plane } (t,z)$$

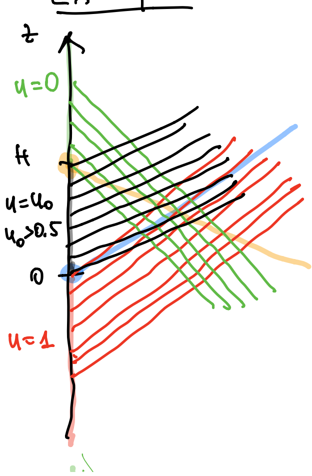

$$\tilde{\lambda} = \frac{1}{\tilde{\sigma}} = \frac{\Delta F}{\Delta M} = -V_0 \frac{\Delta\phi}{\Delta u} \overset{V_0=1}{=} -\frac{\Delta\phi}{\Delta u}$$

from $M(u)$ and $F(u)$ → PDE

↳ write the characteristic eqs.

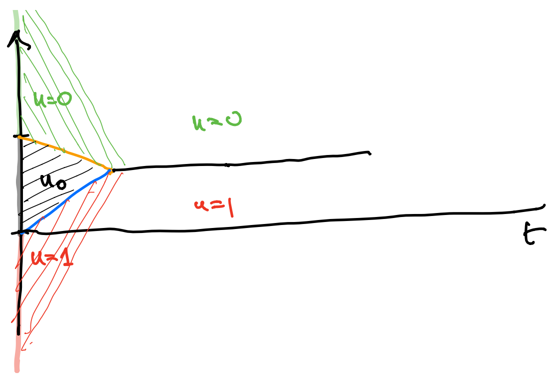
determine $\lambda$

## Example




$u_0 // 0$ $\quad \tilde{\lambda}(u_0, 0) = -\frac{\phi(u_0)}{u_0} = u_0 - 1 < 0$

$u_0 // 1$ $\quad \tilde{\lambda}(u_0, 1) = -\frac{\phi(u_0)}{(u_0 - 1)} = u_0 > 1$




- $\tilde{\lambda}(0,1) = 0$