PDE $\to$ criterion on existence and uniqueness of the solution (surface)

① ② ③

$$P(t,z,w)\,\frac{\partial w}{\partial t} + Q(t,z,w)\,\frac{\partial w}{\partial z} = R(t,z,w)$$

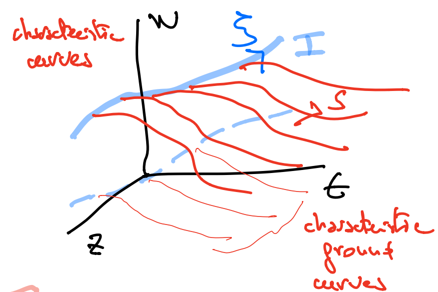
$$I:\begin{cases} t = \varphi(\xi) \\ z = \psi(\xi) \\ w = \omega(\xi) \end{cases}$$

① characteristic differential eqs

$$\begin{cases} \dfrac{dt}{ds} = P(t,z,w) \\ \dfrac{dz}{ds} = Q(t,z,w) \\ \dfrac{dw}{ds} = R(t,z,w) \end{cases}$$

initial conditions

$$\begin{cases} t(0) = \varphi(\xi) \\ z(0) = \psi(\xi) \\ w(0) = \omega(\xi) \end{cases}$$



$$\begin{array}{l} t = t(s,\xi) \\ z = z(s,\xi) \end{array} \xrightarrow[\text{locally around } I]{\text{inversion}} \begin{array}{l} s = s(t,z) \\ \xi = \xi(t,z) \end{array}$$

$$w = w(s,\xi)$$

$w = w(t,z)$, as required

$y = f(x)$

$x = f^{-1}(y)$

if $f'(x_0) \neq 0$

$$\underline{J} = \frac{\partial(t,z)}{\partial(s,\xi)} = \begin{bmatrix} \dfrac{\partial t}{\partial s} & \dfrac{\partial t}{\partial \xi} \\ \dfrac{\partial z}{\partial s} & \dfrac{\partial z}{\partial \xi} \end{bmatrix}$$

$$t(s,\xi) - t(0,\xi_0) = \frac{\partial t}{\partial s}(s-0) + \frac{\partial t}{\partial \xi}(\xi-\xi_0) + \dots$$

$$z(s,\xi) - z(0,\xi_0) = \frac{\partial z}{\partial s}(s-0) + \frac{\partial z}{\partial \xi}(\xi-\xi_0) + \dots$$

$$\det\left(\underline{J}\Big|_I\right) \neq 0 \qquad \det\left(\underline{J}\Big|_I\right) = \frac{\partial t}{\partial s}\frac{\partial z}{\partial \xi} - \frac{\partial t}{\partial \xi}\frac{\partial z}{\partial s} =$$

$$= \boxed{P(\varphi(\xi),\psi(\xi),\omega(\xi))\,\psi'(\xi) - \varphi'(\xi)\,Q(\varphi(\xi),\psi(\xi),\omega(\xi)) \neq 0}$$

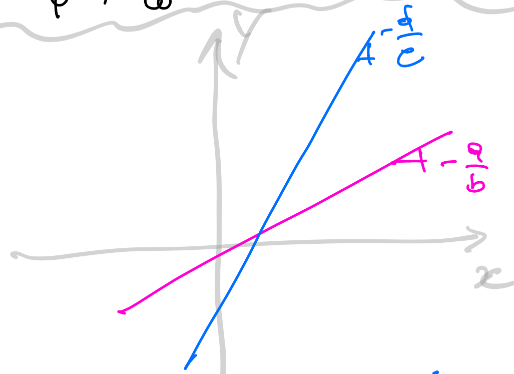
② $$\begin{cases} P(\varphi(\xi),\psi(\xi),\omega(\xi))\dfrac{\partial w}{\partial t} + Q(\varphi(\xi),\psi(\xi),\omega(\xi))\dfrac{\partial w}{\partial z} = R(\varphi(\xi),\psi(\xi),\omega(\xi)) \\ \dfrac{\partial w}{\partial t}\varphi'(\xi) + \dfrac{\partial w}{\partial z}\psi'(\xi) = \omega'(\xi) \end{cases}$$

$w = w(t,z) \Rightarrow \omega(\xi) = w(\varphi(\xi),\psi(\xi)) \xleftarrow{\frac{d}{d\xi}}$

$$\left|\underline{\underline{A}}\right| = \begin{vmatrix} P & Q \\ \varphi' & \psi' \end{vmatrix} = \left(P\psi' - Q\varphi'\right)\Big|_{I} \neq 0 \qquad \text{Q.E.D.} \Rightarrow \text{1 and only one solution}$$

$$\left|\underline{\underline{A}}\right| = 0 \iff \frac{P}{\varphi'} = \frac{Q}{\psi'} \Rightarrow \begin{cases} \dfrac{P}{\varphi'} = \dfrac{Q}{\psi'} = \dfrac{R}{\omega'} \Rightarrow \infty \text{ solutions} \\[2ex] \dfrac{P}{\varphi'} = \dfrac{Q}{\psi'} \neq \dfrac{R}{\omega'} \Rightarrow \text{no solutions} \end{cases}$$

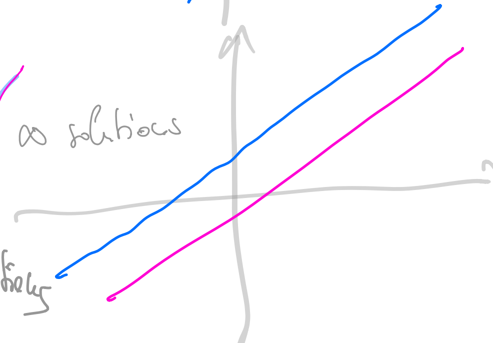
$$\boxed{\begin{cases} ax + by = c & -\dfrac{a}{b} \\ dx + ey = f & -\dfrac{d}{e} \end{cases}} \qquad \boxed{\begin{aligned} x &= \frac{\partial w}{\partial t} \\ y &= \frac{\partial w}{\partial z} \end{aligned}}$$



$$+\frac{a}{b} \neq +\frac{d}{e} \qquad \frac{a}{d} \neq \frac{b}{e}$$

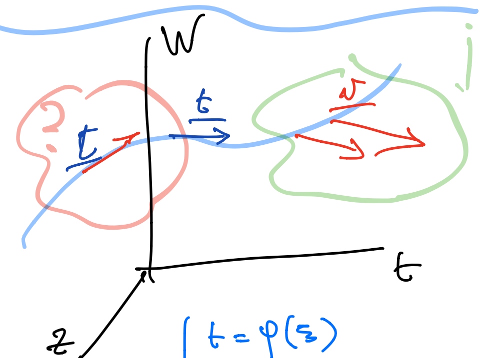
$$\frac{a}{d} = \frac{b}{e} \begin{cases} \dfrac{a}{d} = \dfrac{b}{e} = \dfrac{c}{f} & \infty \text{ solutions} \\[2ex] \dfrac{a}{d} = \dfrac{b}{e} \neq \dfrac{c}{f} & \emptyset \text{ solutions} \end{cases}$$

lines are parallel



3

$$\boxed{\underline{v}} = \begin{bmatrix} P(\varphi(\xi), \psi(\xi), \omega(\xi)) \\ Q(\quad) \\ R(\quad) \end{bmatrix}$$



$$\boxed{\underline{t}} = \begin{bmatrix} \varphi'(\xi) \\ \psi'(\xi) \\ \omega'(\xi) \end{bmatrix} \qquad \boxed{\frac{P}{\varphi'} = \frac{Q}{\psi'} = \frac{R}{\omega'}} \qquad \boxed{\frac{P}{\varphi'} \neq \frac{Q}{\psi'}}$$

$$I \begin{cases} t = \varphi(\xi) \\ z = \psi(\xi) \\ w = \omega(\xi) \end{cases}$$

2

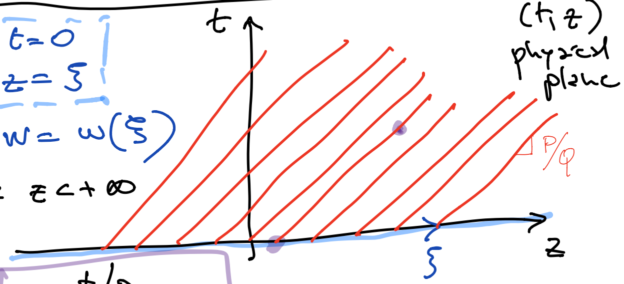
Linear and semi-linear equations

$$\boxed{P \frac{\partial w}{\partial t} + Q \frac{\partial w}{\partial z} = R(t,z)} \qquad \boxed{P \frac{\partial w}{\partial t} + Q \frac{\partial w}{\partial z} = R(t,z,w)}$$

$$\begin{cases} \frac{dt}{ds} = P \\ \frac{dz}{ds} = Q \\ \frac{dw}{ds} = R(t,z) \end{cases} \qquad \begin{cases} t(0) = 0 \\ z(0) = \xi \\ w(0) = \omega(\xi) \end{cases}$$

$$I \begin{cases} t = 0 \\ z = \xi \\ w = \omega(\xi) \end{cases}$$

$-\infty < z < +\infty$



$$\boxed{\begin{aligned} t &= Ps \\ z &= Qs + \xi \end{aligned}} \longrightarrow \boxed{\begin{aligned} s &= t/P \\ \xi &= z - \frac{Q}{P}t \end{aligned}}$$

$$\int_{\omega(\xi)}^{w} dw' = \int_0^s R(Ps', Qs' + \xi)\, ds'$$

constant

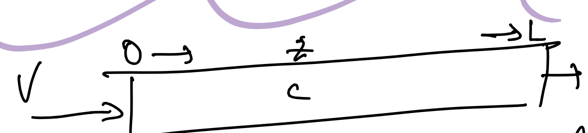
$$w = \omega(\xi) + \int_0^s R(Px, Qx + \xi)\, dx$$

$$\boxed{w = \omega\left(z - \frac{Q}{P}t\right) + \int_0^{t/P} R\left(Px, Qx + z - \frac{Q}{P}t\right) dx}$$

$w = w(t,z)$

slope: $\sigma = \frac{dt}{dz} = \frac{P\,ds}{Q\,ds} = \frac{P}{Q} = \sigma \Rightarrow$ constant

tubular reactor
homogeneous
$A \rightarrow$ products



C: concentration of the reactant A

$c \left[\frac{mol}{m^3}\right]$

$r = kc$

$$\Rightarrow \boxed{\frac{\partial c}{\partial t} + \boxed{V \frac{\partial c}{\partial z} = -kc}}$$

$V \left[\frac{m}{s}\right]$

steady state

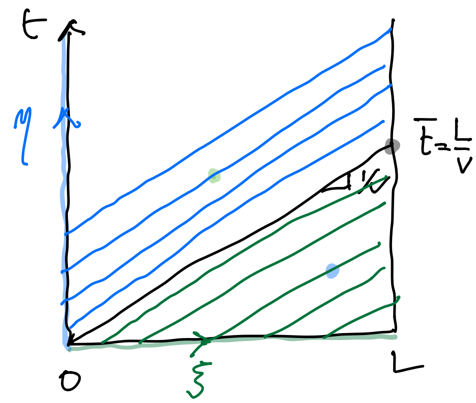
$k \left[\frac{1}{s}\right]$

transient behavior
from start up to S.S.
semi-linear equation

$$\begin{cases} \dfrac{dt}{ds} = 1 \\ \dfrac{dz}{ds} = V \\ \dfrac{dc}{ds} = -kc \end{cases}$$

$$\sigma = \frac{1}{V} = \frac{dt}{dz}$$

$$\lambda = \frac{1}{\sigma} = \frac{dz}{dt} = V$$



I $\begin{cases} t = \eta \\ z = 0 \\ c = g(\eta) \boxed{(= c_F)} \end{cases}$

I $\begin{cases} t = 0 \\ z = \xi \\ c = f(\xi) \boxed{(= c_i)} \end{cases}$

$$\left.\begin{aligned} t &= s + \eta \\ z &= Vs + \cancel{\phi} \end{aligned}\right\} \qquad \eta = t - z/V \qquad s = z/V$$

$$c = g(\eta)\exp(-ks) = g\left(t - \frac{z}{V}\right)\exp\left(-kz/V\right)$$

$$\left.\begin{aligned} t &= s + \cancel{\phi} \\ z &= Vs + \xi \end{aligned}\right\} \qquad s = t \qquad \xi = z - Vt$$

$$c = f(\xi)\exp(-ks) = f(z - Vt)\exp(-kt)$$

$$\tau = \frac{z}{V}$$

special / normal cases

$$c = c_F \exp\left(-\frac{kz}{V}\right) \qquad t > z/V \qquad z < Vt$$

$$c = c_i \exp(-kt) \qquad t < z/V \qquad z > Vt$$

for $t > \dfrac{L}{V}$

$$\boxed{c(z = L) = c_F \exp\left(-\frac{kL}{V}\right)}$$

steady state solution