#### **Belousov-Zhabotinsky reaction**

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# Experimental evidence (Cerium)

- Malonic acid oxidation by bromate in acidic solution, catalyzed by a metal ion (Ce<sup>4+</sup>)
- Long series of oscillations in the concentration of the intermediate species (with a chromatic effect)
- The main reactants are irreversibly consumed (but very slowly)
- Finally, the oscillations vanish and chemical equilibrium is monotonically approached



## Chemistry

The main reaction driving the Belousov-Zhabotinsky system forward is given by the following equation:

 $2Br^{-} + BrO_{3}^{-} + 3CH_{2}(COOH)_{2} + 3H^{+} \rightarrow 3BrCH(COOH)_{2} + 3H_{2}O$ 



### Experimental evidence (ferroin)



#### **Detailed mechanism**

#### TABLE I: Mechanistic Model of the Belousor-Zhabotinskii Reaction ([11]0] = 55 M included in the Rate Constants)"

			rate constant	teletence
		1. Inorganic Subset		reterence
		HOBr + Br + H - Br + H,O	2.3E+9 M-1 s-1	
	÷	$Br_1 + H_2O - HOBr + Br^* + H^*$	2.0 1"	71
		Br" + HBrO, + H" - 2HOBr	2.0E+6 M-1 s-1	374
		HOBr - Br + HBrO, + H*	2.0E-5 MT 5"	22. 23
	· 2 ·	br + bro, + 2H - HOBr + HBrO,	2.0 M <sup>-1</sup> s <sup>-1</sup>	22*
	1	1000 + HBIO, - Br + BIO, + 2H	J.3 M <sup>-1</sup> s <sup>-1</sup>	32*
	ś	BOL + HOR + HT - HUR + H	1.0E+1 M-1 s-1	22
	ğ		7.5E-9 M-1 s-1	22*
	10		13.0 M <sup>-1</sup> s <sup>-1</sup>	22*
	11	8:0 - 18:0 -	2200 s <sup>-1</sup>	22
	12	19:0.1 8:0	7.4E+4 s*	22
	÷.	$C_{r}^{(1)} + B_{r}O_{r} + H^{*} = HB_{r}O_{r} + C_{r}H^{*}$	1.4E+9 M <sup>-1</sup> s <sup>-1</sup>	22
	14		6.2E+4 M-1 5-4	22'
		10101 · Ct - Ct + BrO; + H	7.0E+3 M-1 5-1	22*
		2. Reactions Involving Organic Speci	c1	
	15	MA - ENOT	Radicals	
	16	ENOL - MA	1.0E-1 s-4	25.38
	17	ENOL + Br BrMa + Brt + MT	200.0 s <sup>-4</sup>	25, 38
	13	MA + HOBr - BrMA + H.O	1.91E+6 M-1 5-1	394
	19	BrMA + HOBr - Br MA + H O	8.2 M <sup>-1</sup> s <sup>-1</sup>	39
	20	TTA + HOBr - BrTTA + H.O	0.1 M-1 1-1	/
	21	BrO-MA + H.O - HBrO. + TTA	3.0 M 5.	39*
	22	BrO,MA - HOBr + MOA	1.0 5	47
	- 23	BrO.TTA HBrO. + MOA	1.0 1	17
	24	BrTTA - Br + MOA + H*	1.0 **	47*
	25	Ca** + BrMA - Ce2* + BrMA + 4*	0.00.1011	
	26	Cet + MA - Cel + MA: + H	0.09 M** s**	39, 48
	27	Cet + TTA - Cel + TTA + U*	0.23 M* s*	39, 57
	28	HOBr + MOA = Br' + OA + (COOH)	0.66 M <sup>-1</sup> s <sup>-1</sup>	39, 57
	29	$C_{4}$ + MOA + H.O - $C_{4}$ + OA + COOH + V	140.0 M <sup>-1</sup> s <sup>-1</sup>	10-1
	30	$HOB_1 + OA = Br_1 + TOOP_1 + CO_1 + HO$	10.0 M <sup>-1</sup> s <sup>-1</sup>	30-3
	31	$C_{4}$ + $O_{4} - C_{4}$ + $C_{0}O_{1} + C_{0} + H_{1}$	1-0.0 Mer 5-	30*
	32		10.0 M- s-	30*
			1.6E-5 MP4 5*	30*
	11	c. Reactions Consuming Radicals	λ.	
· · ·	1.5	20r - Br	1.0E+8 M-1 s-1	30
	15 .	Br + Bran - Brank	1.0E+9 M-1 s-1	
	16	Bella's Mars HO	1.0E+\$ M <sup>-1</sup> s <sup>+1</sup>	
	37	Bella' + TTA' + H O - TTA + BETTA	1.0E+9 M-1 s-1	
	35	Brith + Crit + HO - C Is - D - TIA	1.0E+9 M-1 s-1	
	19	BrMA' + BrO' + H O - HBrO + BrTTA + H	1.0E+7 M-1 s-1	30
	40	Briat + 100 + HO - HBRO, + Brita	5.0E+9 M-1 s-1	14*
	41	7MA1 + H.O - MA + TTA	5.0E+3 M-1 51	
	42	$MA' + TTA' + H_0 - 2TTA$	3.2E+9 M-4 s-4	. 13
	23 ·	MA' + COOH $\rightarrow$ MA + CO.	1.0E+9 M-1 s-1	
	44	MA" + Br" - BrMA	LUE+9 MIN SH	· · · · ·
	-45	$MA^* + Ce^{i*} + H^* - MA + Ce^{i*}$	1.06+9 11-3	
	16	MA" + BrO BrO.MA	LIE+4 MI- SH	\$7
	47	2TTA' - TTA + MOA	LOE+9 Met and	14
	-3	TTA" + 'COOH - TTA + CO.	705-9 41-1	134
	49	TTA" + Br - BrTTA	0E+9 X(** **	
	50	TTA" + Cel" + H" - TTA + Ce"	1.7E+4.5(-1.+1	674
	51	TTA" + BrO," - BrO,TTA	5.0E+9 M-1 +1	1.14
	52	2 COOH - 0A	1.2E+9 M-1 +1	10
	53	"COOH + Ce" - Ce1" + CO. + H"	1.0E+7 M-1 -1	10*
	34	COOH + Br - Br + CO1 + H	1.0E+9 M-1 s-1	10
	55	COOH + BrO: - HBrO: + CO.	5.0E+9 M-1 5-1	14
		d Reparing Base		
	56	MA" + Br, - BrMA + Br	1 (E-1 1/-1 -1	
	- 57	MA' + HOBr - TTA + Br	1.05+7 \1-1	1.27
	53	MA" + BrO," + H" - TTA + BrO."	10.0 111 -1	1.4
	59	MA' + TTA - MA + TTA'	-0.0 M - 5"	14"
	60	TTA' + MA - TTA + MA'	LOE+S MEL	
	- 61	MA" + BrMA - MA + BrMA"	10F+5 \14 +4	E
	62	BrMA" + MA - BrMA + MA"	5.0E+2 MT	4
	63	TTA" + BEMA - TTA + BEMA	2.0E+5 M-1 +1	
	6-	BrMA' + TTA - BrMA + TTA'	5.0E+3 M-1 s-1	
	65	TTA' + Br - BrTTA + Br	1.0E+3 M** s**	1.4
	.66	TTA" + HOBr - MOA + Br" + H-O	1.0E+7 M-1 c-1	124
	67	11A - BrO, - H - MOA + BrO, + HO	40.0 M-1 1-1	14**
	63	BrMA' + Br; - Br;MA + Br	1 0E+6 M" 1"	144
	69	BrMA' + HOBr - BrTTA + Br	1.0E+5 M*1 **	1.4
	70	BrMA" + BrO1" + H" - BrO1" + BrTTA	40.0 M-1 +1	
	71	'COOH + 3-MA - 8- + MA' + CO. + H-	1.0E+7 M-1 -1	
	72	COOH + Br Br + Br + CO. + H*	1.5E+X M*1.c*1	1.14
	73	COOH + HOBr - Br' + CO. + H.O	2.0E+7 M-1 -1	1.0
	74	COOH + BrO, + H BrO. + CO. + H.O	2.1E+3 M-1 s-1	10
	75	Br" + MA - Br" + MA" + H"	1.0E+5 M-1 5-1	10
	16	Br' + TTA - Br' + TTA' + H	1.0E+6 M-1 5-1	
	17	Br" + BrMA - Br" + BrMA" + H"	5.0E+6 M-1 5-1	
	78	$Br + MOA + H_{10} \rightarrow Br + OA + COOH + H_{-}$	2.0E+3 M-1 5-1	10*
	78	$Br' + MOA + H_1O - Br' + OA + COOH + H^*$ $Br' + OA - Br' + COOH + CO_1 + H^*$	2.0E+3 M <sup>-1</sup> s <sup>-1</sup> 2.0E+3 M <sup>-1</sup> s <sup>-1</sup>	10* 10

#### Detailed mechanism – Inorganic subset

Mechanistic Model of the Belousov-Zhabotinskii Reaction ([1110] = 55 M Included in the Rate Constants)\*

		rate constant
	1. Inorganic Subset	
1	$HOBr + Br' + H' - Br_{+} + H_{-}O$	7 15+0 11-2 -1
2	$B_{f_1} + H_0 - HOB_f + B_{f_1} + H_1$	1.36+9 MI-3-
1	Br + HBrO. + Ht - 1400-	2.0 5
		2.0E+6 M-1 s-1
	THORE - BL + HBLO' + H.	2.0E-5 M*1 s*1
3	$Br^{*} + BrO_{1}^{*} + 2H^{*} \rightarrow HOBr + HBrO_{1}$	20 M*F **
6	HOB: + HBrO, Br + BrO, + 7H-	1.1.1.4-11
7	$2HBrO_{2} \rightarrow BrO_{2} \rightarrow HOBr + H^{2}$	3.3 /41 - 5
*		3.0E+3 M** s**
š	Broi + HOBr + H - 1HBrO.	7.5E-9 M-1 5-1
9	$BrO_1 + HBrO_2 + H^* - Br_2O_1 + H_2O_2$	11.0 M-1 e-4
10	$Br_0 + H_0 - Br_0 + H_0 + H_1$	1200 -1
117	Br.O 2BrO.*	1100 5
12	19:0 2: 0	1.4E+4 5"
		1.4E+9 M <sup>-1</sup> s <sup>-1</sup>
12	Cer + BrO: + H HBrO. + Cer	6.2E+4 M-2 +4
14	HBrO: + Ce - Ce + BrO: + H-	7.0E+3 M-1 5-1

#### Detailed mechanism – Radicals and organic species

	2. Reactions Involving Organic Spec	cies
	a. Reactions Not Consuming or Producing	Radicals
15	MA - ENOL	1.0E-1 s*
16	ENOL - MA	200.0 44
17	ENOL + Br BrMA + Br + HT	
13	MA + HOBE - BEMA + H.O.	1.91 6 +6 M
19	BrMA + HOBr - Br MA + H O	6.2 M 5
10		0.1 M <sup>-1</sup> 1 <sup>-1</sup>
	HA + HOBT - BILLA + HO	5.0 M <sup>-1</sup> s <sup>-1</sup>
	BIOLNIA + H.O - HBIO. + TTA	1.0 s <sup>-1</sup>
	BIO MA - HOBE + MOA	1.0 5
	BrO, ITA HBrO, + MOA	1.0 s <sup>-1</sup>
24	Britta - Br' + MOA + H*	1.0 5**
	b. Reactions Producing Radicals	
	C:" + BrMA Cel" + BrMA' + H"	0.09 51-1 5-1
26	Ce + MA - Ce + MA + H	0.21 M <sup>-1</sup> e <sup>-1</sup>
27	Ce** + TTA Ce** + TTA* + H*	0.66 MH cH
28	HOBr + MOA - Br' + OA + COOH	
29	$C_{4}^{**} \pm MOA \pm H_{*}O = C_{*}^{**} \pm OA \pm COOH \pm H_{*}$	1-0.0 M 5
30	$HOB_r + OA = Br' + COOH + CO + HO$	10.0 .4-4 5-4
31		1-0.0 M-1 5-1
17		10.0 M- s-
		1.6E-5 M-2 5"
	A Renation Court of the	
11	28et - Re. Consuming Radicals	
1.	Bet & Balting D. Les	1.0E+8 M-1 s-1
16	Br - BrMA - BrMA	1.0E+9 X(-1 s-1
12	SIMA + HO - BIMA + BITTA	1.0E+3 M-1 s-1
10	BrMA' + MA' + HO - MA + BrTTA	1.0E+9 M-1 -1
37	BrMA" + TTA" + H,O - TTA + BrTTA	105-49 14-1
38	BrMA" + Cet" + H.O - C-1" + BrTTA + H-	LOC 17 MIL ST
39	BrMA" + BrO." + H.O - HBrO. + BrTTA	1.0E+7 M- 57
40	BrMA' + COOH - BrMA + CO	3.0E+9 M-1 s-1
41		5.0E+3 M-1 5-1
		3.2E+9 M <sup>-1</sup> s <sup>-1</sup>
	$H_{0} + 11A + H_{0} - 21TA$	1.0E+9 M-1 s-1
	MA' + COOH - MA + CO.	2.0E+9 M-1 -1
44	MA' + Br' - BrMA	10F+9 M-1 -1
-15	$MA^* + Ce^{i*} + H^* - MA + Ce^{i*}$	1754131-1-1
-46	MA" + BrO." - BrO.MA	
47	$2TTA^* \rightarrow TTA + MOA$	3.0E+9 M- 5-
-3	TTA: + COON - TTA + CO	1.0E+9 M** s**
10	TTA + Bat - Batter	2.0E+9 M-1 s-1
60	The form of the	1.0E+9 M** s**
50	11A" + Ce" + H" TTA + Ce"	1.7E+4 M-1 s-1
31	TTA" + BrO," - BrO,TTA	5.0E+9 M-1 +-1
32	2°COOH - OA	1 75+9 11-1 +1
53	"COOH + Ce" - Ce" + CO. + H"	LOEAT MEL OF
54	"COOH + Br" - Br" + CO. + H"	
55	"COOH + 8:0." - HR:0. + CO	1.0E+9 M-1 5-1
		5.0E+9 M-1 5-1
	d Repetions Preservice Restantion	
56	MA" + Br BrMA + Br	
57	MA' + HORE - TTA - Ret	1.3E+3 M" s"
- 53	MAT + BO TA HT - TTI + BO T	1.08+7 .4" 5"
40 Ka	MAN THE THE TIM T BO	40.0 M <sup>-1</sup> s <sup>-1</sup>
10	THE THAT MA + FTA	1.0E+5 M-1 s-1
60	11A' + MA - TTA + MA'	1.0E+5 M" s"
61	MA + BrMA - MA + BrMA*	1.0E+5 M-1 5"
62	BrMA" + MA - BrMA + MA"	5.0E+2 M <sup>-1</sup> c <sup>-1</sup>
63	TTA" + BrMA - TTA + BrMA"	205+5 11-1-1
6-1	BrMA' + TTA - BrMA + TTA'	(00-1) M 5
65	TTA' + Br BrTTA + Br	JUET J M
66	TTA" + HOBE - MOA + Bri + H O	1.0E+3 M- 1
47		1.0E+7 M-1 s-1
42	Brita' + Br - Brits - Brit	40.0 M <sup>-1</sup> s <sup>-1</sup>
40 ·	and a ari - ariwy - Br	1 0E+6 M"' s"
69	BrMA' + HOBr - BrTTA + Br	1.0E+5 M <sup>-1</sup> -
70	BrMA" + BrO," + H" - BrO," + BrTTA	40.0 M <sup>-1</sup> x <sup>-1</sup>
71	"COOH + 9-MA - 8" + MA" + CO. + H"	10E+7 31-1
72	"COOH + Br Br" + Br" + CO H"	t they sent of
73	'COOH + HOBr - Br + CO + H O	100-11-11-1
74	COOH + 80 - + 81 - 8-01 + 60 + 11 C	1.VE+1 M" 5"
75	Rd + Ma = 0.01 + H = 0.01 + CO1 + HO	2.1E+J M-1 s*1 3
74	Bet + TTA - DF + MA' + H	1.0E+5 M=1 5-1
10	Br + 11A - Br + TTA' + H*	1.0E+6 M-1 s-1
11 ^	Br" + BrMA - Br" + BrMA" + H"	5.0E+6 M-1 s-1
73	Br" + MOA + H,O - Br" + OA + "COOH + H"	2.0E+1 M-1 +1
79	Br" + OA - Br" + "COOH + CO. + H"	2.05+1 31-1 -1
30	BrO, + OA - HBrO, + COOH + CO.	10E+7 M-1

#### Mechanism of the oscillations



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$$r_{1} = k_{1} \left[ \mathrm{H}^{+} \right]^{2} \left[ \mathrm{BrO}_{3}^{-} \right] \left[ \mathrm{Br}^{-} \right]$$

$$r_{2} = k_{2} \left[ \mathrm{H}^{+} \right] \left[ \mathrm{HBrO}_{2} \right] \left[ \mathrm{Br}^{-} \right]$$

$$r_{3} = k_{3} \left[ \mathrm{H}^{+} \right] \left[ \mathrm{HBrO}_{2} \right] \left[ \mathrm{BrO}_{3}^{-} \right]$$

$$r_{4} = k_{4} \left[ \mathrm{HBrO}_{2} \right]^{2}$$

$$r_{5} = k_{5} \left[ \mathrm{Ce}^{4+} \right] \left[ \mathrm{MA} \right]$$

 $\begin{array}{l} \mathrm{BrO}_3^- + \mathrm{Br}^- \to \mathrm{HBrO}_2 + \mathrm{HBrO}\\ \mathrm{HBrO}_2 + \mathrm{Br}^- \to 2 \ \mathrm{HBrO}\\ \mathrm{BrO}_3^- + \mathrm{HBrO}_2 \to 2 \ \mathrm{HBrO}_2 + 2 \ \mathrm{Ce}^{4+}\\ 2 \ \mathrm{HBrO}_2 \to \mathrm{BrO}_3^- + \mathrm{HBrO}\\ \mathrm{CH}_2(\mathrm{COOH})_2 + \mathrm{Ce}^{4+} \to f/2 \ \mathrm{Br}^- + \mathrm{prodotti} \end{array}$ 

#### Kinetic scheme: Oregonator

$$\begin{array}{l} \operatorname{BrO}_3^- + \operatorname{Br}^- \to \operatorname{HBrO}_2 + \operatorname{HBrO}\\ \operatorname{HBrO}_2 + \operatorname{Br}^- \to 2 \ \operatorname{HBrO}\\ \operatorname{BrO}_3^- + \operatorname{HBrO}_2 \to 2 \ \operatorname{HBrO}_2 + 2 \ \operatorname{Ce}^{4+}\\ 2 \ \operatorname{HBrO}_2 \to \operatorname{BrO}_3^- + \operatorname{HBrO}\\ \operatorname{CH}_2(\operatorname{COOH})_2 + \operatorname{Ce}^{4+} \to f/2 \ \operatorname{Br}^- + \operatorname{prodotti} \end{array}$$

 $= k_1 [H^+]^2 [BrO_3^-] [Br^-] - k_2 [H^+] [HBrO_2] [Br^-] +$  $+k_3[H^+][BrO_3^-][HBrO_2] - 2k_4[HBrO_2]^2$  $+hk_5[CH_2(COOH)_2][Ce^{4+}]$ 

 $d[\mathrm{HBrO}_2]$ dt  $d[{
m Br}^-]$  $= -k_1[H^+]^2[BrO_3^-][Br^-] - k_2[H^+][HBrO_2][Br^-] +$ dt  $d[Ce^{4+}]$  $= 2k_3[H^+][BrO_3^-][HBrO_2] - k_5[CH_2(COOH)_2][Ce^{4+}]$ 

#### Kinetic scheme: Oregonator

 $A=\operatorname{BrO}_3^-$  ;  $B=\operatorname{CH}_2(\operatorname{COOH})_2$  ;  $P=\operatorname{HBrO}$  ;  $X=\operatorname{HBrO}_2$  ;  $Y=\operatorname{Br}^-$  ;  $Z=\operatorname{Ce}^{4+}$  .

(01)		A	+	Y	$\rightarrow$	Х+	P
	· .						

(O2)	Χ	÷	Y	$\rightarrow$	2 P	

- $(O3) \qquad A + X \rightarrow 2 X + 2 Z$
- $(O4) \qquad 2 X \rightarrow A + P$

(O5)  $B + Z \rightarrow h Y + prodotti$ 

# Oregonator (dimensionless, pool chemical approximation)

$$\varepsilon \frac{dx}{d\tau} = qay - xy + ax - x^{2}$$
$$\delta \frac{dy}{d\tau} = -qay - xy + fbz$$
$$\frac{dz}{d\tau} = ax - bz$$

$$\varepsilon = 0.12$$
  $\delta = 0.0006$   $q = 0.0008$   
 $f = \text{varying parameter} \approx 1$   $a, b = \text{constant}$ 

#### **Reduced Oregonator**



#### **Reduced Oregonator**

