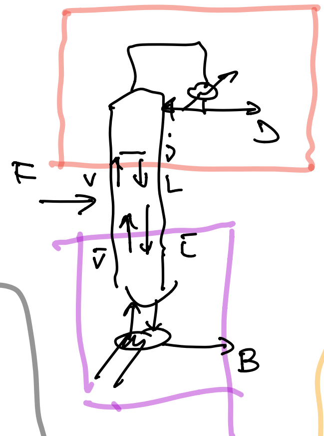
Multicomponent distillation : C components $i = 1, \dots, C$

Constant relative volatility :
$$y_i = \frac{\alpha_i x_i}{\sum_{j=1}^{C} \alpha_j x_j}$$
non-azeotropic model

Treatment of non ideal multicomponent distillation is complex
- detailed simulations using Aspen Plus
- batch distillation → Residue Curve Maps
- distillation ← short-cut methods

$R = \frac{L}{D}$



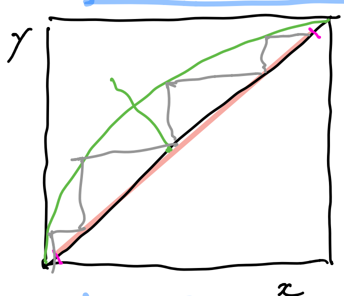
$V = L + D$

$$\frac{V}{L} = 1 + \frac{D}{L} = 1 + \frac{1}{R}$$

$$V y_{i,j+1} = L x_{i,j} + D x_{i,D}$$

$$y_{i,j} = \alpha_i K_2 x_{i,j}$$
$$K_2 = \left(\sum_{j=1}^{C} \alpha_j x_j \right)^{-1}$$

Fenske : $R \to \infty$



$V = L$

$\Rightarrow D = B = 0 = F$

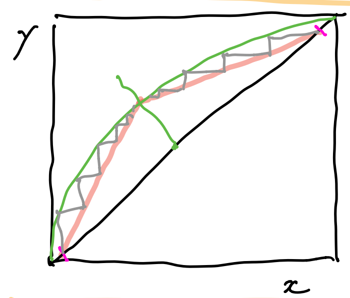
$$y_{i,j+1} = x_{i,j}$$
component material bal.

$$\alpha_i K_2 x_{i,j+1} = x_{i,j}$$

A, B : two generic species

$$\alpha_i = \frac{K_i}{K_2} \Rightarrow \alpha_{AB} = \frac{K_A}{K_B} = \frac{\alpha_A}{\alpha_B}$$

Underwood : $R = R_{min}$ $(N \to \infty)$



$$y_{i,j+1} = y_{i,j} = \dots = y_i$$

$$V y_{i,j+1} = V y_{i,j} = L x_{i,j} + D x_{i,D}$$

$$V y_i - \frac{L}{\alpha_i K_2} y_i = D x_{i,D}$$
component material bal.

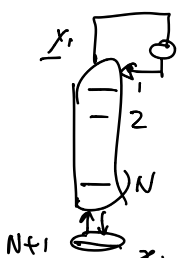
$$\frac{V y_i}{\alpha_i} \left(\alpha_i - \frac{L}{V K_2} \right) = D x_{i,D}$$

$$K_i = \frac{p_i(T)\,\gamma_i(T, P, \underline{x})}{P}$$

A / B

$$\frac{\alpha_A K_2\, x_{A,j+1}}{\alpha_B K_2\, x_{B,j+1}} = \frac{x_{A,j}}{x_{B,j}}$$

$$\alpha_{AB} \frac{x_{A,j+1}}{x_{B,j+1}} = \frac{x_{A,j}}{x_{B,j}}$$



$$\underline{x}_0 = \underline{x}_D$$

$$\underline{x}_{N+1} = \underline{x}_B$$

$$\alpha_{AB} \frac{x_{A,j}}{x_{B,j}} = \frac{x_{A,j-1}}{x_{B,j-1}}$$

$$\alpha_{AB}^2 \frac{x_{A,j+1}}{x_{B,j+1}} = \frac{x_{A,j-1}}{x_{B,j-1}}$$

⋮

$$\alpha_{AB}^{N_{min}+1} \frac{x_{A,N+1}}{x_{B,N+1}} = \frac{x_{A,0}}{x_{B,0}}$$

⋮

$$\frac{Fz_B}{Fz_A}\,\alpha_{AB}^{N_{min}+1} \frac{B}{B}\frac{x_{A,N+1}}{x_{B,N+1}} = \frac{x_{A,0}}{x_{B\,0}}\,\frac{D}{D} \quad \frac{Fz_B}{Fz_A}$$

$$\left(V y_i = \frac{D\,\alpha_i x_{i,D}}{\alpha_i - \phi} \right) \sum_{i=1}^{C}$$

rectifying

$$V_{min} = \sum_{i=1}^{C} \frac{\alpha_i (D x_{i,D})}{\alpha_i - \phi}$$

2nd Underwood equation

$$-\bar{V}_{min} = \sum_{i=1}^{C} \frac{\alpha_i (B x_{i,B})}{\alpha_i - \bar{\phi}}$$

$$\bar{\phi} = \frac{\bar{L}}{\bar{V} K_2}$$

stripping

$$\Rightarrow \quad \phi = \bar{\phi}$$

$$V_{min} - \bar{V}_{min} = \sum_{i=1}^{C} \frac{\alpha_i (D x_{i,D} + B x_{i,B})}{\alpha_i - \phi}$$

$$(1-q)F \qquad Fz_i$$

$$1 - q = \sum_{i=1}^{C} \frac{\alpha_i z_i}{\alpha_i - \phi}$$

1st Underwood equation

Underwood

$$\alpha_{AB}^{N_{min}+1} \frac{FR_A^B}{FR_B^B} = \frac{FR_A^D}{FR_B^D}$$

(I) $A = LK$ $B = HK$

$1 = FR_A^D + FR_A^B$ $FR_B^B + FR_B^D = 1$

$$\alpha_{LK,HK}^{N_{min}+1} = \frac{FR_{LK}^D}{1-FR_{LK}^D} \; \frac{FR_{HK}^B}{1-FR_{HK}^B}$$

$$N_{min}+1 = \frac{1}{\ln \alpha_{LK,HK}} \ln\left(\frac{FR_{LK}^D}{1-FR_{LK}^D} \; \frac{FR_{HK}^B}{1-FR_{HK}^B}\right)$$

(II) $A = i$ $B = HK$

$$\alpha_{i,HK}^{N_{min}+1} = \frac{FR_i^D}{1-FR_i^D} \; \frac{FR_{HK}^B}{1-FR_{HK}^B}$$

→ FR_i^D $i = LNK, HNK$

distribution ← i = intermediate

Fenske

Underwood

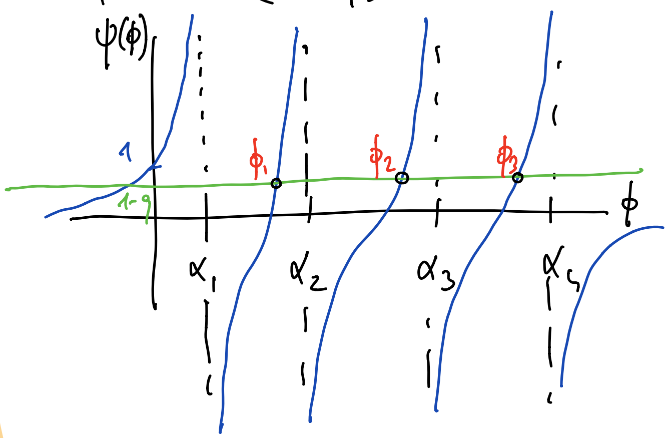
$$1-q = \sum_{i=1}^{C} \frac{\alpha_i z_i}{\alpha_i - \phi} = \psi(\phi)$$

1st Underwood equation

$$V_{min} = \sum_{i=1}^{C} \frac{\alpha_i (D x_{i,D})}{\alpha_i - \phi}$$

2nd Underwood equation

$$\frac{d\psi}{d\phi} = \sum_{i=1}^{C} \frac{\alpha_i z_i}{(\alpha_i - \phi)^2} > 0$$



$$\alpha_{HK} < \phi < \alpha_{LK}$$

(I) sharp split $HK+1 = LK$

$$\alpha_2 = \alpha_{HK} < \phi_2 < \alpha_{LK} = \alpha_{HK+1} = \alpha_3$$

$$D x_{i,D} = FR_i^D F z_i$$

ϕ_2: only one solution of 1st U eq.

one unknown: V_{min}

(II) non-sharp split $HK+1 < LK$, there are intermediate species!

e.g. one intermediate species

$HK = 1$ $LK = 3$ $I = 2$ $\Rightarrow \alpha_1 < \phi_1, \phi_2 < \alpha_3$

in 2nd U eq. there are two unknowns: V_{min}, $D x_I^D = D x_{2,D}$

$$\begin{cases} V_{min} = \dfrac{\alpha_1 (D x_1^D)}{\alpha_1 - \phi_1} + \dfrac{\alpha_2 (D x_2^D)}{\alpha_2 - \phi_1} + \dfrac{\alpha_3 (D x_3^D)}{\alpha_3 - \phi_1} + \dfrac{\alpha_4 (D x_4^D)}{\alpha_4 - \phi_1} \\ V_{min} = \dfrac{\alpha_1 (D x_1^D)}{\alpha_1 - \phi_2} + \dfrac{\alpha_2 (D x_2^D)}{\alpha_2 - \phi_2} + \dfrac{\alpha_3 (D x_3^D)}{\alpha_3 - \phi_2} + \dfrac{\alpha_4 (D x_4^D)}{\alpha_4 - \phi_2} \end{cases}$$