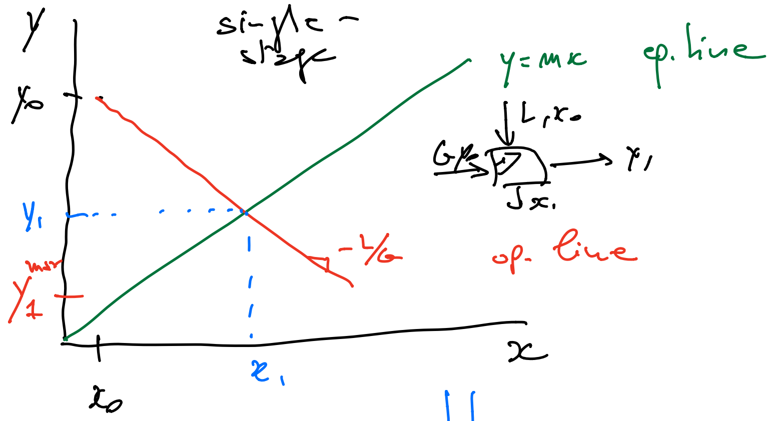


Data

G [mol/s]

y_0 [-]

x_0 [-]

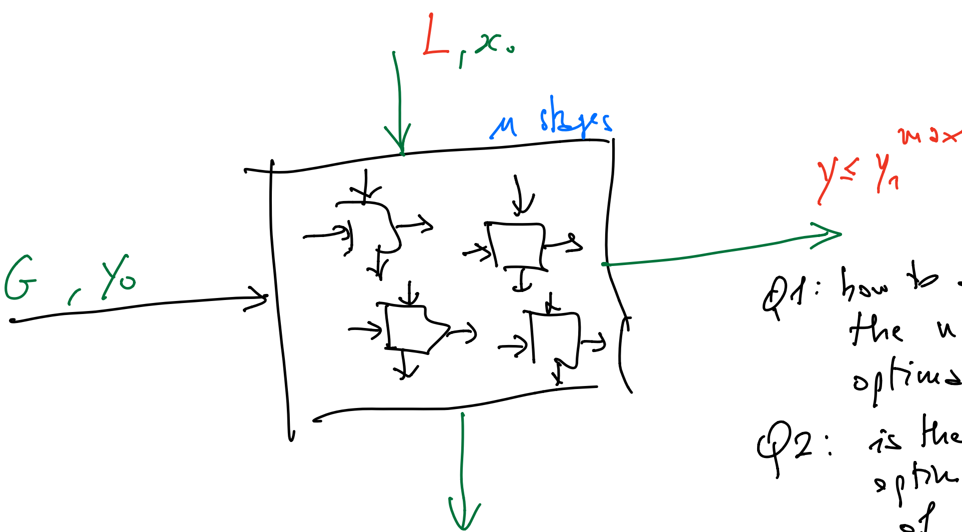
Constraints

y_1^{max} max value

$\boxed{L \text{ fixed}}$

FAILED !!

let's use $n > 1$ stages

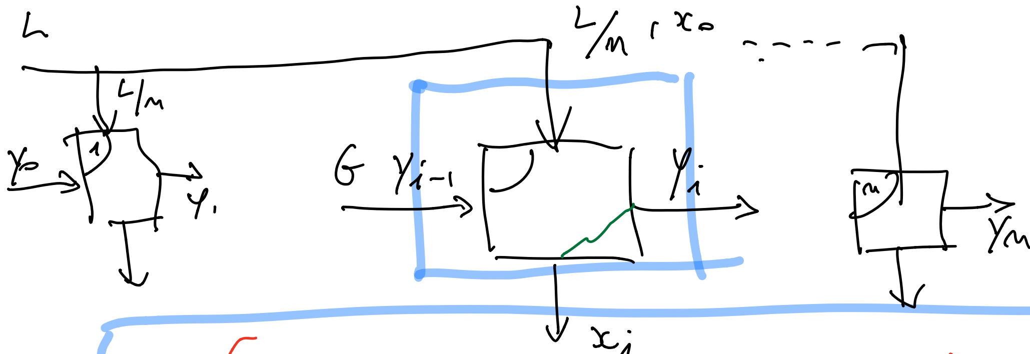


$y = m x$ is known @ T, P

Q1: how to arrange the n stages, optimally?

Q2: is there an optimal value of n?

Q3: can you justify your answers quantitatively?



$$\frac{A}{m} = \frac{L}{mGm}$$

$$y_0^* = m x_0$$

$$\cancel{G} y_{i-1} + \frac{L}{mG} x_0 \frac{m}{m} = \cancel{G} y_i + \frac{L}{Gm} x_i = \cancel{G} y_i + \frac{L}{m} \frac{y_i}{mG}$$

$$y_i = m x_i$$

$$\boxed{y_{i-1} + \frac{A}{m} y_0^* = y_i \left(1 + \frac{A}{m}\right)}$$

$$y_i = \frac{y_{i-1} + \frac{A}{m} y_0^*}{1 + A/m}$$

Background for the lecture on Thursday March 18^{th}, 2021

First order difference equation

Unknown: $y \in \mathbb{R}$
Independent variable: $i \in \mathbb{N}$

y_{i+1} depends on y_i, i.e.
$y_{i+1} = f(y_i, i)$

Linear case:

$\boxed{y_{i+1} = \gamma y_i + \delta}$, γ and δ are constant, with $\boxed{\gamma \neq 1}$ and $\boxed{y_0 = \bar{y}}$ given

(I) homogeneous sub-case: $y_{i+1}^H = \gamma y_i^H$, at every iteration y is multiplied by γ

$y_i^H = \beta \gamma^i$

(II) constant sub-case: $y_i^P = K$ constant

$K = \gamma K + \delta$ $\qquad K = \dfrac{\delta}{1-\gamma}$ (that's why $\gamma = 1$ is forbidden)

General solution:

$$y_i = y_i^H + K \Rightarrow y_i = \beta\gamma^i + \frac{\delta}{1-\gamma}$$

$$y_0 = \beta + \frac{\delta}{1-\gamma} = \bar{y} \Rightarrow \beta = \bar{y} - \frac{\delta}{1-\gamma}$$

$$\Rightarrow \boxed{y_i = \left(\bar{y} - \frac{\delta}{1-\gamma}\right)\gamma^i + \frac{\delta}{1-\gamma} = \bar{y}\gamma^i + \frac{\delta}{1-\gamma}\left(1-\gamma^i\right)}$$

This general solution can be applied to any specific case.

If $\boxed{\gamma = 1}$, then $\boxed{y_{i+1} = y_i + \delta} \Rightarrow \boxed{y_i = \beta + i\delta}$

Second order difference equation

Same as above, but now

$$y_{j+1} = f(y_j, y_{j-1})$$

linear case

$$a\, y_{j+1} + b\, y_j + c\, y_{j-1} = 0$$

Hypothesis : $y_j = r^j$

Substitute $\left(a r^2 + b r + c\right) r^{j-1} = 0$

$$r_{1,2} = \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right)$$

if $b^2 - 4ac > 0$ $\quad r_1 \neq r_2$ and $r_1, r_2 \in \mathbb{R}$

$\Rightarrow \quad y_j = \alpha r_1^j + \beta r_2^j$ with α and β determined through two conditions

Please study these two pages before Thursday, so as we can use this general result!! Thank you