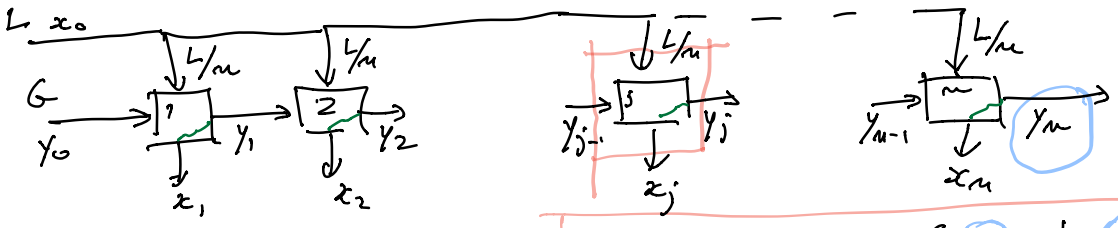
CROSS-CURRENT CASCADE of Eq. stages



generic stage j

mat. bal. eq.

$$\begin{cases} G\,y_{j-1} + \frac{L}{M}\,x_0 = G\,y_j + \frac{L}{M}\,x_j \\ y_j = m\,x_j \end{cases}$$

$$A = \frac{L}{Gm} = \frac{L/G}{m}$$

$y_0^* = m\,x_0$

$y_0 = y_0$!

$$y_j = y_{j-1}\,\underbrace{\frac{1}{1+A/M}}_{\gamma<1} + y_0^*\,\underbrace{\frac{A/M}{1+A/M}}_{\delta}$$

$$y_j = y_0\,\frac{1}{(1+A/M)^j} + y_0^*\left(1 - \frac{1}{(1+A/M)^j}\right) =$$

$$= y_0^* + (y_0 - y_0^*)\,\frac{1}{(1+A/M)^j}$$

$j = M$

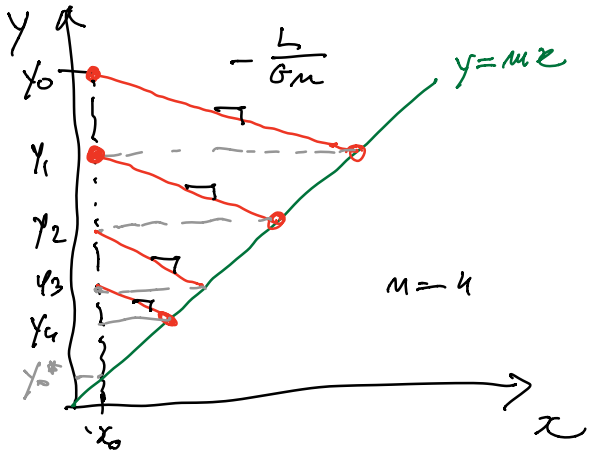
monotonically increasing

$$\alpha = \frac{\text{amount absorbed}}{\text{max amount absorbable}} = \frac{y_0 - y_M}{y_0 - y_0^*} = 1 - \frac{1}{(1+A/M)^M}$$

$M \to \infty$

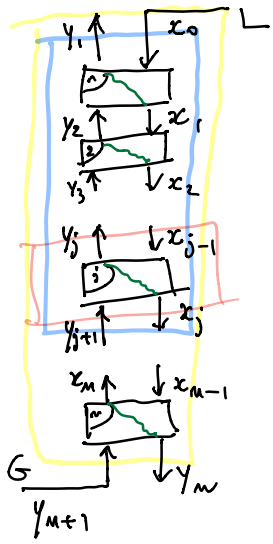
$1 - e^{-A}$

fraction of absorption = absorption efficiency



$$y_j = -\frac{L}{GM}\,x_j + \left(y_{j-1} + \frac{L}{GM}\,x_0\right)$$

COUNTER-CURRENT CASCADE of Eq. STAGES



Specs y_1^{m+1} **Data** G, y_{m+1}, x_0

Mat. balance Eq.

$$G y_{j+1} + L x_{j-1} = G y_j + L x_j$$

$$y_j = m x_j$$

$$y_{j+1} + A y_{j-1} = y_j (1+A) \qquad A = \frac{L}{Gm}$$

2° order D.E.

Mat. bal. Eq.

$$G y_{j+1} + L x_0 = G y_1 + L x_j$$

$$y_j = m x_j$$

graphical representation

$$y_{j=0} = y_0^*$$

$j = 0, m$

$$y_{j+1} = A y_j + (y_1 - A y_0^*)$$

1° order D.E.

overall mat. bal.

$$G y_{m+1} + L x_0 = G y_1 + L x_m$$

$$y_m = m x_m$$

$r \geq 1$ (8)

$$y_j = y_0^* A^j + (y_1 - A y_0^*) \frac{1 - A^j}{1 - A}$$

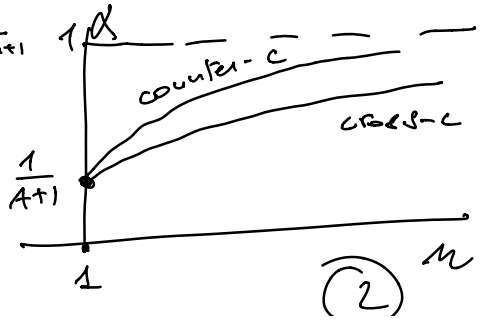
$j = m+1$

$$y_{m+1} = y_0^* A^{m+1} + (y_1 - A y_0^*) \frac{1 - A^{m+1}}{1 - A}$$

$$\alpha = \frac{y_{m+1} - y_1}{y_{m+1} - y_0^*} = \frac{A^{m+1} - A^{1-1}}{A^{m+1} - 1} = 1 - \frac{1-A}{1-A^{m+1}}$$

$$\lim_{m \to \infty} \alpha = \begin{cases} 1 & A > 1 \\ A & A < 1 \end{cases}$$

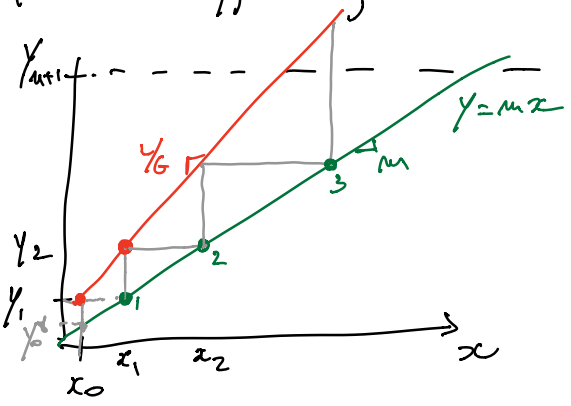
$A = 1 \quad \alpha = 1 - 1/(m+1)$



(2)

mat. bal. $y_j = \frac{L}{G} x_j + \left(y_1 - \frac{L}{G} x_0\right)$ $\Bigg\}$ $y = \frac{L}{G} x + \left(y_1 - \frac{L}{G} x_0\right)$

eq. $y_j = m x_j$ $\quad y = mx$



$m=3$ is enough to close the right pas. !!

What is the real value of y_1 for this L/G ?

$A = \frac{L/G}{m}$ known

$\Rightarrow \quad \alpha = 1 - \frac{1-A}{1-A^{N+1}}$

$\Rightarrow \quad y_1 = y_{N+1} - \alpha \left(y_{N+1} - y_0^*\right) < y_1^{max}$

How to use these results:

- verify how an absorber works
- design an absorber

optimal value of L ?